THE THERMODYNAMICS OF
HEAT–ENGINES

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OF HEAT-ENGINES

BY

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To My Wife

TO WHOSE DEVOTION AND AID

(ALTHOUGH SHE DOESN'T KNOW ENTROPY FROM CARBONIC ACID)

THE EXISTENCE OF THIS BOOK IS DUE

IT IS DEDICATED

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NOTATION

C = the density-constant for a permanent gas, or the number of foot-pounds of external work done during the heating of one pound of the gas one degree Fahrenheit under constant pressure. For air, $C = 53.18$.

C' = the specific density-constant involved in a case concerning a mass of gas other than one pound.

D = the disgregation-energy of dry saturated steam.

E = quantity of energy.

F = efficiency.

H = the total heat of dry saturated steam above that of melted ice.

I = internal energy of a gas or vapor.

J = the mechanical equivalent of heat = 778.

K = the ratio of the specific heats of a gas under constant pressure and constant volume respectively.

L = the latent heat of vaporization of dry saturated steam.

l = the isomorphic heat of water above that of melted ice.

M = mass.

N = entropy.

P = pressure, in pounds per square inch unless otherwise stated.

Q = quantity of heat.

Q_1 = the heat supplied at the upper temperature-level during a cycle.

Q_2 = the heat rejected at the lower temperature-level of a cycle.

Q_w = the heat available for doing work = $Q_1 - Q_2$.

q_w = the heat actually converted into work during a cycle.

R = volume-ratio of gaseous expansion.

S = specific heat.

T = absolute temperature Fahrenheit.

T_1 = the upper absolute temperature of a pair of temperature-limits.

T_2 = the lower absolute temperature of a pair of temperature-limits.

t = temperature Fahrenheit.

V = volume, in cubic feet unless otherwise stated.

W = weight, in pounds avoirdupois unless otherwise stated.

X = the external work involved in the vaporization of dry saturated steam.

x = the unknown or indefinite exponent of the volume-ratio (in the expansion of a gas) in the equation connecting it with the pressure-ratio.

y = the unknown fractional proportion of steam to total mass in a steam-and-water mixture.

PART I

THEORY

THE THERMODYNAMICS OF HEAT-ENGINES

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CHAPTER I

THE GENERAL PRINCIPLES OF ENERGETICS

THE term *thermodynamics* is derived from two Greek words: $\theta\epsilon\rho\mu\acute{o}s$, meaning "heat," and $\delta\acute{v}\nu a\mu\iota s$, meaning "force." It refers to the study of the relations between heat and mechanical energy.

There is scarcely a science so widespread in its applications as thermodynamics. To the student of modern science the source of all things is heat. Wind and weather, all vegetation, all life, take their origin and find their strength in heat. Not only man and all of the higher forms of animal life, but the lowest and simplest of vegetable types as well, depend upon heat for their existence and absorb it as constantly as their food. This heat-energy it is the continuous duty of their lives to convert into mechanical motion, or work. In inanimate nature the same is true. The vigor of the storm, the irresistible motion of the glacier, the erosion of mountains and the filling of valley or lake; the frosted window-pane, the sculptured iceberg, the mirage of the desert, the singing bullet, and the quiet beneficent storage of water in pond and reservoir;

the gun, the rocket, the steam-engine and the gas-engine, —all are manifestations of heat and its transformations into motion, all are more or less thermodynamic in character. For the study of each a comprehension of the principles of thermodynamics is necessary. Each is an illustration of the working of those principles and of the laws which limit them.

The two quantities which form the stock-in-trade of all heat-engines—the raw material and the manufactured product, so to speak—are *heat* and *mechanical energy*. Both of these quantities belong under a common classification title as being both of them *forms of energy*.

According to the modern scientific analysis of natural phenomena, the entire universe is made up of two broad subdivisions of existence—*matter* and *energy*. The conception of matter is familiar to all. It apparently appeals directly to our senses at every turn. It apparently, too, appears in a myriad of different forms. The study of chemistry, however, teaches that in all of these infinitely varying forms there are really to be found only some eighty or ninety distinct kinds, or *elements*, of matter, such as carbon, hydrogen, calcium, etc. Physics, again, tells us that each of these elements, and each of their innumerable chemical combinations, exists in four distinct conditions: solid, liquid, gaseous, and radiant forms of matter.

The fundamental law which underlies both chemistry and physics is that of *the Conservation of Matter*. This law is the statement of the fact that, no matter what physical alterations matter may undergo, whether from solid to liquid or from liquid to gaseous, or the reverse, and no matter what chemical transformation may occur, whether the substance be burnt as fuel in a furnace, decomposed as lime in a kiln, or corroded with acids, throughout every

conceivable change in its form and characteristics the *quantity* of matter in the universe remains always the same. Man may change its locality at will. Working in harmony with the laws which limit all natural action, he can transmute its outward appearance in a myriad of different ways, to suit almost every imaginable whim or purpose. But to alter in the slightest degree the existing quantity of matter, to create one particle, or to destroy a single atom, he has always been and always will be impotent. The universe is eternal. In the face of its steadfast continuity man's momentary existence and evanescent will are as cloud-wreaths against a mountain-side.

The other of the two broad subdivisions of all things natural includes the various forms of energy. These forms of energy are distinguished by external characteristics upon which is based a proper classification. But this classification is not parallel with that of either chemistry or physics. Whereas, so far as the science of chemistry has yet progressed with certainty, the various chemical elements can never be confused with one another, the several forms of energy are separated by lines often quite indistinct to our limited discernment. Whereas one chemical element can never be altered into any other, the various forms of energy are each of them capable of being transformed, under favorable conditions, into any one of the other forms. Nevertheless, under and through all study of any of the forms of energy, or of their mutual transformations, runs one fundamental and immutable law, that of *the Conservation of Energy*, quite parallel to the law of the conservation of matter. It states that, no matter what transformations may occur to any form of energy, the quantity of energy in the universe after the change is quite same as it was before. These transformations may greatly alter its appearance to our senses, or its effect upon other things ;

yet the rule remains unbroken. No energy has been destroyed and none created. Energy, the consort of matter in that union of all things which we call Nature, is eternal, is constant, as is matter.

These two laws, of the conservation of matter and of energy, together constitute the foundation of modern engineering science. It was ignorance of the law of the conservation of matter which led the alchemists of old on their long, costly, hopeless search for the "philosopher's stone," by mere touch of which all matter might be transmuted into gold. It is ignorance of the law of the conservation of energy which has more recently led to the strenuous, fascinating, but hopeless endeavor for the production of "perpetual motion," or mechanical energy produced without the aid of any anterior supply of energy. Volumes of history could be crowded with the list of the lives ruined or lost in the following of these two dreams, and no more forcible illustration could be adduced to prove the indispensable need of every engineer for a complete and accurate grasp of these two laws and of their absolute rigidity.

What is meant by the words *energy* and *transformation?*
Energy is defined as "the ability to overcome resistance." All natural phenomena consist, in one way or another, of an overcoming of resistance; therefore all phenomena known to man are manifestations, in one way or another, of energy. It is a quality inherent in all matter, to some extent.

Yet, so far as we now know, any or all of the energy inherent in any body is quite distinct and entirely detachable from that body. Although human experience has never covered the case of a bit of matter possessing no energy, yet we always conceive of such a case as quite

possible, because some portion of the energy of a mass of matter is always removable from it; nor is there any definite limit to the amount removable. Thus, a weight lifted above the earth possesses energy, for it is able to overcome resistance in falling again toward the earth. Yet the energy resident in the body of the weight is quite distinct from the matter composing the body; for if the weight be removed to a locality where it no longer possesses weight, say to the centre of the earth, the mass of the body will remain unchanged, while the energy has entirely disappeared. Similarly, a moving projectile possesses the ability to overcome resistance, and therefore contains energy; but when it has been brought to rest, if not too suddenly, the matter will remain unchanged, while the energy is gone.

The overcoming of resistance involves two distinct factors: (1) the resistance, and (2) the extent to which it is overcome. The first is most simply defined in terms of *force*, a thing so familiar to us all that no further analysis or definition of it need here be attempted. It does need a unit of measurement, however, and for this will be exclusively adopted, in the following pages, the *pound* avoirdupois. Secondly, the extent or distance through which the overcoming of resistance is effective is another matter of familiar experience and needs no definition. The unit for its measurement which will be used hereafter is the English *foot*.

It is next to be noted that the overcoming of resistance involves the expenditure of energy. The distance traversed by the acting force in the direction of its exertion is the same as that through which the resistance is overcome in a direction opposite to its exertion. This is the modern equivalent to saying that "action and reaction are equal and in the opposite direction." The driver must lose as

much energy as the driven gains, and the driven must gain as much as the driver loses. The distinction between loss and gain is an artificial one and depends solely upon the point of view from which they are regarded; they are, therefore, properly distinguished by positive and negative signs, it being usual to consider the acting force as positive and the absorbing resistance as negative.

From the foregoing considerations is reached the fundamental idea that the *unit of quantity of energy* is that manifested when a force of one pound overcomes a resistance of one pound through a distance of one foot, during which process the acting force has expended and the resistance has absorbed one *foot-pound* of energy.

Work. — As this particular form of energy has its individual name, in the classification of different forms of energy, and is known as *mechanical energy*, or *work*, the foot-pound is also the unit for measuring quantities of work, the word *work* being understood in its mechanical sense. Further, as mechanical energy has always been the most familiar of all of the different forms of energy and the one with which acquaintance is first made, it has become the custom to base the units of the other forms of energy used by the engineer upon the foot-pound, and to define them in terms of it as the fundamental unit.

Power. — Closely associated with the idea of mechanical energy is that of *power*. Power is defined as the *rate* at which work is done. It therefore involves, besides the factors of force and distance which are involved in the idea of work, the additional factor of *time*. The unit of power is the *horse-power*, which is defined as that rate of doing work which will accomplish 33,000 foot-pounds of work if steadily continued for the space of one minute. This same rate would accomplish 550 foot-pounds of work in one second; so the latter is often given as a

definition of the horse-power. In metrical units the horse-power is defined as the rate which would accomplish seventy-five kilogram-meters of work if continued for one second.

It is obvious that power, or the *rate* at which work is done, has nothing to do with the *quantity* of work accomplished. A very slow rate may accomplish a great deal of work, if it only keeps at it long enough, while a very high rate may be represented by a very small quantity of work, provided it be expended in a sufficiently short space of time. Thus, a pump driven by a motor having a power of much less than one horse-power, by working steadily, day and night, for weeks and months, may lift millions of tons of water and thereby perform tens of millions of foot-pounds of work; yet its power is none the less a fraction of a horse-power. Or, on the other hand, a rifle, accomplishing, in imparting motion to the bullet, no more work than does a boy in throwing a heavy stone, by performing the task in an exceedingly short period of time actually does exert a power, or a *rate* of performance, of something like 5000 horse-power.

The Forms of Energy. — Energy, the eternal, manifests itself to the human senses in several distinct aspects, or forms. Familiar instances are those of mechanical motion, light, sound, electricity, heat, etc. In fact, there is no phenomenon under the canopy of heaven which may be defined as constituting a *process*, as being aught else than a dead static *fact*, or condition, which is not a manifestation of energy, in some form or another. The motion of the sun and its planets, the rise of the tide, the shower of rain, the flash of lightning and the peal of thunder, the growth of vegetation and the most intricate processes of animal life, are all to be classed, in their broadest sense,

as manifestations of energy; and all of them must live within its laws.

This myriad of different forms of energy may be subdivided into groups or classes in a variety of ways, each based upon some line of distinguishing characteristics. Of these the broadest classification is that into three main groups, viz.: *Kinetic, potential,* and *vibratory* forms of energy.

The term *kinetic,* from the Greek κινητός, "motion," always implies the presence of motion. Hence, kinetic energy may be defined as that involved in the motion of a mass. It is one of the primary conceptions of mechanics that energy is always required in order to impart motion or change of motion to mass; and, obversely, that mass can never part with its motion except in the course of overcoming resistance. As motion without or separate from mass is a purely geometric conception, finding no counterpart in human experience, it will be sufficient, hereafter, to refer to kinetic energy simply as consisting of *motion.* The most familiar instances of kinetic energy are those of a moving projectile, of a hammer, or of a fly-wheel.

The term *potential,* from the Latin *potior,* "to be able," always implies latent ability. It is, however, the distinguishing characteristic of potential energy that it has always the appearance of having been acccumulated by previous motion, or kinetic energy. It will therefore be most clearly defined to the mind as *stored motion.* This fact may be easily fixed in the mind by association of the words *potential* and *position ;* for potential energy is always involved in mere position, attained as the result of motion, as contrasted with kinetic energy or the motion which gave it birth.

To illustrate, the most familiar instance of potential energy is that of a weight raised from the earth. By rea-

son of its separation from the earth it is able to overcome
resistance in again falling into contact with the earth.
The ability to do work is there, latent, unasserted; yet it
is none the less powerful.   Thus, imagine a six-inch rifle
aimed vertically into the air and fired.   The projectile, as
it leaves the muzzle, possesses an enormous supply of
energy which it has received from the explosion of the
powder.   As it arises, this kinetic energy is gradually and
constantly being expended in overcoming the force, or
resistance, of gravitation.   The projectile must finally reach
an elevation at which, its supply of kinetic energy having
been all expended, it will have come to rest.

Let us suppose that a scaffolding had been built up
from the earth to this point and that a man standing upon
it should catch the projectile, at the instant of its stop, and
hold it there.   The projectile must contain at this instant,
if we neglect the friction of the atmosphere, the entire
amount of energy with which it left the gun; for if it be
released, it will exert, in its fall, exactly that amount and
will reach the earth with as high and destructive a velocity
as it had at the start.   But at the start this energy would
have blown the man to atoms had he been so unfortunate
as to come into contact with the projectile.   At the top he
may handle the deadly thing with entire impunity.

What is the difference?   It is not easily to be explained,
but a broad fact it certainly is, one which may not yet be
stated as an absolute law but which is certainly more than
a widespread coincidence, that the human organism is
extremely sensitive to kinetic forms of energy but takes
almost no cognizance, except through the medium of de-
ductive reasoning, of potential forms.   The distinction is
important, even in the elementary study of engineering
problems, because there is no one source of engineering
failure, or of commercial, economic, and political failure,

for that matter, so common as a lack of proper estimate of the possibilities concealed within the latent, invisible, potential forms of energy.

Of the third group of energy-forms, the *vibratory* energies, little may be said here, in spite of the fact that they are the most widespread and familiar of all the forms. Vibratory energy consists of periodic alternations between kinetic and potential forms of the same kind. The simplest instance is that of the pendulum. Here, supposing the pendulum to be already in swing, is visible at the centre of its arc the kinetic energy of the mass of the pendulum-bob. This carries it up the one side of its arc, the kinetic energy being gradually transformed into the potential form of separation from the earth, until the extreme height of swing is reached. Here the transformation is complete and the energy all potential in form. But it does not remain so a single instant. The bob has no sooner come to rest than it starts upon its return-swing, reconverting its potential energy into motion until the lowest point of the arc be reached, when the process is repeated up the other side.

Here the alternations between the two forms is slow enough, usually, so that the eye, or the ear, as in the case of a clock, might easily distinguish the separate beats and count them. But if we substitute for the force of gravity the elasticity of a spring, and make the pendulum quite short, the rapidity of the beats may easily elude the eye. The apparatus becomes blurred in appearance, and we know the rapidity of vibration only as a degree of smoothness of blur. If the vibrations be well above sixteen to the second, the ear can no longer count the separate sounds emitted by the pendulum in passing the centre; they become blurred into a continuous sound, or note, and the rapidity of vibration is known only as *pitch*. Yet each

vibration is none the less a complete cycle of alternate transformations of kinetic into potential energy, of potential back into kinetic.

If we substitute for the gravity-pendulum an absolutely fluid mass, to which water is a close simile, we shall find that each particle of water near the surface is easily set moving in a vertical circular orbit which will carry it alternately above and below the mean level of static equilibrium. Its extremes above and below, the crests and troughs respectively of the water-wave, correspond to the extremes of the pendulum's swing, its energy there being all potential in form, either positive or negative. Its alternations between the two constitute a new form of vibratory energy. Here, however, we find a strange and unwavering tendency to transmit the energy from one particle to another, and that almost without loss, so that the form of the vibration, or wave, as it is called, progresses continuously from one locality of the surface of the fluid mass to another. This is a faculty possessed not at all, or very slightly, by the vibratory motions of single solid bodies, such as the pendulum. It is a property of all fluids, however, and of all elastic, continuous solids, so far as transmission within their own confines is concerned. In air, which is elastic as well as fluid, these (or very similar) waves are called sound.

If we substitute for these well-known substances the less familiar ones of the luminiferous ether and the electric atom, the vibrations will take the form of light or electricity. If it be light, the number of vibrations, always numbering in the hundreds of millions per second, is known to human sense only as color. But there are many series of vibrations in this medium which the eye cannot count, just as there are sounds too high in pitch for the ear to hear. Those too slow are known as radiant heat; those too high

are known as actinic rays, and can be perceived and counted only by the photographic plate.

There is another broad method of classification than this distinction between motion and stored motion. It is based upon the medium in which the energy finds expression, upon the particular kind or size of mass which happens to be in motion. Thus, we make a very easy and useful distinction between light and sound, though both are vibratory; or between the latent energy of gunpowder and that of a lifted tilt-hammer, though both be potential in form. Of the many different kinds of energy-forms thus arising the following discussion will make use of only the four (together with their distinctions between kinetic and potential) which are most commonly met with in the engineering of heat-engines, viz. : —

> **Mechanical**: *Kinetic* and *Potential;*
> **Electrical**: *Kinetic* and *Potential;*
> **Chemical**: *Potential;*
> **Thermal**: *Kinetic* and *Potential.*

Mechanical Energy is defined as that occurring in the motion or the stored motion, relatively to other similar bodies, of bodies of some appreciable size and mass. Such is the energy of weights, springs, bodies of water, etc.

Electrical Energy may be defined as that embodied in the relative motion or position of separation of the particles of some atomic medium, as yet not well defined or understood, but whose characteristics are so markedly distinctive from those of the other forms of energy that no more scientific definition is necessary here. The atoms embodying this sort of energy are known as *electrons*.

Chemical Energy is defined as that inherent in the relative stored motion or position of molecules of two or more

different chemical natures; that is, such as that between a molecule of carbon and one of oxygen, of hydrochloric acid and iron, etc.

Thermal Energy is defined as that inherent in the molecules of a body by virtue of their motion and position relatively to other molecules of the *same* chemical nature within the same body.

Concrete illustrations of these several forms may be drawn from daily experience, viz.: —

Mechanical, kinetic:	A hammer, a fly-wheel, or a projectile.
Mechanical, potential:	A lifted weight, a distorted steel spring, or the water of a mill-pond.
Chemical, potential:	The energy stored in fuel, in gunpowder, or in corrosive acids.
Electrical, kinetic:	An electric current.
Electrical, potential:	An electrical charge, in a Leyden jar or similar condenser.
Thermal, kinetic:	The sensible heat or temperature-heat of a body, as of red-hot iron.
Thermal, potential:	The latent heat involved in a change of physical state, as of the melting of ice into water or of the boiling of water into steam.

It is easily apparent that this list of the forms of energy would need to be expanded only to include magnetism, light, and sound to make it comprehend almost every possible phenomenon occurring on the face of the earth or in the surrounding universe.

One cannot ponder upon this absolute constancy in amount of energy, underlying the most complex alterations in its external appearance, without being finally impressed with the fact that the alteration *is* merely external and that the real essence of the energy continues to exist, unchanged and uninterrupted; and that what we regard as a change does not occur in the thing observed, but merely in its locality and in its relation to the observer. For all of these forms of energy consist of motion or of stored motion; and both motion and position are purely relative.

Without any need for an attempt at building upon this fact a system of philosophy, it is nevertheless necessary that the engineer, of all persons, should realize this fact to his finger-tips. Otherwise he will frequently be deceived or caught napping. No pantomime actress was ever quicker or more capricious in her lightning-changes of costume than is energy. It is anything and anywhere. At one moment heat in the hundred-thousand-mile flames of the sun's surface, ninety millions of miles away; the next moment light in a man's eye; the next sealed in his brain as an optic impression and memory which he carries to his grave. Now the cold, black inertness of the coal-seam in the bowels of the earth; soon the glare of the boiler-furnace; then the invisible energy of the silent steam; a moment after, the battle of a mighty ship with the fury of the gale, or the peaceful glow of an electric arc miles away. But the actress remains the same. No matter what the guise, on the stage or in the world the impersonator retains the individuality of a continuous existence; and in the latter case the impersonator is always *motion*, — the motion or the stored motion of mass.

In illustration of this truth instances of such a change of costume, from every different one to each of the others, can

easily be found, not only in the wide field of scientific experiment but in the much narrower one of the useful arts. They are all possible; they all occur in the everyday life of the universe; they are nearly all of them familiar to the average observer.   Most of them are turned to account in the production of wealth.   Thus : —

From Mechanical to Thermal :
> The heating of a body subjected to friction or impact.

From Mechanical to Electrical :
> The dynamo or the glass-plate electric machine.

From Mechanical to Chemical :
> The setting-off of a detonating compound, such as fulminate of mercury or nitroglycerine.[1]

From Electrical to Thermal :
> The heating of any non-conductor by the passage of an electrical current.

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[1] This is one of the most obscure and unfamiliar of energy-transformations. The energy revealed in the decomposition of an explosive, which finds expression in heat, light, sound, and violent mechanical movement, is the equivalent of the chemical energy originally stored in the explosive; the phenomenon should be classed as a transformation of chemical into one or more of these other forms of energy. But every explosive, like a row of blocks stood upon end, needs some initial impulse or supply of energy from without in order to set in motion the process of energy-transformation. With the majority of them, as with gunpowder, this preliminary supply of energy must take the form of heat, in the process of ignition. But in many others, such as those mentioned in the list, heat will not initiate the explosion; a sudden jar will alone suffice. In this sudden shock to the explosive a minute quantity of mechanical energy is absorbed in upsetting the equilibrium of structure of a few molecules; the consequent release of energy from the fall and crash of these few into their more stable components upsets the rest, and the explosion ensues. This preliminary microscopic transformation of mechanical energy into chemical energy is the only instance of its class which has yet occurred to the writer; but it is of considerable importance in the arts, for the detonation of the ordinary rifle-cartridge and of most heavy explosives depends upon it.

From Electrical to Chemical:

> The electrolysis of a chemical compound by the passage of current, as in electroplating, or in the charging of a storage-battery.

From Electrical to Mechanical:

> The electric motor; the solenoid.

From Chemical to Mechanical:

> The evolution of a gas from the combination of two solids or liquids.[1]   The chemical fire-engine.   Animal activity.

From Chemical to Electrical:

> The evolution of an electrical current from a primary or secondary battery.

From Chemical to Thermal:

> The combustion of fuel.   Animal heat.

From Thermal to Mechanical:

> The expansion of a heated body; the steam-engine.

From Thermal to Electrical: The electropile.[2]

From Thermal to Chemical:

> The lime-kiln; the growth of vegetation.

Not only the material arts, but the entire daily life-experience of man as well, swarms with a myriad of such

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[1] This transformation is a very common but, outside of the processes of animal life, not a very obvious one.   The mechanical work performed by the cold evolution of gas from a combination of solids or liquids is usually only that of displacing the atmosphere.   It is utilized in the arts, however, in the operation of chemical fire-engines and extinguishers.

[2] The electropile is a true representative of this class of transformations, but one very narrowly limited in its applications.   The invention of an apparatus which will perform this transformation upon a large scale, without the intermediate transformation into heat, as through the medium of a furnace, boiler, steam-engine, and dynamo, is one of the unsolved problems of the day which attracts inventors to its solution by the most unlimited promise of pecuniary reward.

phenomena, all of which consist of and reveal transforma-
tions of energy from one outward aspect to another. With
energy not only is frequent alteration of form possible, it is
apparently inevitable. Not only do all material phenomena,
but life itself — anything, in fact, which can be defined as
an event or occurrence — consists of a series of energy-
transformations following one another in ceaseless conti-
nuity. There is, so far as the human mind can grasp, no
possibility of a stop. With many forms of energy the
transformations follow one another with a rapidity which
the mind is entirely unable to comprehend; with others,
such as those visible in moving machines, they are more
deliberate and are easily traced; with others, such as those
occurring in the heavenly bodies, the changes are so slow
or infrequent that human experience is too brief to cover
and perceive them directly, though their occurrence is
known by scientific deduction.

Without doubt, light is one of the most evanescent of all
forms of energy. Its natural speed of 186,000 miles per
second carries it across all terrestrial distances in the
twinkling of an eye, and it cannot meet an opaque body
without being transformed into heat. Yet the light travel-
ling from the sun to the earth exists unchanged for over
eight minutes; that coming from Sirius remains unchanged
for some twenty years; while astronomers tell us that
there are fixed stars, or suns, so distant that their light
which we perceive to-day must have left its source before
the birth of Christ and therefore must have existed as light
for some two thousand years without alteration. Sound,
too, is short-lived. Though much slower than light in
transmission, the resistance which it meets in its denser
medium, air, is so great that only a few minutes may elapse
before it too becomes heat. Electricity is quite as short-
lived as light.

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The other forms of energy change less frequently. Familiar types show the opposite extreme and apparently never change. The potential energy stored in the "everlasting hills" appears to be eternal; but the hills are but the creatures and toys of a childish ocean not yet full-grown, which will yet be the death of them with its cloud-bursts and tidal waves. The aged ocean is itself but the first-born of Mother Earth; she herself is but the baby of the solar system; and even the sun, patriarch and ruler of all the planets, sure to outlive in his enormous might the biggest of them, is having his age estimated by modern astronomers as by his teeth and the probable date of his death foretold in serious discussion. There is nothing known which is stable. There is nothing which will not, eventually, change into something else.

To the untrained observer this endless procession of alterations of form is as a kaleidoscope to the childish eye. All is bewildering confusion,—wonderfully beautiful, but distracting and leading nowhere. But to one who knows that the kaleidoscope consists of nothing more than a few bits of colored glass and three mirrors, its explanation is easy and the mystery is gone. The different bits can be identified and utilized to produce new effects at will. So with this ceaseless chain of energy-transformations visible in nature; we must first learn to identify the component parts, the several forms of energy. We watch a bit of sun-heat grow a tree; we fell the tree for fuel; fed into a boiler furnace it makes steam; with the help of an engine the steam makes motion, which drives a dynamo, etc. At each link of the chain the form of energy must first be identified. It then becomes clear that only the form, and not the quantity, of the energy has changed. In each case the senses are so deceived by the complete contrast between former and latter appearance, and by

the apparently complete lack of continuity underlying the transition from one to the other, that the mind is with difficulty persuaded that there has been no change in reality and that the transformation has been, to that extent, a deception. Nevertheless, the unalterable fact is as has just been stated. The existence of the energy has been continuous; its locality and its effect upon us are what have changed. Its amount, its power, its possibilities, are just what they were before. And in each alteration of form the direction of the step shows no evidence of chance, of indecision, of mystery. Each is guided by absolute law, — by a law which can be identified, defined, utilized.

In order to trace this law, attention must next be turned to the question of the *dimensions* of energy.

The Dimensions of Energy. — If each of these diverse forms of energy be examined more closely, it will be perceived that it does not assert itself to our senses as a single simple thing, to be comprehended, as to amount and characteristics, at a single effort. Indeed, with all except one or two it will appear that neither our senses nor any instrument which we can construct will be affected by and measure its quantity directly. Thus, in its mechanical form, energy appeals to us, if potential, merely as a force. We cannot directly perceive what amount of motion has been stored therein or through what distance that force will act if released, though we may reason it out. So knowledge as to the force alone is no knowledge as to the amount of energy present. If the energy be kinetic mechanical in form, we usually can perceive it as a velocity. But the mass concerned is not directly visible, and the velocity alone will not measure the energy present. It is true that the ballistic pendulum will measure kinetic energy directly, and so will the electric watt-meter. But in each case the instrument takes cognizance of two factors going

to make up the energy and integrates them accordingly. In reality it does within its own mechanism just what must be done in the measurement of the quantity of any and every form of energy: (1) The measurement of two distinct factors, and (2) their mathematical combination into a result which is a measure of the quantity of energy. In the most familiar case, that of mechanical energy, these two factors are *force* and *distance*, and in every other form of energy the two factors are these same familiar ones more or less disguised.   A list of them is appended.

FORM OF ENERGY	FACTOR OF INTENSITY		FACTOR OF EXTENT	
	Name	Unit of Measurement	Name	Unit
Mechanical, pot.	Distance.	Feet.	Force.	Pounds.
Mechanical, kin.	Velocity.	Feet-per-second.	Mass.	Pounds ÷ G.
Electrical, pot.	Potential.	Volts.	Charge.	Coulombs.
Electrical, kin.	Potential.	Volts.	Current.	Amperes.
Chemical, pot.	Affinity.	—	Mass.	Molecular weight.
Thermal, kin.	Temperature.	Degrees (Fahr.) absolute.	Entropy.	—
Thermal, pot.	Temperature.	Degrees (Fahr.) absolute.	Entropy.	—

In this table the factors for the first five forms of energy are familiar to every college student.   The factors, or dimensions, of thermal energy, however, do not seem to be a part of one's everyday experience, nor to be readily comprehended as factors of energy.   It is for this reason that they are placed last in the list.

The fundamental idea of energy is based upon the combination of force and distance.   In mechanical potential energy, where the idea originated, the two factors are

plainly visible. What, for instance, is there about the factors of kinetic mechanical energy which connects them with those of potential mechanical energy, into which it transforms itself with the greatest readiness whenever opportunity offers?

The amount of energy resident in a moving mass is known to be expressed by the formula $\frac{1}{2} M v^2$, where M is the mass and v the velocity. But a moving mass is known to possess energy only because it will always overcome resistance in coming to rest. If it be brought to rest by means of a constant negative acceleration in the unit of time, one second, it will have traversed the *distance* $\frac{1}{2} v$. If its total energy is to be absorbed in this distance, it must be done by a resistance having a mean value of $\frac{1}{2} M v^2 \div \frac{1}{2} v = M v$. But $M v$ is the *force* which, if applied constantly through the unit of distance to the mass M, will produce the velocity v, and hence will, if applied negatively, destroy the velocity v within the distance $\frac{1}{2} v$. Thus kinetic energy is also a function of force and distance.

In the case of chemical energy the factors of force and distance, though not directly visible, are easily comprehensible. The molecules of certain chemicals possess a mutual attraction for the molecules of other chemicals entirely aside from the mutual gravitation of their masses. This attraction is known as chemical affinity. Thus, carbon and oxygen exercise a powerful attraction the one for the other. Hydrogen and oxygen do likewise; but in the latter case the affinity between the two is much more powerful than with carbon and oxygen. When either hydrogen or carbon burns, developing heat, the process is nothing more mysterious than a falling-toward-one-another of each pair of molecules, and the energy developed is exactly analogous to that developed when a weight and the earth, mutually attracting one another, fall together.

Similarly, when steam is dissociated into its components, hydrogen and oxygen, or carbonic acid into carbon and oxygen, the molecules require to be separated against their mutual attraction. The process is exactly analogous to the separation of a weight from the earth against the mutual attraction, and the energy absorbed in the dissociation is entirely similar to that absorbed in raising the weight. In the infinite variety of combinations and dissociations of chemicals there are as many different degrees of affinity and of stages of more or less complete combination or separation as there are of different weights and levels in the field of mechanical potential energy.

The cases of electrical and thermal energy are not so simple in explanation as the above. Each consists of a series of vibrations, in its kinetic form, and of permanent separation in its potential form. In the case of heat these vibrations and separations concern the molecules of the body itself, or portions of them, in terms of their mutual cohesion; in the case of electricity they concern, supposedly, the electric atoms or electrons. Of the molecular construction of matter very little is yet known, so that it were unsafe to attempt a too close and accurate description of the ultimate natures of these phenomena.

But of this much we can be absolutely sure: If all matter, including the electrical, does possess component molecules, if these molecules be separated from one another by a distance considerable in proportion to their diameter, and if they exercise a mutual attraction for one another, then, in their vibrations within this attraction they must involve alternations of kinetic and potential energy, and are to this extent a dynamic parallel to a pendulum or to waves in motion upon the surface of a sheet of water, requiring a supply of energy from without to be set in motion, capable of transmitting that energy with great ease

and little loss, and necessarily giving that energy out again before rest can again be attained. Similarly, in their static separation from one another against their mutual attraction they must involve a static parallel to a weight and the earth, requiring a supply of energy, or motion, from without for the performance of their separation, giving little evidence of the possession of the latent energy so long as the separation remains unchanged, and re-developing that same energy into visibility when again allowed to come together.

What is more essentially true than that kinetic energy is a disguised function of force and distance, is that potential mechanical energy is a disguised function of mass and velocity. It will be noted that all of the other items in the column of Factors of Extent of which anything definite can be said are mere measures of mass. Now mass, when the source of the idea is once analyzed, appears plainly as that portion or quality of the substance of the universe which remains constant. Indeed, it alone *is* that substance. All other features, such as weight, etc., are mere attributes, due entirely to the position of the mass in question relatively to other portions of mass.

The first law of physics, that of the conservation of matter, states this fact : —

" The mass of the universe remains unchanged and is unchangeable."

The definition of mass is the mere obverse of this : —

" Mass is the unchangeable in matter."

But the first law of energetics, that of the conservation of energy, states that the energy of the universe is unchangeable. Since all forms of energy are reducible to the motion or the stored motion of mass, and since the latter factor has just been defined as a constant, it follows that the former factor must also be a constant, when the total quantity in the universe is viewed as a unit.

The truth of this statement will appear more clearly from further consideration of the question. For the present it is sufficient to note that, of the two factors going to make up each form of energy, one, of the list which the writer prefers to call the "factors of extent," is always a function of *mass*. It, in fact, measures the amount of the universe which is involved in the energy in question. The other dimension, of the list which he calls the "factors of intensity," is always a function of velocity of motion or of remoteness of separation of the mass involved. It is a function of the *mutual relations* existing between any two or more portions of the mass involved in the first factor.

It is quite consistent with a conception of the universe as a system of portions of mass involving various degrees of relative motion and position, that the several masses should be regarded as always remaining constant, from instant to instant, and that their integrated sum be also constant. It is also consistent with such a conception that the instantaneous relative motion or position of the individual component masses be ever varying; indeed, the very conception of motion implies it. And yet, if the centre of mass of the universe be imagined as fixed, or, which is the same thing, if these various motions and positions be regarded as *purely relative*, then the integration of their relative velocities and of their degrees of separation must always together amount to a constant.

If, then, to return to the list of energy-factors, the factors of the first column, those of intensity, be remembered as functions of relative velocity or of relative separation, it can be stated that in individual cases they will be ever found varying and variable, but in their summation or average they will always be found constant in quantity.

The three basic laws of energetics, approached in this way, may be stated as follows : —

   I. **The Conservation of Matter:** The mass of the universe ever remains constant in quantity.

  II. **The Conservation of Motion:** The total intensity or potential of the universe ever remains constant in quantity.

III. **The Conservation of Energy:** The total energy of the universe, the product of the two foregoing factors, ever remains constant in quantity.

It has already been noted that there exists throughout the universe a tendency for every form of energy to transform itself into some other, and that although this tendency is in some cases resisted for periods of time seemingly very long, when compared to human standards, yet it always finally occurs. It is natural to expect that these relative motions and positions of the myriad of portions of mass which constitutes the known universe should be continually altering themselves into other motions and positions. It would seem that there must be some laws controlling these transformations, now permitting some transformations to take place with instantaneous rapidity and now delaying others for ages.

In the tabular list of page 20 the classification into factors of intensity and factors of extent was not made primarily upon the basis of the broad theoretic postulates enunciated above. It had been observed that the factors of energy possess two distinct sets of characteristics. Upon these characteristics the classification was based. The names *intensity* and *extent* were chosen as best representing those characteristics.[1]

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[1] Mr. George Richmond, in treating of thermal phenomena only, makes use of the terms *height* and *breadth* instead of *intensity* and *extent* respectively, and there are many reasons why, especially in connection with graphic representation of energies by coördinate diagrams, his choice is to be pre-

In all forms of energy it is the prime characteristic of the factor of intensity that it ever and always tends to a decrease and disappearance, by the fall of the energy of which it is a part to and through lower and lower degrees of intensity, until ultimately the zero of intensity might be reached.   Thus, taking the most familiar form of energy-fall first, mechanical potential energy always tends to decrease its factor of intensity, distance of separation, to lower and lower degrees of separation until there be no separation left.   Water, attracted by the earth's gravitation and separated from it by evaporation into a rain-cloud, tends to fall nearer and nearer to the most intimate pos-sible association with the earth, from rain-cloud to hilltop, from hilltop into mill-pond, thence to the tail-race, and from the tail-race to the sea; and if there were a passage from the sea to the centre of the earth, it would flow thither.   Similarly, a steel spring, stretched by previous distortion to a distance of separation from its condition of equilibrium, tends always to return, through various intermediate degrees of separation, to the position of the closest possible approach to equilibrium.

In a like manner the mechanical kinetic energy of a moving body tends always toward a reduction in velocity, whether through the medium of friction, of impact, or of the imparting of a portion of its energy to other bodies in the form of motion.

Electrical energy, whether kinetic or potential, always tends to flow from the point of higher voltage to the point of lower, reducing its own potential as it goes.

In chemical energy the tendency exhibited in reactions and combinations is always away from the compounds of maximum chemical affinity and minimum chemical stability

ferred.   The writer's reasons for preferring the terms he has adopted are for the sake of breadth of definition.

to those of less and less intense affinity for one another until a level is reached where no further combination is possible.

In thermal energy is exhibited one of the most familiar illustrations of all, the constant tendency of heat to flow from the body or locality of higher temperature to that of a lower temperature. But this illustration does not fully cover the law. Heat tends strongly to alter itself to a lower temperature even when there is no opportunity for flow out of the body in which it lies into another colder one. Even if the body be completely isolated from the rest of the universe, by an insulation so perfect that both radiation and conduction are absolutely impossible, its heat will alter itself to a lower temperature by other processes when opportunity occurs. Discussion of the nature of this opportunity must be deferred until later in the argument.

The converse of this law is also true, viz.: That energy never will alter itself spontaneously and without transformation from a lower to a higher degree of intensity. Water will not flow uphill, nor any weight rise from the earth, nor a spring distort itself. A moving mass will not spontaneously increase its velocity, nor impart its energy to a mass moving at a higher velocity. Electricity will not flow against a superior potential, nor will a charge spark from one jar to another more highly charged. Chemical compounds will not combine, unaided, into compounds having a higher affinity than the originals. Heat will never flow spontaneously from a cold body to a hot one.

This being true, how is it that there ever exist any higher degrees of intensity, to maintain by their fall the current phenomena of the universe from which this law has been observed? It is unimaginable that they should all have come from an original store and that the universe

is engaged in slowly "running down." It is obvious that all of these at first sight unattainable degrees of intensity in energy must have been somehow attained, and that by means of processes which must be as current and as natural now as at any previous time.

There are two methods by which rise of intensity is accomplished, and only two, and of these the first-mentioned is only local and temporary in its effect : —

(1) There comes from some outside source an additional supply of energy to force the apparently unnatural process ; or

(2) There occurs an intermediate energy-transformation into some other form and back again.

Indeed, it is only so long as the energy remains in its original form that the spontaneous tendency to drop in intensity is manifested. Thus, water will not flow uphill ; but it can be pumped up with the aid of some external supply of power, or, its potential energy transformed into kinetic and back again through the medium of the hydraulic ram, it will pump a portion of itself to a higher level than the supply. The kinetic energy of a moving mass cannot of itself take on a higher velocity ; but enforced by an additional supply of energy from without it will take on a higher velocity, it will become a portion of a larger total quantity of kinetic energy having a higher degree of intensity ; or, if inherent in an elastic body and meeting a body having velocity in the opposite direction, it will, by conversion into potential energy of distortion and back again, convert a portion of itself into a higher velocity. Electricity will not spontaneously flow to a higher voltage ; but its voltage may be increased by being electrically reënforced ; or it will raise its own voltage by means of a double transformation through a converter. Similarly, heat will not voluntarily alter itself

from a colder to a hotter temperature; but it may be forced to do so by an external source of power acting through a refrigerating-machine; or it may be transformed into heat of any degree of temperature by means of a double transformation into some other form of energy, as into electricity, and then back again by way of the electric arc.

It is especially to be noted, in considering this question as to how the at first sight apparently unattainable degrees of intensity originated, that in the preceding illustrations reliance was placed upon the two methods of increase of intensity, viz.: —

(1) The simple addition of energy from some other portion of the universe;

(2) The transformation, into the form under discussion, of some other form of energy already existent.

It next appears that whenever the latter method prevails, whenever such an energy-transformation occurs, there appears to be no broad natural limit to the degree of intensity of the secondary form of energy. Thus, there is no broad law connecting the muzzle-velocity of a projectile with the chemical affinity of the gunpowder. There is a relation between the two, but it depends upon the relative mass of powder and ball; the relative diameter and length of gun-barrel, etc., etc. These are all accidental circumstances, not broad principles. The attainment of the highest muzzle-velocities has not resulted from the discovery of explosives of extreme chemical affinity. Similarly, in the electric arc, the temperature attained is not a basic function of the voltage producing the arc. Very high voltages apparently produce no higher temperatures than quite low ones may do, under favorable conditions. In any such case the intensity of the secondary form of energy appears

to depend upon circumstances which may be called acci-
dental in their character, and not at all upon the intensity
of the primary form.   It is plain that the phenomenon is
such as to permit the attainment, in the secondary form of
energy, of the highest degrees of intensity known to man,
and that from comparatively low degrees of intensity in
the primary form.

Upon further inspection it appears that these two pro-
cesses by means of which higher degrees of intensity are
attained are in every way the obverse of those by which
intensity lowers itself.   For when the energy of a body
obeys the familiar tendency to fall in intensity, its quantity
must always decrease.   But, since annihilation of energy
is impossible, decrease is possible only in one of two
ways : —

(1) **Dissipation**, or the distribution of energy to other
masses of *similar* character ;

(2) **Conversion** into other forms of energy, or the dis-
tribution of energy to other masses of *dissimilar*
character ;

and these two methods are plainly merely the obverse of
the two which are listed on the preceding page.   Thus,
the energy of a moving mass of appreciable size, such as
that of a projectile, always exhibits a tendency to decrease
its intensity, or in other words its velocity, of motion.   It
will always do this in one of two ways : —

(*a*) By meeting other bodies which are perfectly elastic
and which will partake of the motion of the original mass,
the result being a greater total mass in motion of a lower
average intensity ; or (*b*) by meeting other bodies which
are more or less inelastic, in which case friction and impact
occur ; and friction and impact are merely names for the
conversion of mechanical energy into heat.

Similarly, chemical energy will always, when permitted to free itself, either dissociate a larger mass than its own to a lower degree of chemical affinity, or it will convert itself into heat, light, electricity, sound, etc., which are merely the motions or positions of masses dissimilar to chemical mass.

Electrical energy will always fall in potential when permitted, and in so doing will convert itself into either heat, light, sound, chemical energy, or mechanical motion ; or, if in static form and free to conduct itself to a larger mass of surface, it will do so, but with a corresponding reduction in potential.

Heat, in the same way, will always tend to alter itself to a lower intensity, and in doing so either other masses of matter will be heated to temperatures lower than the original or new forms of energy, such as chemical dissociation or mechanical motion, will be produced.

If attention then be turned to the secondary bodies which are affected by these processes, it will be plain that they are undergoing, in each case, the obverse of the processes under discussion. That is, the intensity of their energy is being raised by one of two processes : —

(a) By the simple addition of energy from the primary mass ;
(b) By the conversion of the primary form of energy into their own.

These three statements of the two processes, on pages 29, 30, and 31, show them to be identical ; on each page they were approached from a different point of view.

Finally, if it be remembered that all forms of energy consist of the motion or the stored motion of masses of one sort or size or another, it becomes plain that even the conversion of energy into other forms is merely a transfer

of motion from one mass to another, but in this case to a mass of a different kind. Thus, when the energy of a moving mass of appreciable size is converted into heat by friction, the parallel simultaneous motion of the molecules forming the body is altered either into other sorts of motion of those same molecules, in so far as the moving body itself is heated, or into varied motions of the molecules of the bodies with which it comes in contact. Similarly, the production of electricity or chemical dissociation from mechanical energy is merely the transfer of the parallel motions of the particles of the original body into various peculiar motions or separations of the numerous masses of electrical or chemical atoms of the bodies affected. These bodies may remain stationary in relation to the surrounding universe, but they are in possession of active internal motion ; and the original source of energy has been brought to visible rest.

Summary

The universe, viewed in this light, reduces to the following simple statement, which includes all known phenomena, whether simple or complex, whether inanimate or animate, from the motions of the heavenly bodies to the action of dust floating in the sunlight, from the quiescence of the clod to the most intricate and delicate processes of human life and thought, viz.: —

An infinite series of portions of mass, of infinite variety of size and form, linked together by mutual gravitational attraction, each associated with the others in a certain degree of separation and possessing a certain amount of motion. Both separation and motion are purely relative. Motion cannot exist without accomplishing separation, positive or negative. Separation cannot exist, appear, or disappear, without accomplishing motion, positive or nega-

tive. Motion, therefore, cannot exist without causing change in both separation and motion. The same is true of separation.

Newton's "First Law of Motion" is in no conflict with the above statement, because the former, in its application to a given mass, assumes the non-existence of all other mass. The law stated in the preceding paragraph assumes in its premises the existence of *all* other mass, — not merely a few portions of it, but an infinity of portions, extending to unthinkable limits.

Motion and separation are purely relative and are interchangeable. They together constitute the *intensity* of the energy of the universe. The mass embodying the motion or separation constitutes the *extent* of the energy of the universe. Both intensity and extent ever remain constant in quantity.

The intensity of the energy of a given mass always tends to decrease. When that energy consists of separation, the separation tends to disappear, producing motion ; when it consists of motion, the motion tends to disappear, producing separation.

When the motion is transferred from one portion of mass to another (which constitutes what is known as a "transformation" of energy), the proportion of intensity of motion or separation to extent of mass involved in the new form *is independent of the degree of intensity in the old ;* the degree of intensity in the new is always a maximum controlled by environment.

The laws which govern the when and how of such conversions, and which dictate the limit which has been cloaked under the expression "the maximum permitted by environment," are of the most intricate. Their discovery and statement are the work of all the existing sciences and of all those yet to be constructed, in combination.

D

These principles may be stated in more concrete and popular language in the following form.

The primary law of energetics is that of the

Conservation of Energy

" Energy is eternal.   It can never be either destroyed or created; whenever one form of energy disappears it must reappear in another form in equal quantity."

This law comprises two minor principles:—

I. *The Conservation of Mass*

Matter is eternal.   It can never be destroyed or created; only the mutual relation of its parts can be altered.

II. *The Conservation of Motion*

Motion is eternal.   The total amount and the average intensity of the motion or the separation (the "stored motion") of the universe remains ever constant and can never be destroyed or created; it can be altered only in the mutual relations of its parts.

Mechanical energy and thermal energy, by virtue of their existence as energy, come within the above law as special illustrations.   Hence the primary law of thermodynamics, the so-called

First Law of Thermodynamics

Heat can be neither destroyed nor created.   When converted into motion or derived from motion, the energy disappearing is always the equivalent of that reappearing.[1]

The second great law of energetics is that of the

[1] Of the two minor principles noted above only the first applies in the study of thermodynamics, and it is of passive rather than of active interest. The second, which covers all the motion of the universe, naturally cannot apply to the one peculiar portion of that motion which we call heat. Any one portion of the motion of the universe, such as heat-motion, may decrease or increase ; it is the sum total only which remains constant.

INTERCHANGE OF ENERGY

The form of energy is fickle and evanescent.   Neither motion nor separation can exist in mass without alteration, either into the other one of the pair or into the motion or separation of other portions of mass.   This alteration always takes place from a condition or locality of higher to one of lower intensity, and never the reverse.

Thermal energy, in obedience to this generalization, presents the

Second Law of Thermodynamics

Heat is fickle and evanescent.   It ever tends to alter itself from higher to lower temperatures whenever opportunity offers, either by dissipation or by conversion into other forms of energy.   It will never spontaneously alter itself from a lower to a higher temperature, but may be forced to do so in either of two ways, viz.: —

(1) By a supply of energy from without;

(2) By a double intermediate transformation into some other form of energy and back again.

When the tendency of heat to fall in temperature was first noted and formulated into a law, it was also noted that its dissipation amongst larger and larger masses was the commonest method of temperature-fall.   No return of the heat to higher temperatures seems to have been noticed, although it is occurring upon every hand.   The situation was therefore covered by the so-called "law of the dissipation of energy," viz.: —

The entropy of the world tends to a maximum and the temperature to a minimum.

The modern statement of both sides of the law, in similar language, would be: —

The total entropy and the average temperature of the universe remain constant.

But no such law may probably be true, for entropy and temperature are respectively the mass and the intensity of only one sort of energy.   Since there is as yet no evidence accumulated which reveals any fixity of proportion between the several sorts of energy in the universe, although in all probability that proportion is rigidly controlled by natural law, no law similar to the above may be stated which confines itself to a single form of energy, such as heat.

CHAPTER II

THE CYCLE

THE tendency of energy to fall in intensity, whatever be the form of energy concerned, almost always results in the more or less complete transformation of the energy into some other form. But in each of these cases of transformation it is to be noted that the continuity of transformation is limited. Thus in the case of a weight suspended above the earth, it may continue to fall and to convert its energy into a kinetic form only until it reach the earth. That once reached its potential energy is, in one sense, all gone; its intensity of separation is simultaneously all gone, and the progress of conversion must cease. The matter which had recently served as a medium for the desired transformation of energy is no longer useful for that purpose.

This fact is a serious obstacle in the practical application of the process in the arts. A supply of energy must be continuous in order to be useful. Further, it is not ordinarily possible to look to a continuous, inexhaustible supply of matter containing the particular form of energy available for conversion. We must convert an indefinite amount of energy from the available to the desired form by means of a finite amount of matter, which must be utilized and relied upon as a carrier for the energy. The only known method by which such a carrier can perform continuous energy-transformation is by endless repetition of the process. Thus, water endlessly repeats the process of fall from the rain-clouds to the mill-pond and thence to the sea, and of evaporation from the sea to the cloud again.

The Working-substance. — The particular form of matter used as a carrier in any such repeated conversion is called the *working-substance*. The repeated process by which it effects a continuous conversion of energy is called a *cycle*. The latter may be defined as follows : —

A cycle *is a series of processes, usually four in number, by the repetition of which a finite amount of matter may effect a continuous or infinite conversion of energy from one form to another.*

It is in the application of thermodynamics to the design of heat-engines that the study of the form of cycle involved becomes of prime importance, much more so than in most other classes of energy-conversion. But as the conception of heat as a form of energy, and particularly as its two dimensions, temperature and entropy, are more difficult of comprehension than is the case with some more familiar and concrete forms of energy, the study of the thermodynamic cycle will be presented in the following pages by a running parallel between it and a similar conversion of potential mechanical energy, as of stored water-power, into some other form, and of heat-energy into some mechanical form.

In every such case the primary form of energy is composed of two factors, viz.: *height* and *weight* in the first case, *temperature* and *entropy* in the second. If the weight of a given mass be assumed to be constant over all ordinary variations in height, which is very nearly true, it would follow that the potential energy involved in the weight was equal to the height multiplied by the weight. In fact, in all hydraulic engineering we rely upon the formula that $E = H \times W$, where E is the energy, H the height, and W the weight. But this formula is based upon the assumption that all changes of weight take place at a

constant fixed height, and is therefore special in its character. To state a general equation based upon the same idea, recourse must be had to the calculus. The equation then becomes

$$dE = HdW. \tag{1}$$

This is the fundamental equation of potential mechanical energy.

In the case of heat-energy the situation is not quite so simple; at least it is not so apparent. Temperature has been defined as some function of the square of the velocity of translation of the molecules composing a mass of matter. But entropy has not yet been defined to the comprehension with even this degree of accuracy of conception. Its origin arose from the fact that the early founders of the science of thermodynamics found that in any alteration in the quantity of heat in a body the ratio $\frac{dQ}{T}$, wherein Q is quantity of heat and T the absolute temperature at which the alteration takes place, was a very frequently recurring and a convenient and useful function for a variety of calculations. They therefore gave it a name, **entropy**. The fundamental equation of all thermodynamic calculation therefore is

$$dN = \frac{dQ}{T}, \tag{2}$$

where N is entropy, Q the quantity of heat involved in the change, and T the absolute temperature at which the change occurs.

This equation gives the mathematical definition of entropy. Its definition in general terms is:—

Entropy *is that quality of a body which increases when heat is added to the body, which decreases only when heat is abstracted, and consequently remains constant only when heat is neither added nor abstracted.*

Equation 1 easily converts itself into $dQ = TdN$, and in this equation we see great similarity to the equation connecting the two dimensions of potential mechanical energy, just given with water-power as an illustration. It is the integration of this equation, or

$$Q = \int_{T_1}^{T_2} TdN, \tag{3}$$

which gave rise to Zeuner's name for entropy, *heat-weight;* for in the energy developed by the fall of heat from the condition T_1N_1 to the condition T_2N_2, as given by the above equation, the entropy plays a very close parallel to the function of weight in the energy developed by the fall of a mass from a condition H_1W_1 to a condition H_2W_2.

These two definitions, the mathematical and the general, must suffice for the identification of entropy with mathematical accuracy.

But an approximate conception of its nature which is much more useful than the foregoing is to be had from the table of dimensions of energy given on page 21. It will become very clear, as the study of thermal phenomena proceeds, that entropy belongs in the column of factors of extent. When a thermic body is left thermally isolated, its entropy never alters in quantity, as its temperature may. When the body meets another and exchanges energy of like sort, there appears no law of connection between the direction of the interchange and the degree of entropy present. In all such ways entropy shows none of the characteristics which appear so plainly as the distinguishing characteristics of the factors of intensity and which form the basis of the Second Law of Energetics. Clearly entropy is a dimension of extent.[1]

[1] As to the idea frequently promulgated at present in a manner far too dogmatic to harmonize with our meagre knowledge of the truth, viz.: that

If so, entropy must be either mass itself or else some close function of mass. There are obstacles in the way of further definition of the first hypothesis; there is no basis which visibly directs one to a determination of the function referred to in the second hypothesis. So that further elaboration of the definition seems impossible. Yet the original statement remains unshaken.

It will develop, upon the most superficial inspection of thermal phenomena, that entropy is an active variable. If entropy be mass, the inevitable deduction must be, since the mass of a body remains constant during heating or cooling, that the proportion of the mass of the body which takes part in thermal vibration must be variable. Since thermal energy is normally distributed evenly throughout the body containing it, this must mean that *there is a variable portion of the mass of each atom which enters into the motion which constitutes thermal energy*. This variable portion, according to the first hypothesis given above, is the entropy of the body. If not, it is at least an active factor in that entropy.[1]

entropy is a mere mathematical ratio, heat divided by temperature, it is a brief task to point out its untenability. Heat is energy, an eternal, indestructible reality. Temperature is a mere dimension, quite destructible, of that reality. Their natures are totally incompatible. There is no more possibility of a ratio between heat and temperature being nothing more than a ratio than there is of a ratio between area and length being nothing else.

Nor does the hypothesis that temperature is itself energy (based solely upon assumptions which apply only to a single form of matter, the gaseous) help the situation. Both the isothermal and the constant-heat processes (see pp. 47 and 105) deny the tenability of the hypothesis. In the first, energy is absorbed with no increase in temperature; in the second, temperature disappears with no reappearance of temperature. Both processes show an irreconcilable conflict between the hypothesis and the First Law of Thermodynamics.

[1] This hypothesis is in no way untenable in the face of the present aspect of molecular physics. The word *atom* has completely lost its pristine significance of something akin to a microscopic billiard-ball. The atom has never been anything more than a concept, based upon certain assumptions in order to

Let the two pairs of factors which go to measure energy-quantities of the potential-mechanical and the thermal types respectively be represented by ordinates and abscissæ of two respective sets of rectilinear coördinates, the former as in Fig. 1 and the latter as in Fig. 2. Let OW be an axis upon which is measured *weight*, OH an axis on which is measured *height*, OT an axis on which is measured (absolute) *temperature*, and ON an axis of *entropy*.

Let us suppose that the point A, in Fig. 1, represents the condition of a mass having the height of separation from the earth of H_1 and the weight W_1; then its inherent potential-mechanical energy must be $E = H_1 W_1$. Similarly, we may suppose that the point A of Fig. 2 represents the thermal energy of a mass possessing an absolute temperature T_1 and an amount of entropy N_1; but we cannot similarly assume that its total thermal energy is $Q = T_1 N_1$.

clarify certain obscure phenomena. These assumptions have had to grow in complexity with the growth of knowledge, until it is now accepted that what we attempt to define as an atom is merely a mass of matter very much smaller than human dimensions, but which, instead of being an indivisible, elementary sort of thing, may be and is probably as complex within itself as are the largest known masses of matter. So that it is quite feasible to suppose that varying portions of its mass may enter into those motions and positions which we lump under the title of thermal energy.

Moreover, it happens, rather unfortunately, that practically all of the brilliant investigations of the mathematical limits of molecular action have included in the premises the assumption that the mass involved was constant. The sole justification of this is the familiar fact that heating or cooling does not alter the total weight of a body.

For this reason, too, the hypothesis that entropy is merely mass has not progressed in limitation. It has neither been proved nor disproved. To the student who does not find that it aids his comprehension, complete progress through mathematical thermodynamics may be made by means of the two exact definitions of entropy already given; but it will be purely mnemonic in its character and correspondingly limited in usefulness. To the student who takes the trouble to study the real significance of thermodynamic phenomena this hypothesis that entropy is thermal mass, although not demonstrably exact, will be found to be a great and proper aid.

For of mechanical energy it is known that it must always equal distance times force, but of thermal energy it is only true that $dQ = TdN$.

In Fig. 1 suppose that the weight at A be released from its position. It will immediately fall toward the earth, or along the vertical straight line AB. Suppose that at B further fall be impossible, by reason of some practical limitation. The weight, now being useless for the further development of power, must be gotten rid of. If it be water, it is easily poured away, as into a tail-race. In fact, the processes depicted in Fig. 1 are most clearly imaginable as being carried out by a water-wheel made up of an endless chain of buckets passing over four pulleys, one at each corner of the rectangle. Each increment of departure from the axis OH should then be understood as involving a corresponding increase in weight, as of water poured into the buckets; each approach toward OH should be taken to indicate a proportionate decrease in weight, as of water poured out of the buckets. In this sense the line BC represents the emptying of the buckets at the tail-race level.

It may happen, from practical limitations, that when the point C is reached no more weight can be disposed of. Either no more water will flow out; or H_2C may be taken as measuring the weight of the empty buckets themselves. If so, this weight must be raised to the H_1-level before more water can be taken in for the performance of more work. So the weight H_2C is lifted along the line CD to D.

The empty buckets can now be filled, at head-race level, along the line DA, after which the process may be repeated indefinitely.

In this cycle the fall of the weight W_1 along the path AB develops the work $W_1(H_1 - H_2)$. In other words, the secondary form of energy is abstracted, after formation at the expense of the primary. Along BC no work is done.

Instead, energy of the primary form is abstracted. Along
CD the negative work $W_2(H_2 - H_1)$ is done upon the work-
ing-substance by some exterior source of power, or, in other
words, the secondary form is supplied to the system, being
at the same time transformed back into the primary form.
Along DA no work is done, positive or negative. Instead,
the primary form is supplied. (DA is most naturally taken
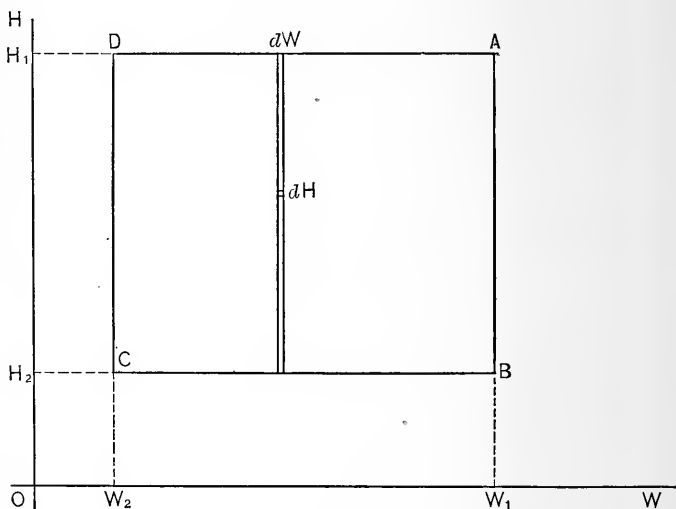
FIG. 1

as the initial process of the four.) Therefore the total net
work done by the working-substance must be the algebraic
sum of the two quantities visible along AB and CD, or

$$\text{Work} = (W_1 - W_2)(H_1 - H_2). \qquad (4)$$

But in this expression the first parenthesis is the width of
the rectangle $DABC$, and the second is the height. There-
fore, in a rectangular cycle such as this *the net quantity of
energy transformed is measured by the area enclosed.*

It is evident that this same result could be reached by subtracting from the primary energy entering the system along DA that rejected along BC. The difference, which disappears during the cycle, must, according to the First Law of Energetics, reappear in the secondary form.

If attention be turned from this cycle of finite dimensions to the infinitesimal rectangular cycle having sides of
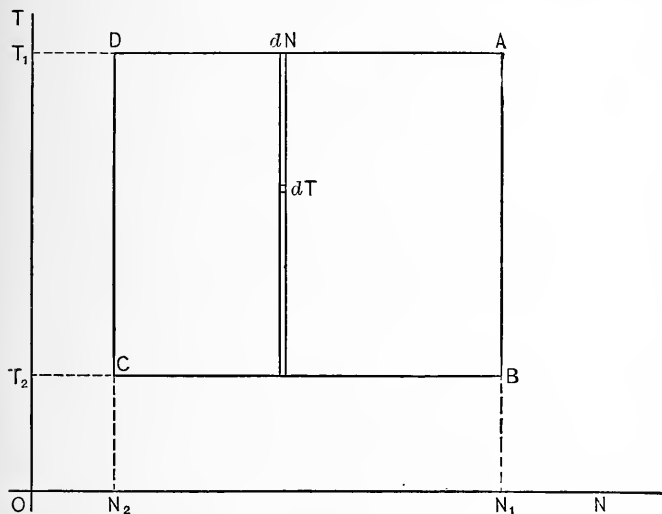
FIG. 2

lengths dW and dH respectively, shown in the centre of Fig. 1, it is plain that this argument must equally apply; or, in other words, the work done in this infinitesimal cycle must be measured by its area, given by the expression

$$d(d\,\text{Work}) = dW\,dH. \tag{5}$$

If this equation be true, the sum of any number of such equations must be true; that is, the equation may be integrated. Such integration will result graphically, from the

second member of the equation, in the development of a
closed area, its form depending upon the particular func-
tions existing between W and H.   It will result physically,
from the first member of the equation, in the net total of
energy transformed from the primary to the secondary form
by the summation of rectangular infinitesimal cycles; and
the physical significance of this summation of cycles, since
their contiguous sides must always represent two identical
processes in reverse directions and hence cancelling each
other, is the same as that of a series of processes repre-
sented by the free sides.

It is plain that any closed area may be considered as the
summation of such infinitesimal rectangular cycles.   Hence
the conclusion is drawn that —

*If, in any series of height-weight processes forming a closed
cycle (that is, one bringing the system back to its original
position and weight) these processes be represented by lines
forming a closed diagram, it must be true that the work
done during the cycle will be given by the integration, to the
proper limits, of the equation*

$$\text{Work} = \int \int dW \, dH. \qquad (6)$$

But this integration will also result in a measure of the
area enclosed by the diagram, from which may be drawn
the proof that *in any height-weight diagram the work done
by a closed cycle is measured by the area enclosed by the
figure depicting that cycle.*

If attention be returned to Fig. 2, it will be obvious that
the rectangle $ABCD$ represents no such familiar series of
processes as does that of Fig. 1.   Nevertheless, it is just
as true to life and just as frequent in occurrence; but it
is much more obscure.   If heat of the condition depicted
at A (Fig. 2) be given the opportunity, it will fall in tem-

perature down the line AB just as promptly and just as accurately as will water at A (Fig. 1) fall down the plumb-line AB; and in falling it will just as surely develop work. At B (Fig. 2) it may have reached the temperature of surrounding objects. If so, it will be prevented thereby from falling farther, at any rate usefully; just as the water-weight at B had reached a point where it could fall no farther advantageously. It must therefore be rejected.

This is supposedly done along the line BC. The line BC therefore indicates the abstraction of heat at constant temperature, just as in Fig. 1 it indicated the abstraction of weight-energy at constant level.

At C it may be supposed that no more heat can be disposed of. If so, the rest must be lifted again, up the line CD, to the original temperature-level, by an expenditure of energy brought in from without. The heat-carrier, whatever it may be, is then ready to receive another charge of heat along the line DA and to repeat the process indefinitely. This addition of heat necessarily implies increase in entropy. The increase is here supposed to take place at constant temperature.

In the case of the water-power of Fig. 1, it is comparatively easy to follow with the imagination the several processes of taking in weight, letting it fall, emptying the residue, and returning the empty bucket. In the case of the heat-engine of Fig. 2, however, the processes are not so easy to follow. They are entirely unfamiliar, and it is only with the help of the water-wheel simile and the other analogies which have been presented that any useful comprehension of them may be had.

Isothermals. — The processes DA and BC are called isothermals, *i.e.* occurring at constant temperature. They necessarily involve, since the entropy increases or decreases, the addition or abstraction of heat. The most

familiar instances of isothermal action are found in the melting of ice or the vaporization of water under constant pressure.   In each case large quantities of heat enter or leave the body, yet the temperature remains unchanged.

Adiabatics.[1] — The processes AB and CD are called adiabatics.   Their location upon the diagram, the entropy remaining constant while the temperature alters, expresses graphically what the definition of the adiabatic does in words: *An alteration in temperature without addition or abstraction of heat*, or, in other words, *while completely isolated from heat-exchange with the surrounding universe.*

There lies herein a seeming paradox which must be cleared from the student's mind before further progress may be made, viz.: If heat be neither added to nor abstracted from the body, how can the temperature of the body, and also (as is clear from the diagram) its quantity of heat, be altered?   The answer is: By conversion into work.   During the adiabatic fall of heat no heat is lost *as heat* to the surrounding universe; but heat is steadily lost by conversion into the form of work.   There is no heat abstracted; but there is heat lost, for work has been abstracted.

Similarly, during adiabatic rise of heat in temperature no heat is added to the body; but heat is gained, in the form of work, from the outside universe.   In the adiabatic processes there is no exchange of heat between the body and its surroundings; there *is* an exchange of energy, however.

The process may be perfectly paralleled by a man who goes down town with his pockets full of money.   He may not have been robbed of a cent, yet he may return with his pockets empty; but the supposition is that his arms are full of bundles which he has purchased.   Not a cent has

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[1] The word *adiabatic* signifies "without exchange."

been abstracted from him, yet he has ceased to possess his money; it has been transformed into another form of economic energy.

The adiabatic process may be said, in one very true sense, to be the only natural heat-process, for it is the only one in which the body acts freely and spontaneously of itself, without thermal influence from without. It is the process which every quantity of heat would follow if left to itself. Hence it is of prime importance in thermodynamic work as being the process from which all heat-processes depart only so far as they are forced to do so by purely external and often artificial forces. The adiabatic fall of temperature, which every portion of heat spontaneously undertakes when permitted to do so, is as natural a specimen of gravitation as that which called Newton's attention to the force which holds the universe together. **The adiabatic is the plumb-line of heat-energy.**

In this cycle of heat-processes the quantity of primary energy or heat supplied along the path DA can be found by the integration of Equation 3. Since in this case T is a constant, T_1, the integration becomes

$$Q_1 = T_1(N_1 - N_2). \tag{7}$$

Similarly, the quantity of heat extracted along BC becomes

$$Q_2 = T_2(N_1 - N_2). \tag{8}$$

Subtracting, according to the First Law of Thermodynamics, the total quantity of secondary energy, or work, appearing as the result of the complete cycle must be

$$Q_w = Q_1 - Q_2 = (T_1 - T_2)(N_1 - N_2). \tag{9}$$

For along AB and CD no primary energy is added to or abstracted from the system.

But in this expression the first parenthesis is the width of the rectangle $DABC$ and the second is the height.

E

Therefore, in a rectangular entropy-temperature cycle such as this, the work done is measured by the area enclosed.

From this idea can be developed the proof that —

In any closed entropy-temperature cycle the quantity of heat transformed into work is measured by the area enclosed, in a manner similar to that used with the height-weight cycle. In this case, however, it is important to establish one of the
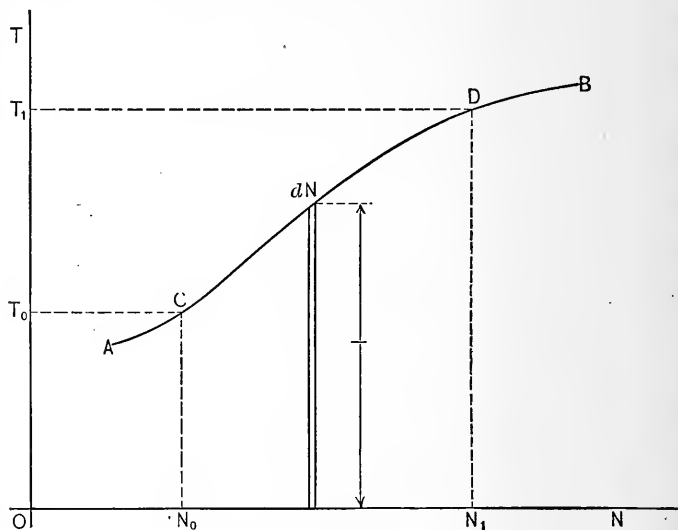
FIG. 3

intermediate steps as a general principle. This can be done by means of Fig. 3.

In Fig. 3, let AB be any heat-process whatever. Equation 2, transformed, gives

$$dQ = TdN. \tag{10}$$

The right-hand member of this equation is represented geometrically by the elementary vertical strip having the

width dN and the height T. The integration of the above equation to any limits, such as T_0 and T_1 or N_0 and N_1, gives, algebraically, Q, the quantity of heat involved in the process, and geometrically, the area between the curve AB and the zero-axis of temperature and between the limiting ordinates, or the area N_0CDN_1. Since AB is any curve and the limits chosen were any limits, the following law may be deduced: —

If any heat-process be depicted by a curve in an entropy-temperature field of coördinates, the heat handled during the process is measured by the area between the curve, the zero-axis of temperature, and the limiting ordinates.
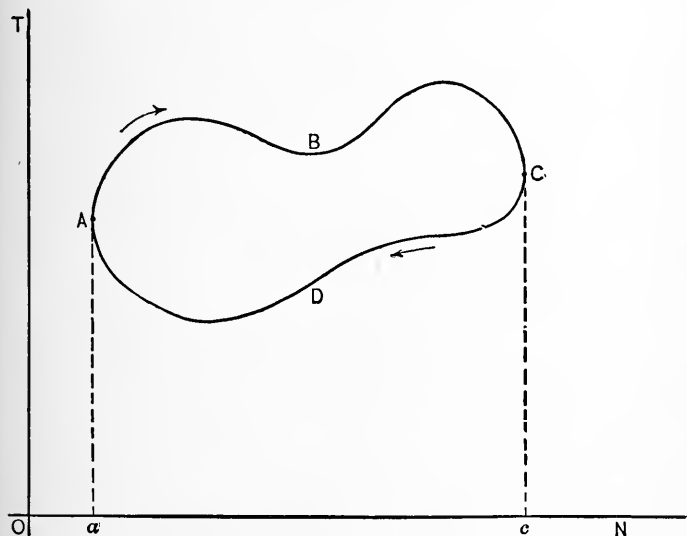
FIG. 4

If attention be next turned to any closed diagram on an entropy-temperature field of coördinates, such as that shown in Fig. 4, it is plain that it may be considered as made up of two portions, viz.: —

(1) The portion ABC in which the entropy changes from a minimum at A to a maximum at C. This portion must indicate the addition of heat to the system. This quantity of heat supplied will hereafter be referred to by the symbol Q_1.

(2) The portion CDA during which heat is abstracted from the system. The quantity of heat abstracted will be represented by the symbol Q_2.

By the preceding argument, Q_1 is measured by the area $aABCc$. Similarly, Q_2 is measured by the area $cCDAa$. The heat converted into work must be their difference, or

$$Q_w = Q_1 - Q_2 = \text{the area } ABCD. \qquad (11)$$

Since Fig. 4 represents any closed diagram, the argument is a general one.

Therefore, *if any closed cycle of heat-processes be represented on the entropy-temperature field of coördinates by a closed figure, the heat converted into work at each revolution of the cycle is measured by the area enclosed.*

Conditions Essential to the Production of a Cycle. — The truth of the following statement of the conditions which must exist before a closed cycle such as that just discussed may come into existence will be obvious from the preceding argument. The original statement, made in reference to thermodynamic cycles only, was defined so long ago as 1824, by Sadi Carnot, then a lieutenant in the French army. The modern, corresponding statement, which applies broadly to all forms of cycles, conducting transformations between any possible pairs of energy-forms, is the following : —

THE THIRD LAW OF ENERGETICS

For the continuous, cyclical transformation of any primary into any secondary form of energy, within the con-

fines of any limited portion of matter, four things are essential : —

    I. A source of supply of the primary form;
    II. An absorbent of the primary form;
    III. A positive difference of intensity between the two;
    IV. An absorbent of the secondary form.

(In the third condition the word "positive" is understood to apply when the intensity of II is subtracted from the intensity of I.)

As with the other general laws of energetics, when the two forms of energy concerned are heat and work respectively, this general law becomes one of the special laws of thermodynamics, viz. : —

THE THIRD LAW OF THERMODYNAMICS

For the continuous, cyclical transformation of heat into work, within the confines of any limited portion of matter, four things are essential : —

    I. A source of heat;
    II. An absorbent of heat, or "refrigerator";
    III. A positive difference of temperature between the two;
    IV. An absorbent of work.

To illustrate from natural facts, taking first the simplest and most familiar form of useful cycle, that between the potential mechanical energy of water-power stored in a mill-pond and the kinetic mechanical energy of moving machinery, it is obvious that the four essentials here are : —

    I. The mill-pond;
    II. The tail-race;
    III. A difference of level, or "head," between the two;
    IV. The driven water-wheel.

Remove from the list any one of the four and it is obvious that the whole process must come more or less promptly to a stop.

Similarly, in thermodynamic work, the four essentials to the conversion of heat into work are already familiar to the student, although their exact identity and relations have probably never been pointed out to him. Taking the steam-boiler and steam-engine as an example, they are : —

    I. The furnace ;

    II. The "refrigerator," or condenser, which may be the atmosphere or may be a current of cold water ;

    III. A difference of temperature between the two ;

    IV. The steam-engine piston and the resisting machinery which is attached to it.[1]

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[1] It is of further and much wider interest and value to call attention to the application of the same law to transformations between other pairs of energy-forms from the list given on page 20. The study of electrical engineering gives rise to a large number of cycles between electrical energy upon one hand and thermal, chemical, or radiant energy on the other, in each of which the four essentials may be traced. The study of chemistry, and particularly of organic chemistry, is productive of a much greater number of cycles ; for there occur a much larger number of chemical compounds of different degrees of affinity than the number of different degrees of electrical potential which are commonly in use in the arts. Of all such instances by far the most interesting and in one way the most simple, although in other ways it is the most complex, is the cycle between animal and vegetable life. The broadest chemical basis upon which it rests is that of the oxidation of carbon and hydrogen, with the obverse dissociation. It also rests broadly upon the chemistry of the nitrates, and a myriad of other chemical reactions enter into the complete scheme; but none of them could go on without the oxidation which is the basis of the whole.

In this broad cycle it is the office of vegetable life to produce intensity of chemical affinity by the dissociation of the carbonic acid of the air into carbon and oxygen and of the water of the earth into hydrogen and oxygen, by the aid of sun-heat acting through the medium of the chlorophyl of the leaves. The oxygen is set free to the atmosphere and the carbon and hydrogen are stored in the plant in the form of various hydrocarbon compounds.

The Reversed Cycle. — If attention be returned to the rectangular cycles of Figs. 1 and 2, it will be clear that nothing in the discussion of them has limited their dimensions. Vertically or horizontally, their limits may be separated by as great or as small a distance as accidental circumstances may dictate. This being so, let it be imagined that the vertical sides of the rectangle be mutually approached. Carried to the limit this process will result in their coincidence. If the displacement of the sides be still further continued, it must result in the rising side being to the right of the falling side. In order that the horizontal processes may close the cycle they will have to be reversed in direction; in other words, the direction of rotation of the entire cycle will have altered from a clockwise to a counter-clockwise one, as in Fig. 5. This means that at the upper level what was formerly the source of energy has become an absorbent; what was originally the absorbent has become a source of supply.

If computation of the areas measuring the energy-quan-

It is the office of animal life to produce intensity of mechanical energy and of animal heat, to the destruction of the intensity of chemical affinity just referred to, by combining within its body the hydrogen and carbon taken in as food with the oxygen of the atmosphere. At the same time, carbonic acid is rejected into and is stored up by the atmosphere.

Thus the two forms of life work together, to the destruction by each of the form of energy which it consumes and to the production of the form which the other consumes. Neither, so far as we know, could exist without the other. By the balance between the two the equilibrium of earthly conditions is maintained. Energy, intensity, and life are alike conserved.

It is not the purpose of this work to develop farther the study of such cycles. It is all-important, however, that the student should see clearly that the laws which are under discussion in the present chapter are not narrow ones, applying only to certain mechanical devices, but are broad principles which cannot be broken nor varied by any accident of form or locality whatever. In this broad sense, heat-engines are like all other portions of the universe and all forms of life or activity are like heat-engines.

tities be made anew, it quickly becomes evident that the
energy supplied at the lower level is less than that rejected
at the upper level by the area of the cycle.   According to
the First Law, that of the Conservation of Energy, the
difference must be supplied from without.   A comparison
of the energy transformed on the rising side of the cycle
with that reversely transformed on the falling side shows
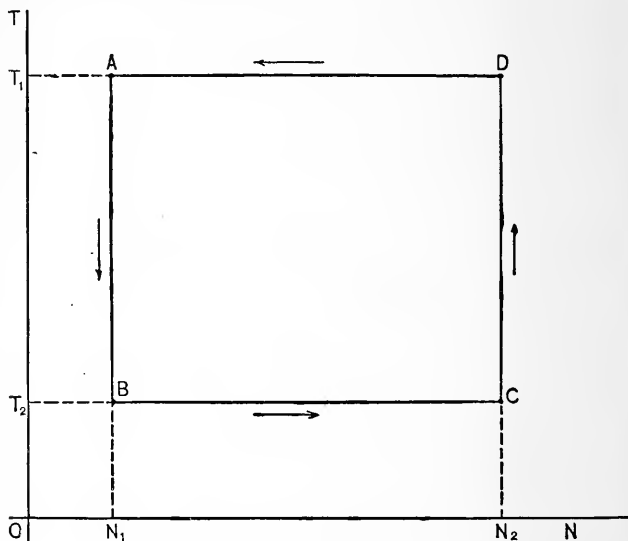
FIG. 5

that the former is now the greater, also by the area of the
cycle.   Plainly, there has been a net transformation of
energy from what was formerly the secondary form back
into the primary form.

Such a cycle is called a *reversed cycle*, and all of the
arguments and laws which were developed for the clock-
wise cycle now apply as rigidly to the reversed cycle if the
terms be properly interchanged to fit the case.   For in-
stance, referring to the Third Law of Thermodynamics,

page 53, it will be noticed that the word *positive* therein must now be changed to *negative.*

In reality, such a cycle should properly be called an "obverse" cycle; for it is merely the portrayal of the cycle of the secondary form of energy instead of the primary. For every "reversed" or counter-clockwise cycle of a secondary form of energy which takes place there must exist a corresponding "direct" or clockwise cycle on the part of a primary form to furnish it motive power. In short, —

A clockwise cycle always portrays the change taking place in a primary, a counter-clockwise in a secondary, form of energy.

This is a direct corollary of the Second Law of Energetics.

In thermodynamic work the reversed cycle is that embodied in the refrigerating or artificial-ice machine. Such a machine absorbs heat along the lower temperature-level *CB*, from the beef to be cooled or from the water to be frozen; it lifts this heat up-temperature against its will, up the adiabatic *BA*; it rejects it at the upper level *AD* into the absorbent circulating-water; the remnant of un-rejected heat at *D* falls down-temperature to *C*. This cycle requires mechanical power to maintain it in motion. Such power is usually derived from a steam-engine.

Efficiency of the Cycle. — It is to be noted, as of especial importance, that if the cycle be rectangular, as drawn in Figs. 1 and 2, the efficiency of conversion of energy from one form to another is given by the ratio between the temperature-range within the cycle and the temperature of the supply of heat. The energy supplied is measured by the area beneath *DA*, the curve of supply; the

energy transformed is the area of the cycle, or $DABC$. Therefore

$$F = \frac{DABC}{\text{area beneath } DA} = \frac{\text{fall in intensity utilized}}{\text{total intensity}}. \quad (12)$$

This gives, for weight-engines,

$$F = \frac{H_1 - H_2}{H_1}, \quad (13)$$

and for heat-engines, $$F = \frac{T_1 - T_2}{T_1}. \quad (14)$$

This equation is of fundamental importance, and permits us to deduce the broad law : —

In any rectangular cycle of energy-transformation the portion of the primary energy supplied which is converted into secondary is given by the proportion existing between the range of primary intensity during the cycle to the initial intensity of supply.

When the two forms of energy concerned in the rectangular cycle are heat and work respectively, the above principle becomes

The Fourth Law of Thermodynamics

In a heat-engine working on the rectangular cycle, the heat converted into work is to the heat supplied as the cyclical range in temperature is to the original absolute temperature of supply.

The Efficiency of the Reversed Cycle. — Approached mathematically, the quantities occurring in a reversed rectangular cycle similar to Figs. 1 and 2 must bear the same relation as they do therein.   It is not possible, however, to deduce therefrom an expression for its efficiency in the

usual sense, similar to equations (13) and (14), because the energy transformed is not now a portion of the energy supplied along the lower level. It may obviously bear any imaginable relation to the latter, according to conditions of relative levels of supply and rejection. If the cycle be rectangular the heat absorbed from the body to be chilled, or Q_2, must bear the same ratio to the energy to be supplied in lifting this heat to the T_1-level, or Q_w, as T_2 bears to $T_1 - T_2$. This ratio may obviously be anything whatever.[1]

Comparative Inefficiency of Cycles other than Rectangular. — Bringing into the argument the law proved on page 51, that the heat handled is measured by the area beneath the curve representing the heat-process, the Third Law will be found *not* to apply when the cycle is any other than rectangular. Thus, let $EFGHIKLM$, Fig. 6, be any rectilinear, but not rectangular, cycle. It may be considered as made up of a series of triangular departures from the circumscribing rectangle $DABC$. In such a cycle all of the heat must be supplied along the line $EFGH$; for only during this portion of the cycle is the entropy increasing. The abstraction of heat must occur only along the line $IKLM$, for here alone is the entropy decreasing. Therefore Q_1, the heat supplied, is measured

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[1] Herein appears the fallacy of a frequently recurring error in publications concerning artificial ice-making and refrigerating. Thus, it is often stated that, since it requires a certain number of B. t. u. to melt a certain quantity of ice into water at a given temperature, to make ice again from that same water would require, in a perfect refrigerating machine, this same number of B. t. u. But between these two quantities of heat: (1) The heat required to melt the ice, which when once in, must be lifted out again in order to freeze it; and (2) The energy required to do this lifting, there is no direct connection. If the heat had to be lifted to a high temperature, as might be the case in India, a perfect machine might require much more than this amount of heat. If it had not to be lifted far, as would be the case in Canada, a quite imperfect machine might require less energy to operate it.

by the area $dEFGHa$; Q_2, the heat abstracted, is meas-
ured by the area $aIKLMd$.   Hence the efficiency,

$$F = \frac{Q_1 - Q_2}{Q_2} = \frac{EFGHIKLM}{dEFGHa}.$$

Comparing the values for this equation with those for
the circumscribing rectangular cycle between the same
limits, $DABC$, it is plain that Q_1 has been diminished and
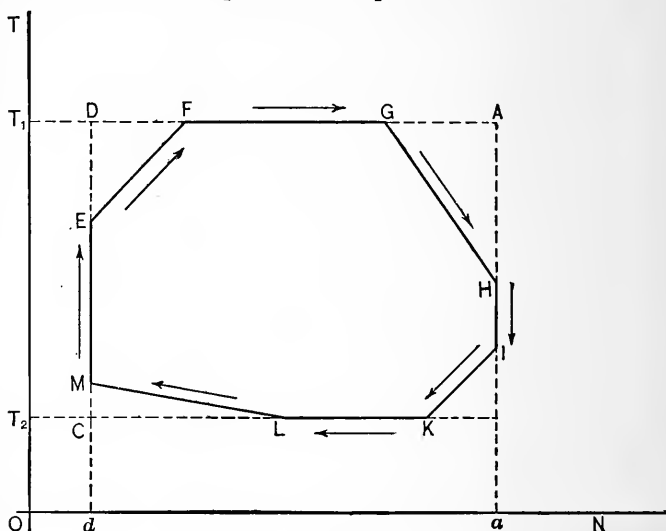
FIG. 6

Q_2 increased.   The effect of either of these changes alone,
or of both in combination, is to diminish the value of the
fraction $\frac{Q_1 - Q_2}{Q_1}$, which expresses the efficiency of the
rectangular cycle.

It is next important to note that: —

*Between any two temperature-limits the only possible ir-
regular clockwise cycle is one lying wholly between those
limits.*

Thus, in Fig. 6 it is quite possible to imagine the irregular clockwise cycle $EFGHIKLM$ operative between the limits T_1 and T_2; for the heat can readily fall into the carrier from DF as it passes along EF, and from GA as it passes along GH, or out of the carrier into BK as it passes along IK, etc. But if we attempt to imagine the cycle $EFGHIKLM$ of Fig. 7 operative in a clockwise
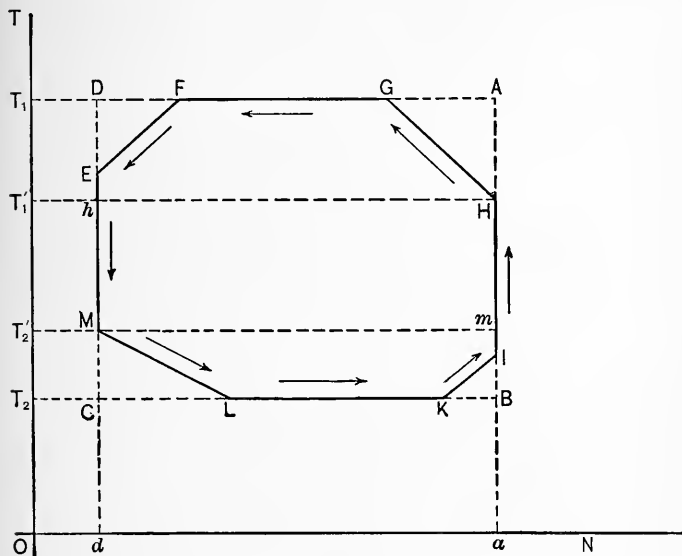
FIG. 7

direction between the limits T'_1 and T'_2 it becomes obvious that the thing is impossible. The carrier cannot pass along $hEFGH$ absorbing heat from hH; it cannot pass along $mIKLM$ rejecting heat into mM. The Second Law forbids.

The irregular counter-clockwise cycle $MLKIHGFE$ of Fig. 7 is quite possible, however, between the limits T'_1 and T'_2. The carrier, as it passes along $HGFE$, can easily

reject heat to Hh; as it passes along $MLKI$ it can absorb it from Mm.   Therefore,

Between any two temperature-limits the only possible counter-clockwise cycle is the rectangular one; any irregular one must vertically circumscribe that rectangle.

But in such circumscribing cycle the energy required to maintain operation is the area of the cycle.   The effective result is measured by the inscribed rectangle.   The efficiency of the former is plainly less than what it would have been had it been rectangular.

By assuming these triangular departures from the rectangle to become infinite in number, the proof can be extended, with the aid of the calculus, to cover a cycle of any non-rectangular form, such as $ABDC$, Fig. 4.   It therefore follows, from pages 60, 61, and 62, that —

Between any two given temperature-limits the rectangular cycle is the one of maximum efficiency.

This proposition may also be proven from general considerations.   If the temperature at which heat enters the engine be at any time lower than that of the heat-supply, the heat must fall into the engine unrestrictedly.   If at any time the temperature at which heat leaves the engine be above that of the refrigerator, or the source of cold, heat must fall out of the engine into the refrigerator.   But since the sole source of mechanical work lies in the fall of heat in temperature, it is obvious that any free or unrestricted fall of the heat down-temperature must represent a lost opportunity.   The efficiency of the process must therefore be less than when no such free fall takes place.   Such a lost opportunity is commonly called a "free fall" of heat.

Conditions for Efficiency. — It is said of water-wheels, and is proven by deduction from a line of argument exactly parallel to the above, and also by experiment, that the

attainment of the maximum possible efficiency depends upon —

    I. The admission of the water at the head-race level and without velocity or impact;

    II. The discharge of the water at tail-race level without velocity;

    III. The handling of the water between the two levels without leakage.

Similarly, it is truly said of heat-engines that the conditions of maximum possible efficiency are —

    I. The admission of all of the heat isothermally at the temperature of supply;

    II. The discharge of all of the heat isothermally at the temperature of the refrigerator; and

    III. The vertical or adiabatic transfer of the heat (*i.e.* without gaining or losing any heat on the way) from one level to the other.

A heat-engine which receives heat from the source of supply after having started down the path to the lower level (as, for instance, through the reheating coils supplying live-steam heat to an intermediate receiver) is like a water-wheel with a leaky flume which lets water dribble into the buckets when they are halfway down to the tail-race; the process is illustrated in *GH*, Fig. 6.[1] The evil of beginning to drop the temperature of the heat before it has all safely arrived in the engine, as by wire-drawing, belongs to the same class.

The line *IK* of Fig. 6 illustrates the loss due to drop-

[1] It is an incidental result of the use of reheater coils that another much larger loss is modified thereby, and therefore the advisability of their use remains an open question; but it is none the less true that the fall in temperature of the live-steam heat to the temperature of the receiver must ever be an irremediable loss, which may or may not be counterbalanced by its beneficial effect upon other inevitable free-fall losses incurred at other points in the process.

ping the heat out of the engine into the refrigerator before it has reached the lower limit by vertical, rectilinear fall, as in an engine working with incomplete expansion; it is like pouring the water out of water-wheel buckets before they have reached the level of the tail-race.

The line LM shows the evil of beginning to raise the buckets before they are completely emptied; it has no familiar parallel in the steam-engine.

The line EF shows how work is lost by pouring water into the buckets as they are rising; the corresponding loss in the steam-engine occurs in the heating of the feed-water in the boiler.

The rectangular cycle was first defined and its efficiency established by Carnot. It is therefore known as the Carnot Cycle. Because of its high theoretic efficiency it has been much sought after. In nearly all such search the limiting portion of the law of its efficiency, the opening words in the above statement, "Between any two temperature-limits," has been entirely lost sight of. The fact was overlooked that higher efficiency might be attained as well or better by extending the temperature-limits than by seeking the rectangular form within certain prescribed limits. For the rectangular Carnot cycle is difficult to reproduce in practice. In fact, it has always proven impracticable of incorporation into an actual working engine. While the task might be possible of solution, yet it is certain that the expense which it would involve would always more than offset any thermodynamic gain which would ensue.

Nevertheless, the study of the Carnot cycle is of prime importance in heat-engine design; for of all of the myriad of variations which may be played by the individual designer upon the dozen known heat-engine cycles, the thermodynamic value of each can be safely and accurately

measured only by its approach to or departure from the rectangular cycle of Carnot. And while the commercial value of a design may be quite another thing from its thermodynamic value, yet the latter must always enter into the question as a factor of prime importance whose estimation by guess is by no means easy or safe.

CHAPTER III

THE THERMAL PROPERTIES OF MATTER

According to the molecular theory, all matter is made up of a vast number of minute particles, or molecules, each of which possesses all of the chemical characteristics of the substance in question. As to the exact size of the individual molecule, there is, of course, much difference of opinion; but in view of their infinitesimal size in proportion to any dimensions which the eye can perceive, even with the aid of the microscope, the various estimates agree more closely than might be expected. One authority offers the simile that if a pea should be magnified to the diameter of the earth each of its component molecules would have attained the size of an orange; or in other words, that the average molecule possesses a diameter of about one-billionth of an inch.

The individual molecules of a body are supposed to be separated by distances very great in comparison with their diameter, and in gaseous matter these distances of separation have been likened to those of the solar system in comparison with planets composing it. At any rate, the component particles or molecules of a body are anything else than rigid, quiescent grains; they have more or less liberty to move actively about, and they are always doing so. Balanced in equilibrium between the mutually attractive forces, called collectively *cohesive*, and the separative forces due to the momentum of their own motion, like the earth between its gravitation toward the sun and its

centrifugal tendency to fly off its orbit on a tangent, they are pictured as being in constant vibration, back and forth, at all times and in all conditions of matter.

Of the form, construction, and attributes of the individual molecule little is known, even to the degree of safe conjecture. Since they must be in constant collision during their vibrations, and as we can conceive of impact only as a lack of elasticity which prevents complete restitution of mechanical energy when two bodies meet, we are forced to conceive the molecule as being absolutely and perfectly elastic; for if it were not so, some of its kinetic energy, when in collision, must be returned as heat instead of as motion. But as our only conception of heat is this same molecular motion, this is a *reductio ad absurdum*.

This does not necessarily imply, however, that they are simple units of matter, — homogeneous spheres, or anything of that sort. On the opposite, while knowing very little as to their absolute construction, we are forced to believe that it is very complex. One scientific authority makes use of the simile that "compared to a molecule a grand piano is simplicity itself." Other authorities conceive the molecule as being a thing as complex in structure and as divisible as an astral nebula. In short, since the entire range of astronomical science reveals no limit to the bigness of the universe, as compared with human standards, there is no basis for imagining any limit to the smallness of things. In either direction the infinite and the infinitesimal stretch into the immeasurable and the unimaginable.

Each known chemical form of matter is found in at least three distinct physical forms: solid, liquid, and gaseous.

In solids the component molecules are in some way so closely bound together, and their vibrations, though still taking place, are so narrowly and rigidly limited in relation

to one another, that the external form of the aggregation, the body taken as a whole, is capable of resisting the action of forces of considerable magnitude which may tend to deform it.

In liquids the molecules, though still held together by a mutual attraction which limits their distance of separation (in so far as internal molecular forces are concerned), they are now quite free to assume any position relatively to one another which they choose, or to roll about one another. This rolling constitutes what we call *flow*.

In the gaseous form the molecules are very much more widely separated than in liquids or solids, they are more actively in vibration, and they tend to separate to the widest possible degree permitted by external resistances, such as the walls of a containing vessel.

There is a condition of gaseous matter, when it is in contact with the same matter in liquid form, which is called vaporous; and the gradation from a vapor into a gas is always more or less gradual. But this does not constitute a proper basis for calling the vapor a distinct form of matter, although its characteristics may differ quite perceptibly from those of a true gas. In fact, all of the so-called "permanent" gases are nothing more than vapors very highly superheated above their boiling points.

A gas existing under so great an attenuation that each molecule travels an appreciable distance without collision with another is said to be radiant in form.

As defined in the tabular list, thermal energy, or heat, is the energy inherent in the motion of these individual molecules, yet integrated for the entire aggregation of molecules, so that we perceive it and speak of it as being the heat of the entire body. But the motion of each particle is continually and repeatedly piling itself up as a momentary

form of potential energy by being brought to rest against molecular resistances, instantly reconverting itself into molecular motion, as does the pendulum against gravitational forces; yet, since there is always a large majority of the molecules in motion (for the rest of each is but instantaneous), and since there is another distinct molecular energy which is purely potential, this vibration of the molecules will be called the kinetic form of thermal energy. It is the energy of periodic, if not continuous, motion, contrasted with the energy of a permanent condition of separation.

It has been demonstrated, by a line of experiment and argument which cannot be repeated here, that the phenomenon of temperature is a function of the translational velocity of this molecular vibration. Since all kinds of matter are solid in form at the lowest temperatures, become liquid when heated, and finally, when heated to a still higher temperature, become gaseous, and since the cohesion of a solid is much greater than that of a liquid, and that of a gas the least of all, it is easy to conceive of the melting of a solid as a breaking-down of the restricting cohesive forces by excessive velocity of vibration, and of the vaporization of a liquid as the same phenomenon carried to a further degree. The velocity of vibration of the molecules of the solid, as it is heated more and more nearly to its melting-point, may be imagined as putting a greater and greater strain upon the molecular, cohesive forces which bind the molecules together in a rigid connection, until finally those forces are first balanced and then overcome, and the molecules take up that freer activity which proclaims the substance a liquid. The vaporization of a liquid into a gas presents a similar series of conditions of equilibrium.

The situation is by no means so simple as has just been

portrayed, and the student must not expect this explanation to clarify all of the puzzling problems of physics. It is perfectly certain that the thermal motion of matter must include and combine several sorts and degrees of motion, in the so-called "different degrees of freedom," involving different portions of the mass of matter present in the molecule. But for the purpose of gaining a clear general conception of how energy is absorbed by water in its variations in temperature and its transformations into ice and steam, and of how such energy may be nothing more than mass in motion, the above will be found to be a material aid.

If heat be kinetic energy of molecular translation and temperature be a function of the velocity of the same, then it is obvious that the temperature of a body cannot be increased without the addition of energy from without. This is true. A body can be warmed by rubbing it, by pounding it, by compressing it, or by simply heating it. In either case the velocity and energy of its vibrations are increased by adding more motion from without. Moreover, a definite increase in velocity of such vibration ought to absorb a definite amount of energy from without; and so it does. This is so true that heat is accurately measured by means of the amount absorbed by some representative substance chosen as a standard, when raised through a definite change in temperature. The substance chosen is water to the amount of one pound avoirdupois, and the rise in temperature is defined as that from 59 to 60 degrees Fahrenheit. This amount of heat-energy is called the British thermal unit (ordinarily abbreviated as "B.t.u.") and is the basis of nearly all heat-measurements in English-speaking countries.

The Mechanical Equivalent of Heat. — As it was stated, as the primary law of thermodynamics, that there must

always be an exact equality between the amount of heat disappearing and of work appearing, or *vice versa*, whenever energy changed costume from one to the other, this equality must be capable of statement in terms of our chosen units of measurements. This is also true. The numerical statement is that

$$1 \text{ B.t.u.} = 778 \text{ foot-pounds.}$$

This ratio, of 1 to 778, is known as *The Mechanical Equivalent of Heat*, and forms the foundation-stone of modern thermodynamics. Long suspected by various physicists, it remained for the Englishman, **Joule**, to definitely prove the fact and determine the ratio. **Joule's** methods were crude as compared with those of to-day, and he obtained results which varied quite widely, as we now judge accuracy. But his results were so consistent, after endless repetition, that the equivalence was proven beyond a doubt and the law of the conservation of energy placed for the first time upon a sound foundation. The ratio 1 : 778 is therefore frequently referred to as "Joule's equivalent."[1]

Specific Heat and Latent Heat. — If the heat absorbed by one pound of water during a rise of one degree Fahrenheit be measured at different temperatures, it will be found to be not always quite the same. Yet it remains very close to unity, varying not one per cent between 60° F. and the boiling-point, and by barely five per cent through the entire range of temperatures common to steam-engine work. Its value at any temperature may be found by consulting the first column of the steam-table.

If one pound of ice at a temperature of 31° F. be heated, with the object of reducing it to water at 33° F., it will develop that it absorbs very much more heat than 2 B.t.u. Indeed, it will be apparent that it absorbs as much heat in

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[1] Grindley (*Trans. Phil. Soc.*, Vol. 194) prefers the value 774.

this slight change of temperature as it does in afterwards being heated up to a temperature of 175° F., or through a rise of 144 degrees. The explanation of the apparent inconsistency lies in the fact that in the first case the water has changed its physical state from ice to water. In fact, if the ice be merely melted, from ice at a temperature of 32° to water at a temperature of 32°, without any rise in temperature whatever, it will be found that it absorbs 143 B.t.u.

This quantity of heat is known as the *latent heat of fusion.* It is apparently absorbed in a separation of the molecules against the cohesive forces which bind them together, the kinetic energy added from without being converted into the stored motion of separation. The process cannot consist of a change of velocity of molecular vibration, for there is no change in temperature involved; the energy absorbed is not revealed by the thermometer. It consists of the attainment of a mean position of greater separation of each molecule, relatively to rest, which has been attained by motion against resistance, by separation against cohesion, and is therefore purely potential in its character. It is, in fact, the potential form of thermal energy.

This sort of heat the thermometer does not perceive. Neither do our nerves. We can feel that one body is hotter or colder than another, or hotter or colder than our fingers. But if ice were not harder to the touch than water, the human perceptions would not reveal the difference between ice at 32° F. and water at the same temperature. Yet the latter possesses 143 B.t.u. more than the former. The energy involved in a change of temperature in a substance is therefore known as its "temperature-heat" or "sensible heat" or "thermometer heat," while that involved in a change in its physical structure is known as its "latent heat" (of fusion or of vaporization as the case may be).

The first is kinetic in character; the second is potential. The first is perceptible by our senses; the second is entirely hidden from them.

So far reference has been made only to water. But the same description applies to nearly all forms of matter, except that the quantities of heat involved are quite different. Thus, if 100 pounds of cast-iron be put into a tub containing 100 pounds of water, the iron having a temperature of 140° F. and the water being at 50°, it will be found, when the two have come to the same temperature, that they are not at 95°, as might be expected, but at 60°. In other words, instead of the iron having fallen 45° in temperature while heating the water 45°, as would seem natural, the iron has fallen 80°, while the heat which it has lost has sufficed to raise the water only 10°. This shows that iron absorbs or gives out, during a given change of temperature, only about one-eighth as much heat as does water.

If the experiment were repeated with other substances than iron, a different result would be obtained in each case, showing that the amount of heat absorbed per degree change in temperature is different with each substance considered. This amount of heat, measured in B.t.u., is called the *specific heat* of the substance. A tabular list of the specific heats of various substances used in steam-engineering will be found in the Appendix.

If the amount of heat absorbed in the melting of one pound of iron, or the vaporization of one pound of alcohol, be similarly measured, the results will be different than with the melting of one pound of ice or the vaporization of one pound of water. It is therefore said that different substances possess different latent heats of fusion or vaporization, as the case may be, as well as different specific heats. A tabular list of these heat-quantities will also be found in the Appendix.

THE FORMATION OF STEAM UNDER CONSTANT PRESSURE

Pressure and Temperature. — If a pound of water be heated in an open vessel, it is found that the temperature rises continuously, as heat is absorbed, without perceptible alteration in its physical condition, until a temperature of about 212° F. is attained. At this point, for some invisible reason, the temperature refuses to rise further. Heat may be added indefinitely, yet the water becomes no hotter. Some result is visible, however, in the formation of vapor, and if the heating be continued long enough, it will be found that the water has quite disappeared, having been completely vaporized. Simultaneously there has occurred the absorption of a very large amount of heat.

Pressure and Temperature. — If, however, the water be placed in a closed vessel and a pressure of, say, 100 pounds per square inch be applied to its surface by means of an air-pump, it will be found that when the water reaches a temperature of 212° it does not vaporize. Instead, the temperature will continue to rise to a point between 337° and 338° before vaporization sets in. There is obviously some connection between the pressure upon the surface of the water and the temperature at which it will vaporize, and this connection is found to be a very rigid and exact one. The explanation is that when the molecules attain a velocity (or, in other words, a temperature) at which they would otherwise break their molecular bonds and take on a wider range of vibration, they find themselves opposed by other resistance than merely that of molecular cohesion; they are pressed together by the external pressure upon the entire mass. They therefore continue to vibrate within their prescribed limits until they attain such a higher velocity that they are able to overcome, not only the molecular forces, but the external ones as well.

The relation between the external pressure and the temperature at which boiling takes place is not a simple one, although it is rigid and exact.   It cannot be expressed accurately by any mathematical equation.   For the sake of both accuracy and convenience it is customary to refer to the columns of a steam-table for its determination.   The data found in the steam-table have been derived from experiments many times repeated.

Volume and Temperature. — If account be taken of the *volume* of steam produced during the evaporation of the water, it will be found in each case that a definite volume has always been developed by the time that one pound of water has been just entirely evaporated.   This volume is called the *specific volume of saturated steam*.   It, too, will be found to have different values under different conditions as to pressure and temperature ; but under the same conditions it is always the same.   This relation between pressure and specific volume, or between temperature and specific volume, is also impossible of exact mathematical statement, though in itself rigid and exact.   It is therefore customary, for the sake of accuracy as well as convenience, to refer to a steam-table for each determination.

Superheat and Saturation. — If the heating of the one pound of water in the closed vessel be continued after all of the water is evaporated, it will be found that the temperature again begins to rise ; and this time it will continue to rise as long as heat be added to it.   Just at the point where evaporation is complete and the final rise in temperature begins, the steam is known as *dry saturated* steam.   At any temperature above that it is known as *superheated* steam.   At any point between the beginning of boiling and complete saturation, when the original one pound is partly water and partly steam, the steam is known as *wet saturated* steam.   In other words, *steam in contact with water is*

*always saturated steam and must always have a definite
temperature and a definite volume when under a definite
pressure.* Steam separated from the water from which it
was derived may have any temperature whatever at or
above the boiling-point, and any volume whatever equal to
or greater than the specific volume.

If heat be added to saturated steam, it will become super-
heated; if heat be abstracted from it, it will condense. If
pressure be released from wet steam, more steam will be
formed; if the pressure upon it be increased, some will
condense.

Suppose that the one pound of water and steam should
originally consist of y pounds of steam and of $1 - y$ pounds
of water, and that the pressure be maintained constant; if
heat be added y will increase and the volume will increase
in proportion, but there will be no change in temperature;
if heat be abstracted, y will decrease and the volume will
decrease in proportion, but there will be no change in tem-
perature. It is therefore obvious that *so long as the pressure
remains constant* the volume can be increased only by the
addition of heat, vaporizing some of the water; the volume
can be decreased only by the abstraction of heat, condensing
some of the steam.

If the volume be maintained constant, the pressure can be
increased only by the addition of heat, which is absorbed
in two ways: (1) By the rise in temperature of the entire
mass; (2) By the vaporization of a portion of the water,
which fills the fixed volume with steam of greater density
and pressure. If the volume be maintained constant, the
pressure can be decreased only by the abstraction of heat,
with the consequent cooling of the entire mass and the
condensation of some of the steam.

In these phenomena the student will note a distinct
difference of action between these steam-and-water mix-

tures and the permanent gases which he has probably studied in his physics.

The total amount of thermal energy which is taken up by one pound of water in changing from water at 32° F. to dry saturated steam at any higher temperature obviously consists, according to the foregoing argument, of three portions, which are entirely distinct in their origin and characteristics, viz. : —

I. *The energy absorbed in increasing the temperature of the mass, or the velocity of vibration of the molecules.* It is kinetic, not potential, in its character; it is purely molecular, or internal, *i.e.* it is dependent upon merely the relative motion of one molecule to another, and is entirely independent of any circumstances or conditions external to the body. It is called "*vibration-heat,*" or "*temperature-heat,*" or "*sensible heat,*" or "*heat of the liquid.*" It is represented by the algebraic character (l), and is presented in the steam-table in the second column.

II. *The energy absorbed during vaporization in separating the molecules one from another against* **molecular** *forces.* It is potential, not kinetic, in character. It is purely molecular, or internal. It is called "*disgregation work,*" the term coming from the same Latin origin as the word *congregation;* the latter signifies the gathering together of a flock, the former the scattering of a flock (of molecules). It is given in the steam-table in the third column, and is represented by the letter D.

III. *The energy absorbed in separating the molecules, one from another, against the resistance due to any* **external** *pressure which may be forcing them together.* It is potential, not kinetic, in its character. It is largely external, *i.e.* it depends upon the amount of this external force and the distance to which it is forced back. Thus, in the ordinary vaporization of water under atmospheric pressure

it consists of the separation of a certain volume of air from the earth against gravitation, and depends in amount upon the atmospheric pressure prevailing at the given time and place. It is called the "*external work*." It is represented by the letter X, and is presented in the steam-table in the fourth column.

Internal Energy. — Upon examination it is to be seen that the first two of these three quantities of energy possess a distinct characteristic in common, viz.: they are both "purely molecular, or internal." They are therefore lumped together into a single heat-quantity which is called the "*internal energy*" or the "*internal work*" of the body. It is given the algebraic symbol I.

It is obvious, from the definitions just given, that the amount of this energy is independent of external circumstances. It depends solely upon the temperature, the physical state, and the peculiarities of the body in question. Thus, with steam, if there be present a certain quantity of steam at a given temperature and pressure, the amount of its internal energy can be known without any reference to the manner or the external circumstances of the steam's production, whether under constant pressure or under any other imaginable set of conditions. The total amount of heat-energy in the steam, or the amount needed to produce it from water of any given temperature, will vary with these external circumstances; but the amount of this heat-energy which is properly to be classed as *internal* energy would be the same in any case. The mathematical definition of this internal energy is given by the equation

$$I = l + D.$$

The Latent Heat of Vaporization. — It is further obvious that the last two of the three quantities of energy also possess a distinct characteristic in common, viz.: they are

both *potential* in form. They are therefore lumped together to constitute the potential portion of the heat-energy of the body under the name "*latent heat*," or "*latent heat of vaporization.*" Its mathematical definition is given by the equation

$$L = D + X,$$

where L is the latent heat of vaporization. Its values are presented in the steam-table in the fifth column.

Total Heat. — Finally, the "*total heat*" of the steam-mass, or the quantity of heat required to produce it from water at 32° F., is given by the equation

$$H = l + D + X = l + L = I + X.$$

The values of H will be found in the sixth column of the steam-table.

The relation between these several heat-quantities and the characteristics which connect them may best be seen by the following tabular exhibit : —

TOTAL HEAT, *or*
TOTAL THERMAL ENERGY.

KINETIC ENERGY.	POTENTIAL ENERGY, *or* Latent Heat of Vaporization.	
(I) Heat of the liquid = temperature-heat = kinetic molecular energy.	(II) Disgregation work = potential molecular energy of separation against molecular forces.	(III) External work = potential energy of molecular separation against external forces.
	Molecular Energy, *or* INTERNAL WORK.	EXTERNAL WORK.

TOTAL THERMAL ENERGY.

Having pointed out that vaporization, or its reversed process, condensation, takes place normally under fixed temperature and pressure, it is only necessary to add that

it need not always be present in full amount. The mathematical definitions given above apply to one pound of dry saturated steam; but of each pound of water on hand it often happens in practice that only a portion has been vaporized. The result is what is called *wet* saturated steam, or a steam-and-water mixture. The portion of the pound which is water, still unevaporated, is usually referred to as "percentage of moisture," and the remainder the "percentage of dryness." As the vaporization proceeds, from 0 % of steam to 100 % of steam, the water appears to take up the heat and become vapor, molecule by molecule; and each particle of water of equal weight, in becoming vapor, takes up an equal amount of heat and produces an equal volume of steam. It therefore follows that if the latent heat of vaporization of one pound of water be represented by L, and the volume produced by its complete vaporization be V, then, if only the fraction y of the pound be already evaporated, leaving $1 - y$ water, the latent heat of the steam-and-water mixture must be yL, and its volume yV. This statement holds true if the original volume of the pound of water be considered zero, and it is so small in comparison with the steam-volumes formed from it that this course seldom introduces a perceptible error.

Finally, if we represent all of the heat in the steam, or, more strictly speaking, all the heat required to convert it from water of a temperature of 32° F. into wet saturation at the given temperature or pressure, by H', and call it the "total heat" of the steam-and-water mixture, we have for wet steam the equation

$$H' = l + yL = l + yD + yX.$$

The Absolute Zero. — In all of these steam-table heat-quantities it must be remembered that an entirely arbitrary zero of heat and temperature has been assumed, viz.: the

melting-point of ice. But water at 32° F. does not contain zero heat. It contains a great deal of heat, as will quickly appear if one attempts to remove the heat in order to convert the water into ice. As noted above, 143 B.t.u. can be removed before it is frozen, and the ice is then no colder than 32° F. Moreover, this ice still has considerable heat, for this heat can be drawn out of the ice and proves itself to be heat. This process may be continued until the ice is at the temperature of 0° F. But even here it is obvious that more heat can still be abstracted from the ice; so that the Fahrenheit zero is plainly no real zero at all, but only a starting-point from which measurements can easily be made.

So the process of heat-abstraction may be continued, while the ice continually falls in temperature as it gives up its heat, until it has reached a temperature about 461° below the Fahrenheit zero. No one has ever cooled ice, or anything else, to this point; but it is known, by lines of physical experiment and argument which need not be produced here, that when this point were reached it would be found that the ice actually contained no more heat to be abstracted, its molecules would actually be at complete rest. This point is called the "Absolute Zero" of temperature, and the total heat of a body, in its exact sense, is always the amount of heat required to bring it from a temperature of absolute zero to its given condition. But for purposes of convenience it is much better to measure all heat-quantities as a surplus above 32° F. So the steam-tables are built on this plan. If the fact of the absolute zero be kept clearly in mind, there is no objection to be found.

The Isomorphic Curve. — The mathematical definition of entropy was given (page 39) as

$$dN = \frac{dQ}{T}. \tag{2}$$

G

Similarly, the mathematical definition of specific heat is

$$S = \frac{Q}{T' - T''} \text{ or } Q = S(T' - T''), \qquad (15)$$

where S is the specific heat of the body in question and Q the quantity of the heat involved in altering its temperature from T' to T''. When the alteration in temperature of the body and the heat concerned therein become less than any assignable quantity, this equation becomes

$$dQ = S\,dT. \qquad (16)$$

Equating equations (2) and (16) gives

$$dQ = T\,dN = S\,dT,$$

or

$$dN = S\frac{dT}{T}. \qquad (17)$$

Integrating,

$$N = S\int\frac{dT}{T} = S\log_e T, \qquad (18)$$

if S be a constant, as it is very approximately with most substances. Taking limits, this becomes

$$N - N_0 = S(\log_e T - \log_e T_0) = S\log_e\frac{T}{T_0}. \qquad (19)$$

This is the general fundamental equation between entropy and temperature, during changes: —

(1) When the specific heat of the body remains constant, and

(2) When the body undergoes no alteration in physical state, all of the heat being absorbed in alteration of temperature.

Should the physical state be altered by addition of heat, the process is isothermal, T becomes a constant, and the integration of Equation 2 gives

$$N = \frac{Q}{T}. \qquad (20)$$

If Equation 19 be plotted on an entropy-temperature diagram, as in Fig. 8, taking A as representing the initial condition of the heat, it must reveal a curve rising toward the right, as along AW; for if the body be heated without change in physical state, increase in temperature must mean addition of heat, and that again must mean increase of entropy. This curve AW might then be called the "curve of heating and cooling" and is frequently referred
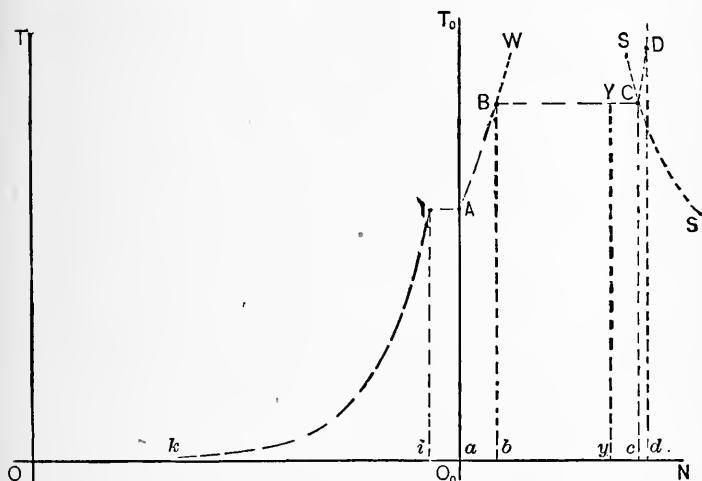
Fig. 8

to by that name; but as it is the curve on the diagram which represents heat-changes in a body only without alteration in physical state (that is, from solid to liquid, liquid to gaseous, etc.), a preferable name is the *isomorphic* line, by which name it will be referred to hereafter in this work. Its equation is always of the form of Equation 19:—

$$N - N_0 = S \log_e \frac{T}{T_0},$$

in which the zero-subscripts refer to the original condition of the body.

If the initial temperature of the heat-change be assumed to be zero, the value of N from Equation 19 becomes infinity for any finite final temperature whatever. The conclusions forced upon us by this fact are:—

(1) That all absolute values of entropy for ordinary temperatures are very large indeed in comparison with the changes in entropy-quantity accompanying all ordinary temperature-changes; and

(2) That Equation 19, which is based upon the supposition of constant specific heat, cannot hold true for those regions of temperature adjacent to the absolute zero which have not yet been explored as to specific heats, etc.

If the point O, Fig. 8, represent the heat-condition of one pound of ice at the temperature of absolute zero,—that is, having no heat, no temperature, and no entropy,—and if heat be imparted to it, the process will be represented by some such line as OkI. The left-hand portion of the curve is indeterminate; but without question the portion near to the temperature of 32° F. follows Equation 19, the value of S being the specific heat of ice. Having attained this temperature, at the point I, the further addition of heat no longer increases the temperature; instead, the ice melts at constant temperature, absorbing much heat in the form of latent heat. This isothermal process is represented by the straight horizontal line IA.

At the point A, therefore, the diagram represents the thermal conditions of melted ice at 32° F. This, for considerations purely of convenience, has been chosen as the arbitrary zero of heat for the construction of the steam-tables. Water at this temperature, or 492.8° F. absolute, is arbitrarily assumed to have no heat. In reality it has much.

Equations 17 to 20 everywhere make use of the temperature-factor as an absolute quantity and never as a relative one. The entropy-factor, on the contrary, always

appears as a relative quantity, its absolute value being
indeterminate. It is therefore quite allowable to measure
all values of N from a purely arbitrary zero, such as that
show by the axis O_0T_0 through the point A representing
melted ice. The zero-axis of temperature, however, must
always appear in its true, absolute position, as at O_0N.

If further addition of heat to the melted ice be made, the
process must again follow the isomorphic AW, and will
continue to do so until the temperature is able to overcome
the resisting pressure and cause evaporation. Should this
point be reached at B, the further addition of heat will
result in the isothermal evaporation of the water, under
constant pressure and temperature, along the line BC to C,
at which point it has all been evaporated into dry saturated
steam. Further addition of heat again causes rise in tem-
perature along the isomorph CD of superheated steam.
This curve will continue unmodified until the temperature
of dissociation is reached, — a point outside the province
of the present discussion.

Each of the curves KI, AB, and CD has the same equa-
tion, viz.: Equation 19; but in each the value of S is dif-
ferent, being in the several cases the specific heats of ice,
water, and steam respectively.

The temperature-level at which the evaporation BC
occurs depends upon the pressure upon the surface of
the water. As there may be an infinite number of such
pressures, differentiated by infinitesimal steps, so there
may be an infinite number of horizontal isotherms such
as BC, separated vertically by infinitesimal temperature-
steps. The infinite number of C-points found in such a
series would all lie in a locus such as SS. Each point in
this locus represents the thermal condition of one pound
of saturated steam of the given temperature. It is there-
fore known as the *Line of Saturation.*

The adiabatics, as Bb, Cc, the isotherms, as IA, BC, and the isomorphs, as AB, CD, all represent natural processes frequently met with in practice. THE SATURATION-CURVE DOES NOT REPRESENT A NATURAL PROCESS. The alteration of steam up-and-down temperature while remaining in a condition of dry saturation does not occur in nature, and could be artificially enforced only by the most delicate and unstable adjustment of heat-supply or heat-abstraction during the process. The locus of saturation-points is, however, a great convenience in geometric calculations.

It is obvious that any point to the right of SS must represent steam having both more entropy and more temperature than saturated steam of the same pressure; it therefore must represent **superheated steam**. Any point between SS and AE must represent a steam-and-water mixture; for it has more entropy than water and less than dry steam. Any point to the left of $kIAE$ must be meaningless (except in the strictly mathematical sense prescribed for all negative quantities); for it has less entropy than water or ice of the same temperature, which is impossible.

To this statement must be noted the unimportant exception that the exact temperature-level of the isotherm IA is somewhat variable, according to the pressure under which the melting or freezing takes place. But as a wide variation in pressure produces only a slight variation in temperature, only a very slight area above IA and to the right of kI produced suffices to cover all of the territory where a geometrical point might represent a physical actuality.

In the passage of the pound of steam-and-water mixture across the isothermal path BC, from all water at B to all dry steam at C, the vaporization takes place particle by particle, each molecule of water absorbing an equal amount of heat, an equal amount of entropy, and producing an equal

increment of volume. This fact, stated mathematically, gives the equation

$$y = \frac{yL}{L} = \frac{n}{N} = \frac{v}{V},\qquad (21)$$

where y is the fractional weight of steam vaporized, yL the latent heat absorbed therein, n the increase in entropy due to the vaporization of w, and v the volume of steam produced. The capital letters represent the specific quantities of the same things applying to the unit of weight. Thus, if the point Y, Fig. 8, represent the heat-condition of a pound of steam-and-water mixture, then the weight of steam present, the heat and the entropy absorbed in its vaporization, and the volume produced can all be known by equating the proportion $\dfrac{BY}{BC}$ to Equation 21; for in the latter all of the denominators can be found from the steam-tables.

If attention be returned to the tabulations of energy-quantities given on pages 77–79, it will be plain that many of them must be visible upon Fig. 8 in the graphical form of areas. Since the heat handled in any process is measurable by the area beneath the curve depicting the process and betweeen ordinates (see page 50), the heat absorbed in raising one pound of ice from absolute zero to the melting-point must be measured by the area beneath the curve OkI. That absorbed in the isothermal melting of the ice must be measured by the area of the rectangle $iIAa$ beneath IA.

This brings consideration to the zero of the steam-tables, at A; for all ice-phenomena lie below its lower limit. Starting from melted ice, the heat absorbed in the raising of the one pound of water from 32° to any known temperature, as at B, which is the heat of the liquid at that point, or the heat-quantity l, must be measured by the area $aABb$.

The latent heat absorbed in the isothermal vaporization of the one pound of water along BC must be measured by the area of the rectangle $bBCc$. There are, of course, an unlimited number of such rectangles between AW and SS, according to the temperature-level of BC. It is further obvious that Equation 21, applying to any given steam-and-water mixture, such as that at Y, shows that the amount of latent heat present in the mixture is measured by the area $bBYy$. This also follows from Equation 3 and from the argument on pages 50–51.

The heat absorbed in superheating the steam above the temperature of saturation, as to D, must be measured by the area $cCDd$.

The total heat, as given by the steam-tables, from water at the temperature of melting ice to either hot water, as at B, to partial evaporation, as at Y, to dry saturation, as at C, or to superheat, as at D, must be measured by the area between a and the point in question and beneath the curve $ABCD$.

In all of the foregoing illustrations and arguments it has been assumed that the substance undergoing thermal alteration was what is known chemically as H_2O. It is obvious that the diagram of Fig. 8 and the equations would apply equally correctly to any other substance which is known to pass successively through the stages of solid, liquid, and vaporous in a manner similar to water, provided the proper specific heats and other constants be substituted. Such diagrams can be applied usefully in the study of the action of ammonia, carbonic acid, ether, etc., in the refrigerating-machines, and of liquefied air, oxygen, nitrogen, hydrogen, etc., in the investigation of low temperatures. For the numerical values of the former series reference may be had to Peabody's "Steam Tables." Of the latter series very little is known empirically; nevertheless, the diagram

is exceedingly valuable in aiding the comprehension of the rather obscure phenomena occurring in the liquefaction of the so-called permanent gases.

PROBLEMS IN THE USE OF THE STEAM-TABLES

1. If a correct steam-gage reveal a pressure of 93.7 lbs. per square inch and the barometer stand at 29.37″, what is the absolute pressure within the gage? (One cubic inch of mercury weighs 0.491 lb.) *Ans.* 108.12 lbs.

2. What is the temperature of saturated steam of an absolute pressure of 172.3 lbs. per square inch? *Ans.* 369.15°.

3. If the temperature of a body of saturated steam be 148° F. and the barometer stand as in Problem 1, what would be a correct reading for a vacuum-gage connected with the same? *Ans.* 22.19″.

4. What is the "total heat" of the steam of Problem 2? *Ans.* 1193.93 B.t.u.

5. What would be the total heat per pound of this steam were it known to be 27% wet? *Ans.* 963.67 B.t.u.

6. How much heat does it require to alter one pound of water from a temperature of 62° F. to 187° F.? *Ans.* 125.5 B.t.u.

7. How much more heat is required to convert it into steam of a pressure of 84 lbs. absolute but 5% wet? *Ans.* 977.5 B.t.u.

8. How much more heat (after the process of Problem 7 is finished) is required to convert it into steam of the same pressure but having a temperature of 468° F.? *Ans.* 128.74 B.t.u.

9. How much external work is done by the heat added in Problem 6? *Ans.* Practically zero. It is so small a quantity that it may always be neglected in ordinary engineering calculations. If the water were heated under atmospheric pressure, it would in this case amount to about 1.07 foot-pounds; under other pressures it would be proportional thereto.

10. How much external work is done by the heat added in Problem 7? *Ans.* 59,320 foot-pounds.

11. What would be the final volume in the case of Problem 7? *Ans.* 4.927 cu. ft.

CHAPTER IV

THE STEAM-ENGINE CYCLE

LET Figs. 9 and 10 represent rectangular coördinate fields measuring temperature and entropy in the first case, and pressure and volume in the second case respectively. Let us trace on these fields the action occurring in an ideally perfect steam-engine.

It is first to be noted that consideration of mere *locality of occurrence* has nothing to do with thermodynamic phenomena. Thus, in the actual steam-boiler and engine the pressure is first supplied by a feed-pump, then the heat is added in the boiler, and finally the steam is removed to the engine-cylinder for further thermal action. In following the thermodynamics of these processes, however, it is much easier if they all be imagined as occurring within the engine-cylinder; that is, that there is first placed therein a pound of cold water to which heat is applied, first warming it and developing pressure, and finally engendering steam and volume. Then let it be imagined that the source of heat be removed while the piston takes its stroke; after which condensation is carried on by the application of cold within the cylinder instead of within a separate condenser.

In order that our diagrams may agree with the steam-tables, it is necessary to confine the attention to the heat-phenomena occurring in a single pound of water, and to assume that the initial condition of this water coincides with the arbitrary zero of the steam-tables, the temperature

of melting ice, or 32° F. At this temperature, or 492.8° absolute, the water is supposed to have zero heat, and therefore zero entropy. Its volume is 0.017 cu. ft.; but this is so small in comparison with the other volumes handled that it will not appear upon the diagram, and may be omitted from all calculations without appreciable error in engineering work. At this temperature, too, its pressure is too small for exhibition upon the diagram. Its condition is therefore properly represented by the point A in Fig. 9, and very closely by the point O in Fig. 10.

As heat is added, both temperature and entropy must increase along the isomorphic curve $A\,W$. The pressure will also increase markedly. The volume will increase slightly, but only by a very small percentage of the original volume, which is itself too small for visibility on the pressure-volume diagram. The process is therefore properly represented by the line OC of Fig. 10, or, with the greatest accuracy, by a line parallel and very close to it.

The point C represents, then, the thermodynamic condition of one pound of heated water under a retaining pressure which is equal to its own vapor-tension; this pressure is therefore sufficient to restrain the water from vaporization. In order to study the different processes which may develop from this point the following series of problems is presented.

The Cycle of the Boiler-explosion. — Let the point C represent the condition of any single pound in a ton of hot water contained in a steam-boiler, or in any other similar vessel. For the time it is permissible to neglect the presence of any steam above the surface of the water, if there be any. Let it be supposed that a rupture of the boiler-walls suddenly releases from the water the pressure which has been upon it. The decreased pressure permits the kinetic energy of the vibrating molecules of water to

expend their temperature in mutual separation. Vaporization and the performance of external work ensue.

Because of the instantaneous character of the process there is no time for heat-interchange with surrounding bodies. The process therefore takes place naturally and actually as the pure adiabatic which it tends to be. Such
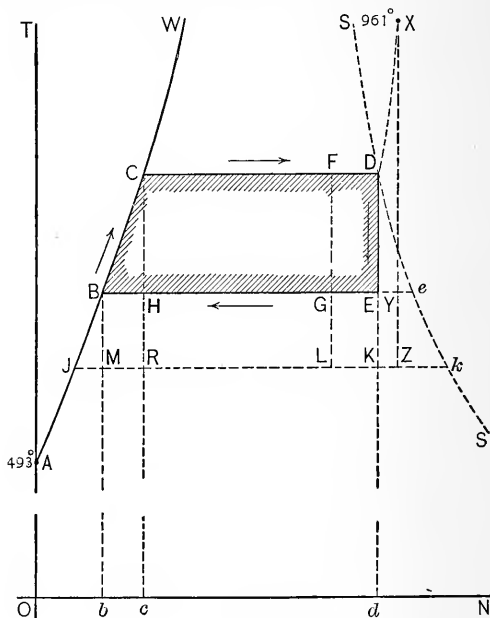
FIG. 9

an explosion is, in fact, one of the few actual instances of a purely adiabatic process known in practice.

Adiabatic fall from C, Fig. 9, develops the plumb-line CH. When H is reached, the kinetic energy of the molecules is in equilibrium with the restraining pressure, which is now the atmospheric pressure, and further fall becomes impossible.

The fact that H lies between the water-curve BC and

the saturation-curve SS shows that the one pound of H_2O is now a mixture of water and steam. The proportion of steam (by weight) present is given by the ratio of $BH \div Be$. The heat still present in the mixture must be that beneath the curve ABH. In other words, the heat which must be rejected if the water is to be returned to its original condition at A must be the area beneath the curve HBA. The heat which has been converted into
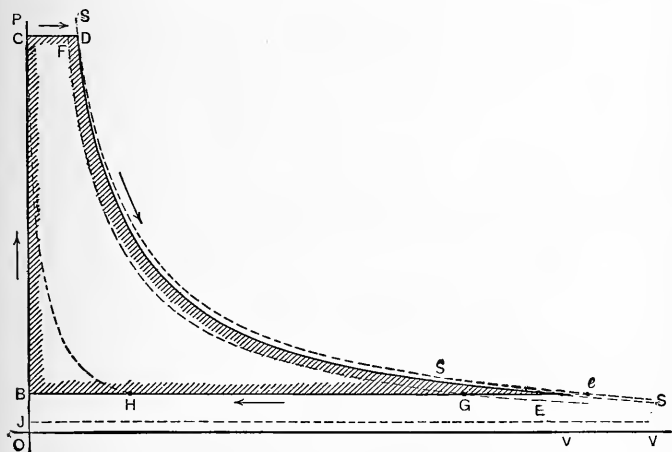
FIG. 10

work must be measured by the area between ABC and HBA. This area is the triangle BCH.

Problem 12. — If a boiler containing one ton of hot water under a gage-pressure of 100 pounds per square inch should explode, how much energy (in foot-pounds) would be released, and what volume of steam would be formed?

Let it be assumed that the barometric pressure is 29.50 inches of mercury. (In actual steam-work the barometer-reading must always be known.) This locates B at the level of 211.3° F., or 672.1° absolute. The pressure at C, being 100 pounds per square inch higher than this, is located

by the aid of the steam-table at the absolute temperature of 798.24°. The entropy of B is 0.3116. That of C is 0.4860. Therefore the entropy BH is 0.1744.

The heat beneath the isomorphic BC must be the heat of the liquid at C minus that at B, or 308.0 − 180.1 = 127.9 B.t.u. The heat beneath the isothermal HB must be the width of this rectangle $bBHc$ times its height, or 0.1744 times 672.1, which is 117.2 B.t.u. The area of the triangle is therefore the difference between these two heats, or 127.9 − 117.2, or 10.7 B.t.u.

This is the energy released per pound expressed in heat-units. The total energy released, expressed in foot-pounds, must be —

$$10.7 \times 2000 \times 778 = 16,649,200 \text{ foot-pounds.}$$

This is about equal to that developed by the firing of a modern 6-inch rifle.

The question as to the volume of steam developed calls for reference to Equation 21, page 87. In applying this equation the entropy of vaporization, Be, at the BH-level, appears from the steam-table to be 1.4372. The entropy BH is 0.1744. Therefore the proportion of steam present is $\frac{0.1744}{1.4372} = 0.1213$ of a pound per pound. The specific volume of vaporization at this same level is 26.98. Therefore the total volume of steam formed is —

$$0.1213 \times 26.98 \times 2000 = 6547 \text{ cubic feet.}$$

This explains why boiler-explosions are so completely destructive to the buildings enclosing them. The building explodes, as well as the boiler, from the immense volume of steam formed. The action is so instantaneous that no weakest point has time to relieve the rest of the structure by giving way first. Each brick is torn from its neighbor, and the building collapses into a heap of rubbish.

The Cycle of the Perfect Steam-engine. — Returning to the store of heated water with which the boiler-explosion began, at the point C, let it now be supposed that the boiler-walls did not give way, but that instead heat continued to be absorbed by the water. The temperature of the water being already in equilibrium with the pressure upon it, further advent of heat must develop isothermal vaporization. The entropy will increase along the isothermal CD (Fig. 9) and the volume will increase along the constant-pressure line CD (Fig. 10). At D vaporization is complete, and the steam is dry and saturated.

In the actual engine, at this stage of the process, the steam leaves the boiler and passes to the engine-cylinder, where D represents in the actual engine the point of cut-off. Here the steam finds itself isolated from all heat-interchange with the rest of the universe but free to impart its molecular energy to the piston in the shape of mechanical work, because the intensity of its kinetic energy exceeds the resisting pressure of the piston.

Finding therein an opportunity to drop in temperature, it promptly does so, falling down the adiabatic DE (Fig. 9) in the process. At E further fall becomes impossible. In the case of the non-condensing engine, the limiting factor is atmospheric pressure; in the case of the condensing engine it is sometimes the pressure and sometimes the temperature in the condenser; that is, either a poor air-pump or a warm condenser will limit the vacuum.

In order to leave the cylinder ready for a repetition of the process the steam in it must be condensed at this bottom temperature-level. The condensation therefore takes place under constant pressure, along the isothermals EB (Figs. 9 and 10), until it is complete at B. The pound of water is then ready for a repetition of the cycle $BCDE$.

By the application of Equation 21 to any point of DE and BH in the same way as was done to the point H of the boiler-explosion the curves DE and BH of Fig. 10 can be developed.   The cycle is then represented in the pressure-volume diagram by the same circuit $BCDEB$, in which CD and EB are isothermals and DE is an adiabatic.

This cycle of processes is called the *Cycle of the Perfect Steam-engine*, or the *Rankine Cycle*.

It will be noticed, first, that the adiabatic process DE, when represented in the pressure-volume diagram, results in a hyperbolic curve instead of a perpendicular drop.   This shows that the adiabatic process is a heat-process, not a steam-process or a volume-process; for the straight line of fall is the natural one.   Indeed, the pressure-volume effects of adiabatic drop upon the various vapors and gases are the most diverse; whereas when viewed simply as heat-processes these diverse adiabatics always appear as vertical plumbline drops.

In the second place, it should be noted that its effect upon the volume and pressure of steam-vapor is so complex as to defy exact mathematical expression.   The curve DE, Fig. 10, is hyperbolic in character, as are all those near it, and asymptotic to the two axes; but there is no known exact equation for it.   The only exact, and the most convenient, method of deriving it is the graphical one through the medium of the corresponding curve in the entropy-temperature diagram described above.

Problem 13.   What are conditions of operation and the efficiency of a perfect steam-engine operating between a boiler-pressure of 100 pounds per square inch by gage and exhausting into the atmosphere?   Represent its operation by means of entropy-temperature and pressure-volume diagrams.

The points B and C have already been located in con-

nection with Problem 12. The entropy of C appears from the tables to be 0.4860 and that of D, to be 1.5837. The entropy of E being the same as that of D and that of B being 0.3116, the entropy BE appears to be

$$1.5837 - 0.3116 = 1.2721.$$

The specific entropy Be at this level being 1.4372, the ratio of the two entropies gives the steam-weight present as 0.8851 pound. The specific volume of steam at this pressure being 26.98 cubic feet, the volume of the wet steam after expansion to D must be

$$0.8851 \times 26.98 = 23.88 \text{ cubic feet.}$$

If the cycles of Problems 12 and 13 as portrayed in Fig. 9 be compared, it appears that the latter is made up of the triangle of the former, BCH, and the rectangle $HCDE$. The heat supplied in Problem 13 is greater than that of Problem 12 by the rectangle beneath CD; the heat rejected is greater by the rectangle beneath HE; the heat converted into work is greater by the rectangle $HCDE$. The heat supplied is

$$Q_1 = l_1 - l_2 + L_1 = 1004.4 \text{ B.t.u.}$$

The heat rejected is the entropy $BE \; (= BH + CD)$ times the absolute temperature at E, or

$$Q_2 = (N_D - N_B)T_2 = 855.0 \text{ B.t.u.}$$

Therefore

$$Q_w = Q_1 - Q_2 = 1004.4 - 855.0 = 149.4 \text{ B.t.u.}$$

The efficiency of conversion of heat into work, or the thermodynamic efficiency, is

$$F = \frac{Q_1 - Q_2}{Q_1} = \frac{149.4}{1004.4} = 14.87\%.$$

H

If the area of the triangle which measures the heat converted into work in the boiler-explosion cycle be expressed analytically, there results

$$l_1 - l_2 - T_2(N_C - N_B) = l_1 - l_2 - T_2\, S \log^e \frac{T_1}{T_2}.$$

If the area of the rectangle $HCDE$ which is added to this in making up the Rankine cycle be expressed analytically, there results

$$(T_1 - T_2)(N_C - N_B) = (T_1 - T_2)\frac{L_1}{T_1}.$$

If these be added, there results

$$Q_w = l_1 - l_2 - T_2\, S \log^e \frac{T_1}{T_2} + L_1\left(\frac{T_1 - T_2}{T_1}\right). \qquad (22)$$

This is the algebraic expression for the utmost work attainable from dry steam between the limits T_1 and T_2. It is inexact in that S is not a constant over any wide range of temperature. The calculation of the graphical areas by means of the steam-tables is therefore to be preferred, for accuracy as well as for convenience. There may arise occasions, however, when the above expression is useful. With a proper choice of value for S its inaccuracy is not great.

The rectangle $HCDE$ must represent a Carnot cycle, of the maximum efficiency, given by the expression $\dfrac{T_1 - T_2}{T_1}$. The last term of Equation 22 is easily recognizable as arising from such a cycle, L_1 being the heat supplied along the isothermal CD and the parenthesis being the efficiency-coefficient by which it is reduced to work done. It will also be noted that the general contour and the efficiency of the Rankine cycle $BCDEB$ closely approach those of the Carnot cycle $HCDEH$. In Problem 13, 14.88% of the heat was converted into work. Had the Carnot cycle

been followed between the same temperature-limits, the portion of heat converted into work would have been

$$(798.3 - 672.1) \div 798.3 = 15.81\%.$$

The difference is not worth striving for. Here can be seen for the first time what will be frequently referred to again : That the inefficiency of the steam-engine is due neither to a poor cycle nor to an improper working-substance, but to narrow temperature-limits. For its cycle is a close approach to the ideally perfect rectangle.

Problem 14. If the engine of Problem 12 were fed with steam 20% wet, what would be the efficiency, etc.?

In this case the boiler-steam, instead of being completely evaporated to dryness at D, leaves the boiler with one-fifth of its weight still unevaporated water. Its condition is shown at the point F. The expansion now follows the adiabatic FG, and the complete cycle is $BCFGB$. At F its entropy is $0.4860 + (0.8 \times 1.0977) = 1.3642$; its volume is $0.8 \times 3.881 = 3.105$; its total heat is

$$308.05 + (0.8 \times 876.35) = 1009.1.$$

In this case

$$Q_1 = 1009.1 - l_2 = 1009.1 - 180.1 = 829.0 \text{ B.t.u.}$$

The entropy of vaporization at G is

$$1.3642 - 0.3116 = 1.0526;$$

the specific entropy of vaporization at this level being 1.4372, the steam-weight present at G is given by the ratio between these two, or

$$\frac{1.0526}{1.4372} = 0.7324. \quad 100 - 73.24 = 26.76.$$

The initial 20% of water has become 26.76% during expansion.

In this problem Q_2 is the area of the rectangle under

BG, or $965.9 \times 0.7324 = 707.42$.   Hence the work done is $829.0 - 707.42 = 121.58$ B.t.u., and the efficiency is 14.66%.

In actual practice the deleterious effect of wet steam is much greater than that shown by a comparison of these two problems.   The trouble is not that the water diminishes the theoretic possibilities for the development of work, but that its presence interferes with the realization of those possibilities, in a manner to be described in Part II.

Problem 15.   If the engine of Problem 13 were to exhaust into a condenser maintaining a vacuum of $25''$ of mercury instead of into the atmosphere, what would be the efficiency, etc.?

Here the lower temperature-limit to which adiabatic drop may take place has been lowered by the decreased back-pressure to $130°$ F., or $590.8°$ absolute.   This temperature-level is represented by the line JK (Fig. 9).   All of the quantities can be calculated as before.

Problem 16.   Find the results of exhausting the engine of Problem 14 into a vacuum of $25''$.

Problem 17.   If the engine of Problem 13 were to be fed with steam superheated to a temperature of $500°$ F., the pressures remaining the same, what would be its efficiency, etc.?

In Figs. 9 and 10 the point D represents steam just at the point of saturation, to be superheated only by additional heat.   The addition of this heat will increase both temperature and entropy according to the equation

$$N - N_D = S \log_e \frac{T}{T_D},$$

where the subscripts indicate the point D and the value of S is the specific heat of steam, or about 0.55.   This process is shown by the curve DX, X being the condition of the steam in the engine just before expansion begins.   The

line DX in the PV-diagram would be a straight horizontal extension of CD, for the superheat causes the volume to increase under constant pressure. Both DX and the expansion-curve XYZ are omitted from the PV-diagram, to avoid confusion, for the latter would be a hyperbolic curve lying close to and crossing SS at a very sharp angle. The efficiency and any volumes desired can be investigated and calculated with the help of the entropy-temperature diagram alone.[1]

When cut off from the boiler the steam-heat at X promptly drops in temperature down the adiabatic XY to Y, at which point no further drop is possible. The rest of the heat must be abstracted along YB. The increase in entropy due to DX is found from the above equation to be $EY =$ 0.1020. Taking the value of 1.2721 for BE from Problem 13, $BY = BE + EY = 1.2721 + 0.1020 = 1.3741$. Since $Be = 1.4372$, the steam-weight present at Y must be 0.9491 pound. The Q_1 of this problem must be greater than that of Problem 13 by the heat added along $DX = 0.55 (500 - 337.4) = 89.4$ B.t.u. $Q_1 = 1093.8$. The Q_2 must now be greater than that of Problem 13 by the rectangle under EY, or it may be determined directly as the area beneath BY; whence $Q_2 = 923.5$. Therefore Q_w, the work done, is $Q_1 - Q_2 = 1093.8 - 923.5 = 170.3$ B.t.u., and the efficiency of the cycle is $\dfrac{170.3}{1093.8} = 15.6\%$.

The beneficial effect of superheat upon the efficiency in the actual engine is much more than what is shown by comparison of this result with Problem 13. The reasons for it are the exact reverse of those processes which make

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[1] The close confusion of these several diverse curves in the PV-diagram, although they are entirely distinct in the NT-diagram, stands witness to the obscurity of the evidence as to what is thermally transpiring in the cylinder which is offered by the indicator-card.

wet steam so deleterious, and will be explained in the Second Part of this work.

Problem 18.   What would be the efficiency and conditions of operation of a direct-acting steam-pump working non-expansively between a boiler-pressure of 100 pounds by gage and the atmospheric pressure?

FIG. 11
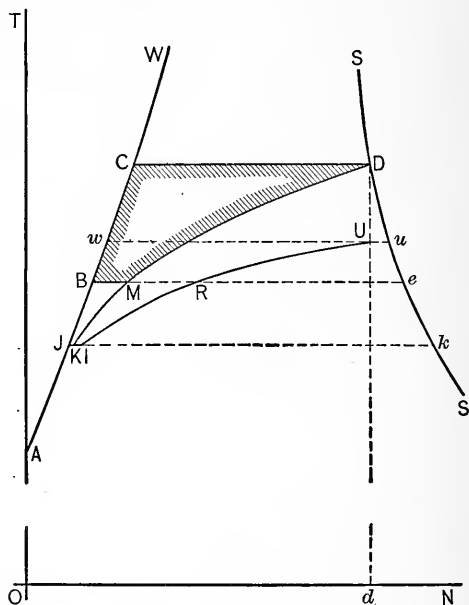
Again, the point D (Figs. 11 and 12) represents the condition of the steam-heat as the supply from the boiler is cut off in the pump-cylinder. But now, instead of being allowed to expand adiabatically, the heat is abstracted at constant volume while the piston stands still at the end of the stroke. In the actual pump the exhaust-valve is opened at the end of the stroke to a separate condenser, but thermodynamically the abstraction of heat by the condenser

may better be imagined as taking place entirely within the cylinder.

The heat is now not free to fall naturally down the adiabatic DU, but is diverted artificially by the limitations of the case into following the path DM, called the *constant-volume curve*, the heat meanwhile dripping from the various points on this curve down to the tail-race level Be by means of "free fall." The curve DM has no known exact
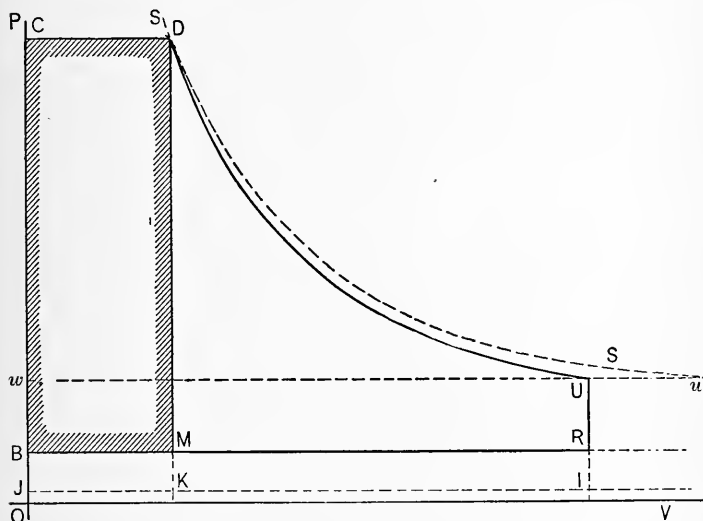
FIG. 12

equation in the NT-diagram; in the PV-diagram its equation is $V = a$ constant. But the NT-curve may be found by determining individual points by the help of Equation 21. In this case it is the volumes which are known and the other variables which are to be found. Thus, at M the volume is the same as at D, or 3.881 cubic feet. At this level the specific volume is 26.98 cubic feet. Therefore there must be present a steam-weight of $\dfrac{3.881}{26.98} = 0.1438$ pound. Its

entropy of vaporization is $0.1438 \times 1.4372 = 0.2067$. Its total entropy is 0.5182, which locates the point M in the NT-diagram. Intermediate points on the curve DM, Fig. 11, may be found similarly.

In this problem the Q_1 is the same as in Problem 13, or 1004.4 B.t.u. Q_2 cannot be found directly from the NT-diagram, but can be deduced from the value of Q_w given by the PV-diagram. The area of the rectangle $BCDM$, Fig. 12, is $100 \times 144 \times 3.881 = 55886$ foot-pounds.[1] Therefore the area of $BCDM$, Fig. 11, is $\dfrac{55886}{778} = 71.83$ B.t.u. This shows an efficiency of $\dfrac{71.83}{1004.4} = 7.1\%$.

Problem 19. What would be the efficiency, etc., were the pump of Problem 18 to exhaust into a vacuum of $25''$?

Here the cycle describes the path $JCDKJ$.

Problem 20. What would be the efficiency, etc., of the engine of Problem 13 if, instead of carrying its expansion completely to atmospheric pressure, it should have reached the end of its stroke by the time the steam had expanded to a gage-pressure of 20 pounds and the exhaust-valve had then been opened?

The cycle now describes the path $BCDURB$ (Figs. 11 and 12), the wu-level being at the temperature corresponding to saturation at 20 pounds by gage, or $258.2°$ F., or $719°$ absolute. The cycle may be divided into two distinct portions: (1) $wCDU$ and (2) $BwUR$. The first portion can

---

[1] In calculating quantities of work from volumes and pressures of fluids, whether the latter be steam, water, air, or other gases or liquids, it is engineering custom for volumes to be always measured in cubic feet and pressures in pounds per square inch. At the same time the resultant work is customarily desired stated in foot-pounds. A slight consideration of the question will make it plain that the only pressure which can be multiplied by volumes in cubic feet, with the resultant product in foot-pounds, is one measured in *pounds per square foot*. The factor of 144 must therefore be entered into all such equations to translate pounds per square inch into pounds per square foot.

be treated similarly to Problem 13; in fact, its Q_1 is the
same.   The second portion can be treated as Problem 18.
For the entire cycle the Q_w is obviously equal to the sum
of the Q_w's for the two portions.   The Q_2, if desired, may
be found from the Q_1 and the Q_w.

Problem 21.   What would be the efficiency of the engine
of Problem 20 if it were to exhaust into a vacuum of 25″
instead of into the atmosphere?

The cycle now describes the path *JCDURIJ.*

WIRE-DRAWING.   THE CURVE OF CONSTANT HEAT

Let A and B, Fig. 13, represent two steam-chambers or
pipes in which the pressure is maintained at P and p pounds
per square inch respectively, connected by a passageway of
relatively much smaller diameter, through which the steam
finds its way from A to B with comparative difficulty.   This
passageway may be a partially opened throttle-valve or
blow-off valve, or it may be a long line of pipe too small
for its task, or it may be the aperture in the diafram of a
Peabody throttling-calorimeter.   In any such case it is
only necessary, in order that the following discussion shall
apply, that (1) the pressure in B be less than in A, and
(2) that the velocities in A and B be small in comparison
with that in the passageway.

Under these conditions let Q, Fig. 14, represent the
heat-condition of a steam-and-water mixture present in
the chamber A.   The diagram shows it to be about 3%
wet; but this is only to illustrate a single application, that
of the throttling-calorimeter used with commercial steam.
It might be quite dry, it might be superheated, or it might
be all water; the argument would apply equally well.

As the steam leaves the chamber A, it finds itself under
a decreased pressure.   As pressure is all which keeps up

the temperature, its diminution allows the temperature to drop; it therefore starts down the adiabatic Qd. This process, however, even when started upon by only an infinitesimal degree, must develop mechanical energy in corresponding amount. There being no other form for this energy to take under the conditions, it takes the kinetic form; or, in other words, the elastic condition of the steam, due to its heat, sets up a flow toward the lower pressure.

The existence of flow immediately engenders friction, and friction is nothing more than a name for the transformation of kinetic energy into heat.   By this double
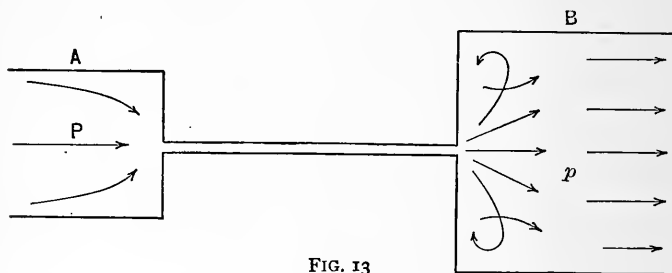
FIG. 13

energy-transformation, therefore, the heat-energy disappearing in the first infinitesimal drop in temperature, which would be measured by the area of the rectangle of infinitesimal height running from CC_1 out to Q, must immediately reappear as heat which is absorbed by the steam which developed it.   For no energy can be lost, either as heat or as work; all must be returned to the steam by friction, eddy, or impact.

The return of this heat, at the instantaneous pressure then prevailing, appears in the form of isothermal vaporization along the line C_1Q_1 to an infinitesimal amount and must be measured by the area under the process-curve. This curve is a horizontal line dN in length.   The area

beneath it is the rectangle of infinitesimal width $Q_1 d_1 d$, the area of which is $T_1 dN$.

By a series of such infinitesimal steps is developed the curve $Q_1 Q_2 Q_3 q$. The law which defines it is that the heat-energy apparent at any point must be equal to that at any other point, because equal to that originally present at the
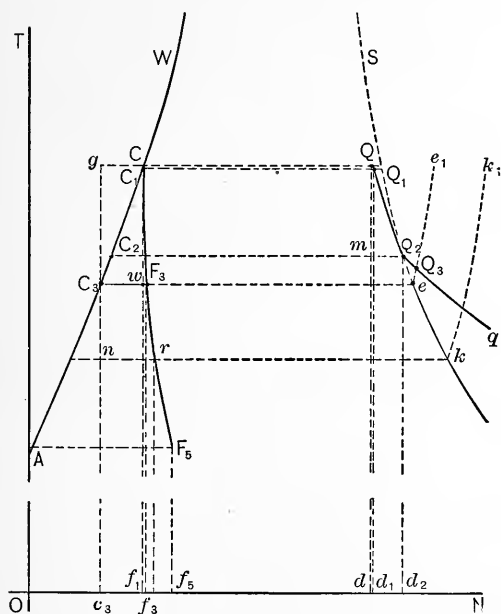
FIG. 14

point Q. This may be stated geometrically by saying that the area under the constant-pressure line to any point of the curve, such as the area beneath $A C_1 Q_1$, $A C_2 Q_2$, $A C_3 e Q_3$, $A C_4 k Q_4$, etc., must be equal to that at the original point; for the integration of the horizontal and vertical infinitesimal rectangles to equal limits must result in equal areas. $C_2 C_1 Q_1 Q_2$ is therefore equal to $Q_1 d_1 d_2 Q_2$. Subtracting the area $Q_1 m Q_2$ from each side and adding the

area, OAC_2md_1 gives $OAC_1Q_1d_1 = OAC_2Q_2d_2$, which is merely the graphical statement of the second sentence of this paragraph.

The heat-quantity of the point Q_1, which is also the heat-quantity of every other point in the curve Q_1Q_2q, is called the *characteristic* of the curve. There are of course as many constant-heat curves on the diagram as there are different quantities of heat imaginable.

It is important to deduce the equation of the constant-heat curve.

The simplest case is where the curve originates in the liquid-line AW, as at C_1, and develops the course $C_1F_3F_5$, etc. Let F_3 be any point and let the subscript 3 be omitted from the algebraic notation. From the nature of the curve, the area $c_3C_3C_1f_1$ must equal the area $c_3C_3F_3f_3$, or

$$S(T_1 - T) - TS \log_e \frac{T_1}{T} = T(N - N_1),$$

whence $\qquad N - N_1 = S\left(\frac{T_1}{T} - 1 - \log_e \frac{T_1}{T}\right).$

Differentiating, $\quad dN = S\frac{1}{T^2}(T - T_1)dT. \qquad (38)$

The area $f_1C_1F_3f_3$ is made up of a summation of elementary vertical strips each having the width dN and the height T. Therefore this area may be expressed mathematically as equal to $\int_{T_3}^{T_1} T dN$. Substituting the value just found for dN, integrating and reducing,

$$\text{area } f_1C_1F_3f_3 = S(T_1 - T_3) - T_1S \log_e \frac{T_1}{T_3}$$

$$= \text{area } c_3C_3C_1f_1 - \text{area } c_3gC_1f_1 = - \text{area } C_3gC_1. \quad (39)$$

(The algebraic sign, in thermodynamic work, indicates merely the direction in which the heat is travelling in relation to the form of energy portrayed by the diagram or

equation in question.   In final results, therefore, it may usually be neglected.)

Stated in general terms, this result just demonstrated becomes : —

When the energy involved in any liquid isomorphic drops in temperature by wire-drawing, the area beneath the constant-heat curve and between ordinates is equal to that between the original isomorphic, the abscissa through its upper end and the ordinate through its lower end.

The Peabody Calorimeter. — In this instrument, invented by Prof. Peabody, the chamber A of Fig. 13 consists of a steam-pipe, etc., from which a sample of steam is to be drawn for the purpose of determining its moisture and heat-value. The small passageway is usually represented by a small hole in a diafram, but it may be a slightly opened valve or a length of very small, well-lagged pipe without affecting the thermodynamics of the question.   The chamber B is made large enough to let the steam come practically to rest and is equipped with a thermometer-well and with every possible precaution against the radiation of heat.   It is best made freely open to the atmosphere, the pressure within it being determined by barometer ; but it may be closed from the air by a valve, if preferred, in which case a pressure-gage must be applied.   The only data necessary for an observation are (1) the pressure in A, (2) the pressure in B, and (3) the temperature in B immediately opposite the steam-entrance.   The first gives the level of the C_1Q_1-line of Fig. 14, the second the level of the C_3e-line, and the third the elevation of the point Q_3, on the line ee'.   Algebraically,

$$l_1 + yL_1 = H_3 + 0.55(t_a - t_3),$$

wherein the quantities in the first member apply to the observed pressure in A, and those of the second to the

barometric or other pressure in B. The unknown proportion of steam in the steam-and-water mixture at Q_1 is represented by y. The observed actual temperature in the chamber B is represented by t_a. The temperature of saturation corresponding to the observed pressure in B is t_3. In this equation y is the only unknown quantity.

Problem 22. Under a barometric pressure of 29.3″, steam is drawn from a pipe under a pressure of 93 pounds by gage and shows in the calorimeter a temperature of 228° F. What was its original percentage of moisture?

Problem 23. In the common marine practice of drawing steam from the boilers at a pressure of 300 pounds by gage and throttling at the engine to 250 pounds, how much of an original 2% of moisture in the steam as it left the boiler will reach the engine, radiation being neglected?

The Fireless Locomotive. — This type of street-railway motor was widely experimented upon about 1870–1880 in France, and somewhat also in this country. It consisted of wheels driven by an ordinary pair of locomotive engines and carrying a plain tank fit for an internal pressure of about 300 pounds to the square inch. At the start this tank was nearly filled with water. Into the water steam was blown from a high-pressure boiler until the water was brought to the saturation-temperature of the boiler and would condense no more steam. From the top of the tank, or from a dome, steam was drawn through a reducing-valve to supply the engines. As the draft of steam reduced the pressure in the tank, the heat of the water vaporized a portion of itself into steam, and thus the engines were driven, the temperature in the tank steadily dropping until its pressure was no more than sufficient to drive the engines, when it must be recharged. The fireless locomotive never reached practical success, by reason of its weight and of the early appearance of its much more

agile rival, the electric trolley-car. But it presents a none the less interesting problem in thermodynamics, which represents a wide class of actual situations arising wherever hot water is stored and drawn upon through a reducing-valve for a supply of steam.

Problem 24. If a fireless locomotive have a cylindrical tank 6 feet in diameter by 15 feet long, with spherical ends of 6-foot radius, filled with water heated to a pressure of 300 pounds by gage; if the pressure be reduced to 80 pounds by gage at the engine; if it requires 40 horse-power to move the train at 15 miles per hour and the general efficiency of cylinders and connecting gear be 50%; how far will the locomotive travel before recharging becomes necessary? (The exhaust is of course atmospheric.)

Referring to Fig. 14, let C_1 be the 300-pound temperature-level, let C_3 be the 80-pound temperature-level, and let C_4 be the atmospheric-pressure temperature-level. Then the heat in each pound of water when the locomotive is freshly charged will be measured by the area OAC_1f_1. When the locomotive has travelled until its tank-pressure has fallen to 80 pounds by gage, each pound of this water contains the heat OAC_3c_3. To be sure, much of the original weight of water has left the boiler during the trip; but as is always the rule, locality does not count. Wherever the water may be, in order to get out of the boiler it must have had at least the latter amount of heat per pound.

The difference between these two heat-quantities, or the area $c_3C_3C_1f_1$, is the energy which has left the tank through the reducing-valve during the trip.

Of this energy the portion C_3C_1w converts itself, by wire-drawing through the reducing-valve and the consequent isothermal vaporization, into the area $f_1wF_3f_3$. The total steam-heat which reaches the cylinders is, therefore, the area $c_3C_3F_3f_3$. Of this, only that above the C_4-level of

atmospheric exhaust is available for doing work. In other words, the heat which represents the work available for drawing the locomotive is the area nC_3F_3r.

From the steam-table, the temperature at C_3 is 784.3, and the heat of the liquid is 293.58. At C_1 the temperature is 882.45, and the heat of the liquid is 396.44. The area $c_3C_3C_1f_1 = 396.44 - 293.58 = 102.86$ B.t.u. This is also equal to the area $c_3C_3F_3f_3$. Of it only proportion $\dfrac{C_3n}{C_3c_3}$ is available for work, or

$$nC_3F_3r = \frac{784.3 - 672.1}{784.3} \cdot 102.86 = 14.714 \text{ B.t.u} = 11,447 \text{ ft.-lb.}$$

At the start the tank contains 23,680 pounds of hot water. The efficiency of transmission is 50%. To move the train 15 miles requires 40 horse-power for one hour, or 79,200,000 foot-pounds. Therefore the distance which the locomotive will travel, with the train attached, before its tank-pressure falls to 80 pounds by gage, is

$$\frac{23,680 \times 11,447 \times 0.50}{79,200,000} \cdot 15 = 25.6 \text{ miles.}$$

Problem 25. When the blow-off cock of a boiler operated at 80 pounds pressure by gage is opened, what proportion by weight of the issuing current is steam?

The foregoing illustrations cover, so far as the author is aware, the ultimate limiting theory of every problem which may possibly arise in any form or variety of steam-engine or of steam-apparatus.

CHAPTER V

THE LAWS OF THE PERMANENT GASES

ALL forms of matter occur in solid, liquid, or gaseous form, according to the pressure and temperature to which they are subjected. With most of them the change from liquid to gaseous is more or less gradual. As heat is added there is first evolved a form of matter called a *vapor*, of which ordinary steam is a good example. Upon further heating this is superheated and gradually becomes a perfect gas. The vapors obey many of the laws of the gases and are distinctly similar to them in their characteristics; but in some points they are quite different, notably in their action under heat. The gases, on the other hand, are distinguished from the vapors by no hard and fast line. They are merely highly superheated vapors.

It is to be noted most emphatically, however, that the laws controlling the mixtures of liquids and their saturated vapors bear a strong contrast to those of the highly superheated vapors called the "permanent gases." This contrast or distinction, although developed by a differentiation which is gradual, is broad and wide. It must never be lost sight of in thermodynamic practice. In the preceding chapter attention was centred upon the laws governing the effects of heat upon liquids, saturated vapors, and mixtures of liquid and vapor; in this chapter are discussed the quite different laws governing the action of heat upon the permanent gases. The laws of the heat itself, underlying both, are invariable.

BOYLE'S LAW. — *When the temperature of a gas remains constant, the pressure must vary inversely with the volume.*

Expressed mathematically, this reads : —

$$\frac{P}{P_0} = \frac{V_0}{V}. \tag{23}$$

CHARLES'S LAW. — *When the volume remains constant, the pressure must vary directly with the absolute temperature.*

Expressed mathematically,

$$\frac{P}{P_0} = \frac{T}{T_0}, \tag{24}$$

where the zero subscript refers to the original temperature.

Suppose the original condition of a gas to be the pressure P', the temperature T', and the volume V'. Suppose that, while the temperature is kept constant at T', the volume is changed to V''. The new pressure, p, must equal $\dfrac{P' V'}{V''}$. Suppose, again, that, the volume remaining constant at V'', the temperature be altered to T''. The latest pressure, P'', must equal $p\dfrac{T''}{T'}$, whence

$$P'' = \frac{P' V' T''}{V'' T'}, \text{ or } \frac{P' V'}{T'} = \frac{P'' V''}{T''} = \text{a constant.} \tag{25}$$

From this, if the pressure be constant,

$$\frac{V}{V_0} = \frac{T}{T_0}. \tag{26}$$

The nature of the constant of Equation 25 can be seen from a consideration of the origin of the idea of the absolute zero of temperature ; viz., that, since gas at 32° F. increases $\frac{1}{493}$ of its volume for each degree of alteration in temperature, a reduction in temperature of 493° must result in a reduction of the volume to zero, beyond which there could be no further abstraction of heat. Any volume of

gas may therefore be imagined as having been derived from heating a zero volume from absolute zero of temperature to the given one. During this process the external work performed must be equal to PV. The external work done per degree change in temperature must be equal to $\dfrac{PV}{T}$. But this is the constant of Equation 25. Therefore : —

The relation of pressure, volume, and temperature of a given mass of any gas at any instant is given by the equation

$$\frac{PV}{T} = C, \tag{27}$$

in which the value of the constant is the **number of foot-pounds of external work done during an alteration of temperature of one degree under constant pressure.**

The volume of one pound of air at 32° F. under a pressure of 14.69 pounds per square inch is 12.39 cubic feet.

$$\frac{12.39 \times 14.69 \times 144}{492.8} = 53.18 \text{ foot-pounds.}$$

Therefore it always holds true for one pound of air that its pressure in pounds per square inch times its volume in cubic feet is 53.18 times its absolute temperature in Fahrenheit degrees, or for air,

$$PV = 53.18\ T. \tag{28}$$

The specific volume of air is usually stated in the handbooks as a certain volume at a certain temperature under a certain pressure. This set of three figures it is impossible to retain in the memory. The single figure, 53.18, however, can easily be so retained. From it the volume of a known weight of air under any given conditions can be calculated, and more readily even than from the data given in the hand-books.

A slight consideration of the method of derivation of the significance and the value of this constant will show that for other gases than air it must be inversely proportional to the density of the gas in question. From a table of specific gravities and the figure 53.18, therefore, can be readily calculated the volume of a known weight of *any* gas under any given conditions.

Specific Heats of Gases. — When gas is heated, one pound of it does not always absorb the same amount of heat per degree rise in temperature, independently of conditions, as is approximately the case with water, etc. If the gas be heated while the volume is kept constant, it is obvious that the only energy absorbed is that required to impart to the vibrating molecules the increased velocity which expresses itself to our senses as an increase in temperature. On the other hand, if the same gas be heated to a like degree under constant pressure, not only must this same energy be supplied in order to increase the velocity of its molecular vibration, but also its volume will increase and external work must be done in overcoming the external pressure through this increase in volume. The heat absorbed in increasing the internal, vibratory energy of the molecules is the same in either case, otherwise the temperature-increase would not be the same. The external work of the latter case is quite additional to this and is peculiar to the condition of pressure kept constant.

If S_v and S_p be the specific heats under constant volume and constant pressure respectively, then, since the constant C of Equation 27 is the external work done in one degree rise in temperature under constant pressure,

$$S_p = S_v + \frac{C}{778}, \qquad \cdot \quad (29)$$

(since the specific heats are measured in B.t.u., and the C in foot-pounds).

If K be the ratio between the two specific heats, then

$$K = \frac{S_p}{S_v} \qquad (30)$$

and

$$S_v = \frac{C}{778(K-1)}. \qquad (31)$$

A table of the values of S_v, S_p, and K for the gases most commonly met with in engineering problems will be found in the Appendix. It will be noticed that they all lie fairly closely together, with the exception of hydrogen; and this gas, in spite of having a specific heat some fourteen times as great as the others, has a value of K very similar to the rest.

Expansion-curves, *PV*-diagram. — When a gas expands without special means being taken to maintain either pressure or temperature constant, its pressure and volume change according to some law which usually can be expressed by the equation $PV^x = $ a constant, or

$$\frac{P_1}{P_2} = \left(\frac{V_2}{V_1}\right)^x, \qquad (32)$$

the latter usually being the more convenient form for use in calculation. When air or similar gases expand or are compressed in the cylinder of a motor or compressor, x may assume any value between unity and about 1.40.

When a gas expands in this way, the external work done is that involved in pushing back the resisting pressure P through the distance represented by the increase in volume. But P not being constant, the amount of external work done is not directly observable.

Let the curve AB of Fig. 15 represent the pressure-volume changes of a gas expanding or being compressed according to Equation 32. The external work done during an infinitesimal amount of expansion, at any point, as

c, must be the area of the elementary rectangle cv, or PdV. The total external work done in expanding between A and B must be the integration of this expression, or the area $V_2 BA V_1$, or

$$W = \int_{V_2}^{V_1} PdV = P_2 V_2^x \int_{V_2}^{V_1} \frac{dV}{V^x} = P_2 V_2^x \cdot \frac{V_1^{1-x} - V_2^{1-x}}{1-x}$$

$$= \frac{P_2 V_2 - P_1 V_1}{x-1} = \frac{C(T_2 - T_1)}{x-1}. \qquad (33)$$

This is the external work actually performed in a mere change of state from the condition $P_1 V_1$ to the condition $P_2 V_2$.
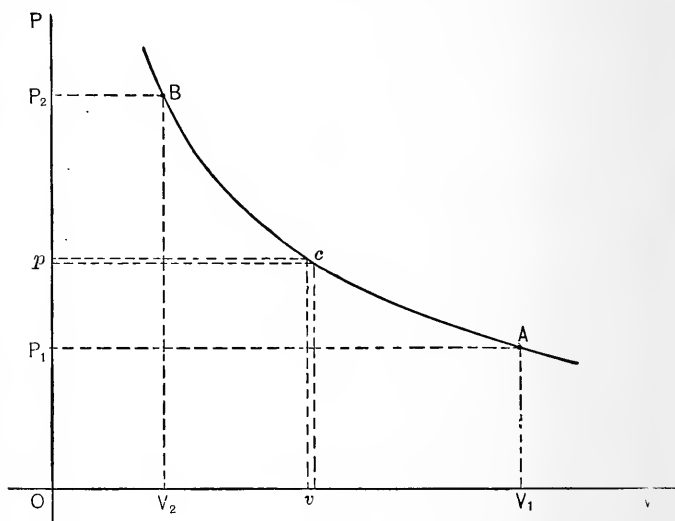
FIG. 15

In the actual expansion or compression of air or gas in the cylinder of a motor or compressor, however, each revolution of the machine involves more external work than merely that of the alteration in pressure and volume;

there is external work involved in first drawing the gas into the cylinder before the transforming process begins, and again in afterwards discharging it from the cylinder. These quantities are in addition to that discussed above. Such a triple process is shown in Fig. 15 by the area P_2BAP_1. In this the rectangle P_2BV_2 is the external work of taking the gas in (if the machine be a motor), and P_1AV_1 is that of discharging it. Of these two quantities, the former always has the same sign as the quantity of Equation 33; the latter always has the opposite sign. Adding these two algebraically to Equation 33, or, what is the same thing, integrating the horizontal elementary rectangle pc, whose area is Vdp, to the limits P_1 and P_2, there results, as the work done between abscissæ,

$$W_a = \frac{P_2 V_2 - P_1 V_1}{\dfrac{x-1}{x}} = \frac{C(T_2 - T_1)}{\dfrac{x-1}{x}}. \tag{34}$$

Equation 33 gives the true measure of transformation of heat into work during the expansion, or of work into heat during the compression, of a permanent gas; it includes no other process than the mere *alteration of condition* of the gas. Equation 34 gives the measure of the work actually given out by a motor, or absorbed by a compressor, in handling a perfect gas; it includes, in addition to the former process, the *displacement of the gas* from a locality of initial to one of final pressure.

The alteration in temperature resulting from compression from A to B, or from expansion from B to A, can be known by combining Equations 27 and 32.

$$\frac{T_1}{T_2} = \frac{P_1 V_1}{P_2 V_2} = \frac{V_2^x V_1}{V_2 V_1^x} = \left(\frac{V_2}{V_1}\right)^{x-1} = \left(\frac{P_1}{P_2}\right)^{\frac{x-1}{x}}. \tag{35}$$

It will be of mnemonic aid to note that the *denominator* of the expressions for the external work are $x - 1$ when

referred to the *volume*-axis and $\frac{x-1}{x}$ when referred to the *pressure*-axis; and similarly, that the *exponent* of the expression for the alteration in temperature is $x-1$ when referred to *volumes* and is $\frac{x-1}{x}$ when referred to *pressures*.

In making arithmetical calculations of the quantities handled by these equations it will be found a great time-saver to keep them always in the form given in Equations 32 and 35; that is, in the form of ratios of initial and final conditions. In doing this, if the subscripts be chosen so as to make the value of the fraction greater than unity, the logarithms will always be positive and so lend themselves to the raising to odd powers with much greater convenience than if otherwise.

Equations 32 and 35 might be stated in the form;

$$\frac{T_1}{T_2} = \left(\frac{P_1}{P_2}\right)^{\frac{x-1}{x}}, \quad \frac{V_2}{V_1} = \left(\frac{P_1}{P_2}\right)^{\frac{1}{x}}, \text{ and } \frac{P_1}{P_2} = \left(\frac{P_1}{P_2}\right)^{1};$$

or Equation 25 might be stated in the form,

$$\frac{P_1}{P_2} \div \frac{V_2}{V_1} = \frac{T_1}{T_2}.$$

From either may be deduced the very convenient rule that *the logarithm of the temperature-ratio involved in a given amount of compression or expansion is always equal to the difference between the logarithm of the pressure-ratio and that of the volume-ratio*, **when such ratios are all kept greater than unity.**

Problem 26. An air-compressor handles 100 cubic feet per minute of atmospheric air at a temperature of 69° F., compressing it to 60 pounds gage-pressure according to the equation: $PV^{1.3} =$ a constant. What is the final volume and temperature of the air and how much power is absorbed?

$P_1 = 14.5$ $\qquad\qquad$ $\log P_1 = 1.161368$

$P_2 = 74.5$ $\qquad\qquad$ $\log P_2 = 1.872156$

Subtracting, $\qquad\qquad\qquad$ $0.710788 = \log \dfrac{P_2}{P_1}$

Dividing by 1.3, $\qquad$ $\log \dfrac{V_1}{V_2} = 0.546760$

Subtracting, $\qquad\qquad\qquad$ $0.164028 = \log \dfrac{T_2}{T_1}$

$T_1 = 66 + 491 = 530$ $\quad$ $\log T_1 = 2.724276$

Adding, $\qquad\qquad\qquad\qquad$ $2.888304 = \log T_2$

$\qquad T_2 = 773.2$ absolute. $\qquad\qquad$ $t_2 = 312.2$ F.

$V_1 = 100$ $\qquad\qquad$ $\log V_1 = 2.000000$

$\qquad\qquad\qquad$ $\log \dfrac{V_1}{V_2} = 0.546760$

$\qquad\qquad\qquad\qquad\qquad$ $1.453240 = \log V_2$

$\qquad\qquad$ $V_2 = 28.39$ cubic feet.

The Work Absorbed $= \dfrac{P_2 V_2 - P_1 V_1}{\dfrac{x-1}{x}}$

$\qquad\qquad = \dfrac{[(74.5 \times 28.39) - (14.5 \times 100)]144}{0.3 \div 1.3}$

$\qquad\qquad = 446{,}000$ foot-pounds (per minute)

$\qquad\qquad = 13.5$ horse-power.

This sample calculation includes every digit essential to the solution of the problem, except that the arithmetical computation of the last equation (for the larger numbers in which logarithms are already noted) is omitted.

Adiabatic Expansion of Gases in the PV-diagram. — In adiabatic expansion the external work done must be at the expense of internal energy; there is no other source whence it may come. In the perfect gases the internal energy is all of the temperature or molecular-velocity form, and its

specific amount per degree alteration in temperature is the specific heat at constant volume, S_v. Whence, if the gas be expanded from a temperature T_1 to T_2, the internal energy lost or gained must be

$$S_v(T_1 - T_2) = \frac{C(T_1 - T_2)}{778(K - 1)}.$$

But, from Equation 33, the external work done during expansion between those same temperatures is, if expressed in B.t.u.,

$$\frac{C(T_1 - T_2)}{778(x - 1)}.$$

If the expansion be adiabatic, these two quantities must be equal, whence $x = K$. Therefore, *the P V-equation for the adiabatic expansion of a perfect gas is*

$$\frac{P_1}{P_2} = \left(\frac{V_2}{V_1}\right)^K. \tag{36}$$

Isothermal Expansion of Gases in the *PV*-diagram. — If the expansion be isothermal instead of adiabatic, T becomes a constant and Boyle's law prevails. The equation of the curve AB, Fig. 15, becomes $P V = $ a constant, and an analysis similar to that by which Equation 33 was derived gives as the external work done during the expansion or compression

$$W = P_1 V_1 \log_e \frac{P_1}{P_2} = P_2 V_2 \log_e \frac{V_2}{V_1}. \tag{37}$$

The work done between horizontals, representing the case of the actual motor-cylinder, is in this case the same as that between verticals.

Regnault's Law. — It was announced by Regnault that *the specific heat of gases, taken under constant pressure, is the same for all temperatures.* This is so nearly true that it will be accepted as such for all engineering pur-

poses.   But the fact that it is not quite true shows it to be a statement of coincidence rather than a law.

Joule's Law. — It was announced by Joule that *when a gas is allowed to expand without doing work its temperature remains constant.* This experiment was tried by placing two vessels containing air at different pressures in a tub of water until all temperatures were alike. Communication was then opened between the two. The gas in the vessel under heavier pressure expanded into the other; but no work was done, the system being considered a unit. The water revealed no alteration of temperature in the air.

The explanation of the result is that all of the internal energy of the gas consists of kinetic molecular energy, or sensible heat, the disgregation work being zero because of the wide separation of the molecules of a gas one from another and their consequent small attraction for one another. But just because their mutual attraction, according to the law of gravitation, can never be quite reduced to zero by separation, though easily made imperceptible, it is obvious that the disgregation work even of a gas must be some positive quantity; and if so it must be supplied during expansion, no matter whether external work be done or not, at the expense of the temperature-heat. Some change in temperature must result. This was proven by **Lord Kelvin** in some very delicate experiments upon the flow of gas through a porous plug. The fact has become of importance in the industrial arts within a very few years by the invention of the modern method for the production of liquid air, which depends entirely upon the absorption of temperature-heat in disgregation work when air expands without doing work.

That this matter may be given further consideration, let attention be turned to Fig. 14, which is reproduced here from page 107. As the curve Q_2q departs farther and farther from

the saturation-curve SS it will plainly become more and more nearly horizontal and straight. It must be asymptotic to ON. At a great distance away from SS it must represent the condition of a very highly superheated vapor at a very low pressure. With steam, which Fig. 14 has been drawn to represent, these conditions would be so
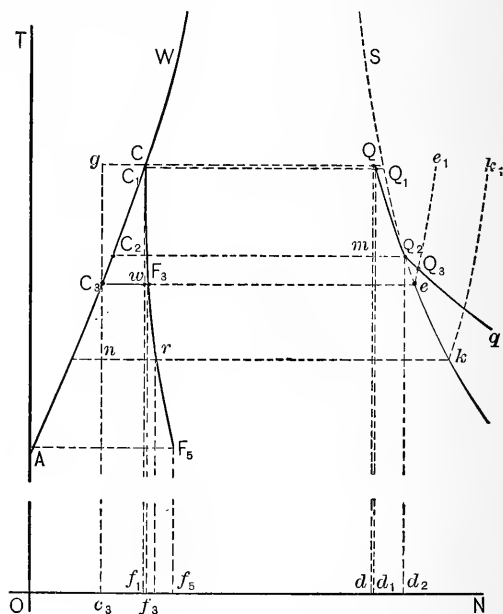
FIG. 14

extreme as never to be found in actual occurrence upon the earth's surface. But if Fig. 14 be used to illustrate the thermal condition of one of the permanent gases, or of air, with which vapor-pressures are very much higher for a given temperature, and vapor-temperatures very much lower for a given pressure, than with steam, such a distant point to the right of SS might well represent the thermal energy of air under quite familiar conditions.

If two adjacent points, both quite distant from SS upon the now nearly horizontal constant-heat curve Q_2q, be considered, it is plain that transition from one to the other must involve alteration of pressure, since Q_2q crosses all of the constant-pressure isomorphs. But drop in pressure without alteration in total fund of heat was the process with which **Joule** experimented in establishing his so-called law. It is now evident, from the graphical argument involved in Fig. 14, that such a process could not possibly take place without drop in temperature, because following the curve Q_2q, which is asymptotic to ON. But Fig. 14 at the same time reveals the reason why **Joule** noticed no drop in temperature; the curve Q_2q had become so nearly horizontal that the maximum variation in pressure possible developed a drop which was at that time imperceptible.

It is now plain, in the light of accumulated knowledge, that **Joule's** free expansion of air was necessarily an illustration of the second law of thermodynamics. It could not have taken place, there would have been no motive-power to initiate it, had there been no concomitant drop in temperature. Drop of energy in intensity is the sole motive-power for all natural phenomena.

Liquid Air. — In Fig. 16 let the curves W and S represent the entropy-temperature relations of liquid air and saturated vapor of air respectively. Since the critical temperatures of oxygen and nitrogen are $-181°$ and $-231°$ F. respectively, or 280 and 230 degrees absolute Fahrenheit respectively, the entire diagram must be imagined as lying below these very low temperatures. The boiling-points under atmospheric pressure of oxygen and nitrogen are 165° and 144° F. absolute respectively. Let CCC' be the constant-pressure curve at atmospheric pressure. Let AA and BB be two isothermals of vaporization at pressures considerably above atmospheric; in practice they are com-

monly 30 and 10 atmospheres respectively.   Let AA' and BB' be isomorphs of superheated air-vapor at these same pressures respectively.   The ordinary thermal condition of atmospheric air would then be represented by some such point as C'.

In Fig. 17 let C be the diagram of an air-compressor taking in atmospheric air and compressing it, as nearly

FIG. 16

isothermally as possible, to the pressure of BBB', Fig. 16, and discharging it into the reservoir B, Fig. 17. Let A be another compressor taking its supply of air from B and discharging it into the conduit aaa at the pressure of AAA', Fig. 16. This high-pressure conduit aaa passes into the interior of the larger pipe bbb. The two together

are formed into a coil presenting a large amount of sur-
face and leading to an end at *V*.   The coil is encased on
all sides with thermal insulation *III*, which is constructed
with great care.   At the terminus of the pipe *aaa* is a
valve *V* opening into the larger medium-pressure pipe *bbb*
and fitted to be adjusted by hand from without.   From the
terminus of *bbb* is a drain-line passing downwardly through
the insulation and ending in a drain-cock *D*.
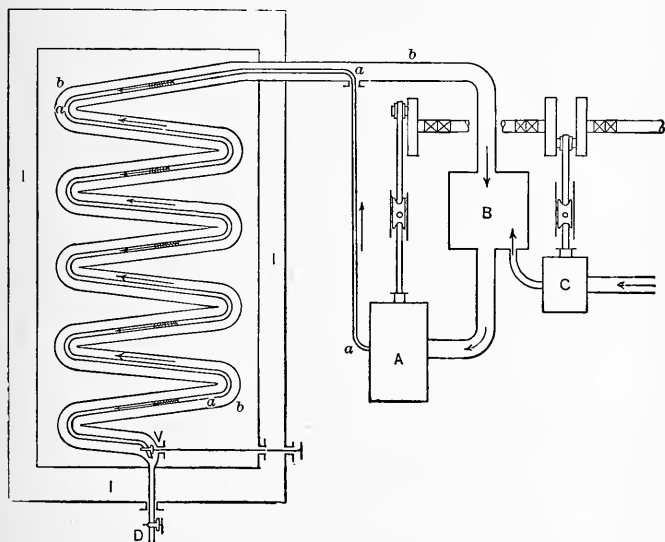
FIG. 17

The thermal activity of the apparatus illustrated in
Fig. 17 may now be traced by means of Fig. 16.

The compressor *C* supplies air at atmospheric tempera-
ture to the reservoir *B* at the same temperature.   The
compressor *A* supplies air to the pipe *aaa* at the higher
pressure *AAA'*, but still at atmospheric temperature; for
temperatures may be supposedly kept down to atmospheric
by cold-water jackets.   This double process may be shown

in Fig. 16 by the line $C'B'A'$, of isothermal compression (although properly at a higher level than drawn). The highly compressed air then traverses the pipe aaa and arrives at the valve V in the condition A', Fig. 16. It finds the valve V slightly open and is wire-drawn through it into bbb, its pressure falling to BBB' and its temperature to b_2, Fig. 16, by passage down the constant-heat curve $A'b_2$.

In traversing the pipe bbb, Fig. 17, this current of air brings the A-pressure air coming in along aaa down to its own temperature, so that the next quantity of the latter to arrive at V finds itself there in the thermal condition a_2, Fig. 16. Its wire-drawing through the valve V must then be represented by the constant-heat curve a_2b_3. The B-pressure air then traversing the pipe bbb at the temperature b_3 necessarily brings the next incoming air to the temperature a_3, and its wire-drawing in turn develops B-pressure air of the condition b_4.

Lord Kelvin's formula for the drop in temperature D involved in each of these passages across from the AA'-isomorph to the BB'-isomorph is

$$D = 0.497 (P_A - P_B) \left(\frac{493}{T}\right)^2, \qquad (41)$$

wherein the pressures are measured in atmospheres and the temperatures in degrees Fahrenheit. T is the initial absolute temperature at the departure from the upper pressure.

So the process goes on, all temperatures within III falling steadily meanwhile, until finally the steady fall of the point successively represented by b_2, b_3, b_4, etc., results in its coincidence with B. As the processes which resulted in temperature-fall due to abstraction of heat still continue, their further action must result in the condensation of some of the air at the BB-pressure. This condensation drains

to the valve D, and may be there drawn off into an external vessel.

It will be obvious, from a slight consideration of the geometrical laws controlling the curves SB and ab, that at these very low temperatures they must be much nearer to parallelism than is the case at ordinary steam-temperatures. How nearly they approach parallelism, whether a reaches A before b reaches B, or the opposite, cannot now be said. If a does reach A before b reaches B, it must indicate that liquid is first formed in the pipe aaa, just above the valve V, before it collects at D. If so, this liquid is vaporized by wire-drawing through V just as the moisture in a throttling-calorimeter is evaporated by a similar process. But whether this be the case or not does not affect the thermodynamic explanation of the final result.

Carnot's Cycle with Air as a Working-Substance. — Carnot's cycle, consisting as it does of two isothermals and two adiabatics, must always appear as a rectangle upon the NT-diagram. But the form of its PV-diagram depends entirely upon the working-substance which serves as a carrier for the heat. When steam performed this office it appeared that the PV-diagram was compounded of two horizontal rectilinear isotherms and two hyperbolic adiabatics, the area enclosed being quite considerable. Indeed, the actual steam-engine, whose success has always been founded chiefly upon its large amount of power developed per unit-weight and unit-volume of working-substance, works a close approach to the Carnot cycle. But if air or any other permanent gas be the working-substance, the isotherms and adiabatics are both hyperbolic in form and are very nearly parallel. The PV-diagram enclosed between any intersecting four of them, as shown in Fig. 18, is very long and narrow in proportion to its area. The range of both pressure and volume per unit of work devel-

K

oped is enormous.   Entirely aside from the obstacles of
bulk and heating-surface which have opposed the success
of all air-engines, this additional one of wide range of
pressure and volume per unit of work has always prohibited
all hope of seeing a successful Carnot-cycle air-engine.
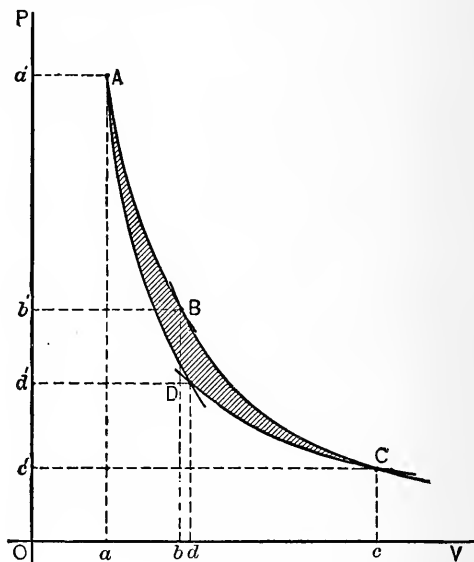
FIG. 18.

The obstacle to efficiency in the steam-engine is not the
poor form of its cycle, but is the narrow practicable range
in temperature.   It already begins to appear that the ob-
stacle to efficiency in the use of the permanent gases as
working-substances in heat-engines is not any limitation in
temperature-range, but in the poor form of their practicable
cycles.   This contrast will appear more strongly in the
following chapters.

CHAPTER VI

THE GAS-ENGINE CYCLES

THE history of the public distribution of artificial illuminating-gas opened with the century which has recently come to its close. Murdoch, Watt's famous assistant, had an active hand in the matter, and the shops of Boulton & Watt were amongst the first buildings upon which the experiment was tried. The presence of the new fuel very quickly led to attempts at its utilization for the development of power. Patents for gas-engines begin to appear upon the records before 1825. The second quarter of the century is thickly sprinkled with them. It was not until 1860, however, that an engine was produced which was sufficiently successful in a mechanical sense to attain a commercial basis, and for a long time thereafter the price of gas and the fuel-consumption of the engines both remained so high that very little progress was made in their adoption as a standard prime mover, even in the small sizes to which they were then restricted. The present position of the internal-combustion engine is due more to Dowson's invention of a cheap power-gas generator, to the natural-gas supply of the United States, to the utilization of the waste-gases from blast-furnaces for power-development, and to the wide and cheap supply of the lighter petroleum distillates, than it is to the development either of illuminating-gas or of the engine itself.

The Lenoir Cycle. — The first gas-engine sufficiently successful to command a market was the Lenoir, brought

out by a Frenchman of that name in 1860. The Lenoir engine much more closely resembled a single-cylinder horizontal steam-engine of the throttle-governed type than has any gas-engine since then. It was double-acting and controlled by a fly-ball governor. There the similarity ended, however. Its cylinder was water-jacketed, the valves were quite different, and the boiler was absent.

The cycle of processes in the Lenoir engine was as follows : —

(1) From dead-centre the fly-wheel drew the piston out, sucking in as it did so a charge of gas and atmospheric air in the correct proportion to form an explosive mixture ;

(2) Somewhat before half-stroke was reached the inlet-valve was closed and the charge was ignited, producing an explosion which elevated the pressure in the cylinder considerably above atmospheric ;

(3) The remainder of the stroke was devoted to expanding this pressure to atmospheric, or nearly so ; and

(4) The return-stroke exhausted the burnt gases. Thus there was produced an impulse upon each side of the piston at each stroke, or two for each revolution within a single cylinder.

The objections which led to the abandonment of the Lenoir engine were (a) the great consumption of fuel, (b) the violent overheating of parts, especially the piston, and their consequent rapid deterioration, and (c) the slight power developed.

Hugon was the only other builder of a true Lenoir engine, but the thermodynamic cycle survived in the *free-piston* engines of *Barsanti & Matteucci* and of *Otto & Langen*, who tried to improve Lenoir's fuel-consumption on the theory that it was the rapid loss of heat to the jacketed walls immediately after explosion which was the source of trouble. They therefore built vertical engines

each with a long cylinder in which a heavy piston was driven rapidly upward by the explosion; but the piston instead of being attached to a crank and limited in speed by its motion was free to rise to the top of the cylinder at the maximum velocity possible. It carried with it a rack meshing with a pinion which rotated loosely on the overhead fly-wheel shaft. The shaft normally revolved in the direction opposite to that of the pinion during the upward stroke of the piston. When the force of the explosion and of inertia had been overcome by gravity, the piston descended, aided by the partial vacuum formed beneath it by the cooling of the spent charge, and the motion in this direction engaged the pinion with the shaft by a pawl-and-ratchet device. It was on this return-stroke that power was imparted to the shaft.

These free-piston engines were better than the Lenoir as to fuel-consumption; they also remained cooler and worked more reliably. On the other hand they were fearfully noisy, too noisy for toleration, and in their turn they have passed away.

The only surviving representative of the Lenoir cycle is the Bischoff, a very small vertical engine whose sole recommendation is simplicity.

Problem 27. What are the conditions of operation and the efficiency of a Lenoir engine using at each stroke one cubic foot of an explosive mixture consisting of nine-tenths air and one-tenth gas of a calorific power of 650 B.t.u. per cubic foot, the atmospheric pressure and temperature being 14.5 pounds and 59° F. respectively?

The first operation, that of drawing in the charge, is shown at AB, Fig. 22. Since this is not a thermodynamic process, but merely a change of location, it does not appear in the NT-diagram. Hence, $V_B = 1.00$, $P_B = 14.5$ and $T_B = 520$.

As to the entropy, since we are no longer relying upon the steam-tables, we need no longer adhere to their arbitrary zero of 32° F. It will be preferable to assume an arbitrary zero-axis for entropy which passes through the initial point of this problem, B. Hence, $N_B = 0$.

Although the piston is really in quite rapid motion at B, and although an appreciable amount of time is required
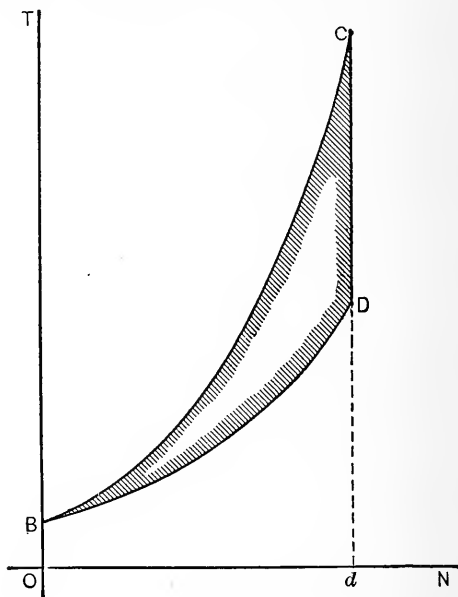
FIG. 19

for combustion to take place, yet theoretically the explosion is instantaneous and the piston accomplishes no motion while it is taking place. The next process therefore consists of the development of heat within the working-substance at constant volume. This process may be regarded as an addition of heat from without, for the heat is just as truly added to the substance when coming from its own

chemical alteration as when coming from outside. The process is therefore properly represented by the curve BC, Fig. 19, whose equation is

$$N - N_B = S_V \log_e \frac{T}{T_B},$$

and by the line BC of Fig. 20, whose equation is

$$V = \text{a constant} = V_B.$$
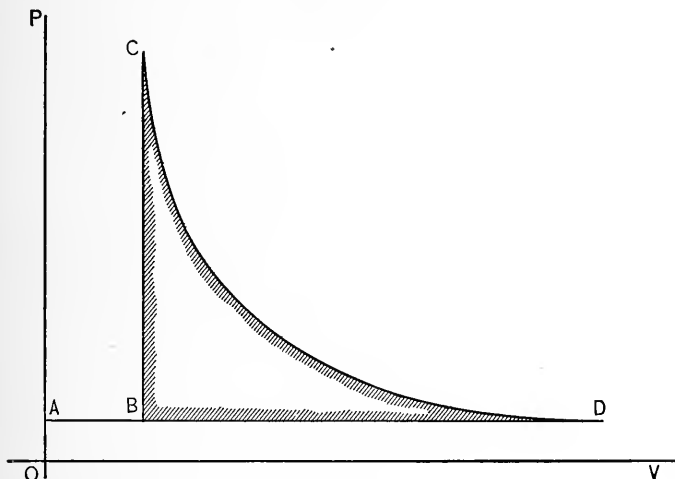
FIG. 20

Moreover, the volume being constant,

$$\frac{P_C}{P_B} = \frac{T_C}{T_B}.$$

The available heat per unit of mass, measured by the area $OBCd$, Fig. 19, is $S_V(T_C - T_B)$. The mass present, M, may be found from the equation

$$PV = CT \cdot M, \text{ or } 14.5 \times 144 \times 1 = 53.18 \times 520 \times M,$$

whence $M = 0.0758$.[1] But since one-tenth of the cubic foot
consists of gas having a density which may be assumed to
be 0.6, the true mass is $0.96 \times 0.0758 = 0.0727$. The
available heat is 65 B.t.u. From these equations are
derived

$$T_C - T_B = 5315,$$

$$T_C = 5835,$$

$$N_C = 0.02044,$$

and $$P_C = 162.7.$$

The process CD must be an adiabatic, unless artificially
distorted therefrom. Therefore in Fig. 19 the equation of
CD is

$$N = 0.02044,$$

and in Fig. 20

$$P V^{1.408} = \text{a constant.}$$

From Equation 35 is derived the value of T_D as 2915.

In Fig. 19 the areas beneath the curves give the heat
handled, and in either diagram the enclosed area gives the
work done. Q_1 is known to be 65 B.t.u.; Q_2, or the area
$OBDd$, $= MS_p(T_D - T_B) = 41.3$ B.t.u. Hence $Q_W = Q_1
- Q_2 = 18.7$ B.t.u. and the efficiency is 28.8%.

It is needless to say that no such temperatures or effi-
ciencies were ever attained in the actual engine; the walls
absorbed too much heat.

The Beau de Rochas Cycle and the Otto Engine. — At
about the time when Otto & Langen were experimenting
with their free-piston engine, M. Beau de Rochas enun-
ciated the principles upon which a gas-engine might be

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[1] In the majority of cases, however, the accuracy of conclusions will not be
affected if the assumption be made, for convenience, that $M =$ unity, the
quantity of heat being calculated per unit of mass instead of per unit of
volume.

operated successfully and economically.   A few years later, in 1876, Dr. Otto brought out his "Silent" engine embodying them.   The name was designed to signify that the objectionable noise of his earlier engine was no longer present.

This cycle, usually but erroneously referred to as the Otto cycle, consisted of the following processes : —

(1) The drawing into the cylinder throughout an entire stroke of a charge of explosive mixture at atmospheric pressure.

(2) The return of the piston under the influence of the fly-wheel, compressing this charge into the clearance space, which is much larger than in steam-engines.

(3) The ignition and explosion of the charge while the piston is at rest on dead-centre, the process being now truly under constant volume.

(4) The expansion of the heated gases throughout the entire stroke.

(5) The exhaust of the burnt gases during the back-stroke.

The Otto Silent engine had a great and immediate success.   The bulk of the gas-engines built since 1876 have closely followed the principles of mechanical design which it embodied, as well as its thermodynamic cycle.   It has nearly always been built in single-cylinder, single-acting form, in order to keep one side of the piston open to the air and thus avoid the overheating of the Lenoir.   In consequence, it gets but one impulse every four strokes, or every two complete revolutions.   Although at first but slightly better in fuel-consumption than the free-piston engine, owing to too little compression, it was quiet, powerful, and reliable in operation, and in every way an improvement over what had preceded.   It is now being built in two-, three-, and four-cylinder form, in order to multiply

impulses.   One or two builders have produced double-acting engines.   For many years its size was limited to an average of less than five horse-power, partly by reason of the cost of fuel and the consequently limited field of industry in which it was profitable, and partly by reason of mechanical obstacles which multiplied rapidly with increase in size.   To-day the natural-gas supply of the Mississippi Valley, and the use of the waste gases from the blast-furnace for driving gas-engines in Continental Europe have raised the practicable limit in size of a single engine to 4000 horse-power, with no reason apparent why further progress in capacity may not be made.

The Clerk-cycle or "Two-cycle" Gas-engine. — The obvious mechanical objections arising from the limitation of the standard form of the Otto gas-engine to one impulse for each two revolutions led to its modification by Mr. Clerk so as to obtain an impulse once each revolution.   To accomplish this a smaller preliminary or metering cylinder was added to the engine.   Its office was to draw in the explosive mixture of atmospheric air and gas to the amount required. Its piston was so timed in its motion that it had partially accomplished its return-stroke, slightly compressing the contained charge, when the main piston arrived at dead-centre after the completion of its expansion-stroke.   Communication between the two cylinders was then opened, allowing the slightly compressed fresh charge to enter the main cylinder, driving before it the burnt gases out the exhaust-valve.   At the moment when the fresh charge had practically displaced the burnt one both exhaust- and admission-valves were closed, and the return of the main piston prepared the fresh charge for ignition without the mediation of a pair of exhaust- and suction-strokes.

As built now, chiefly for marine and locomobile work, or in the smaller sizes for stationary work, the "two-cycle"

engine is nearly always a single-acting vertical engine, with the connecting-rod and crank working in an enclosed crank-case. This crank-case is employed for the preliminary metering of the charge, instead of a separate cylinder. As the piston rises, the fresh charge is drawn into the crank-case, below the piston. At upper dead-centre the admission-valve is closed and the downward stroke slightly compresses the charge. At lower dead-centre communication opens between the crank-case and the cylinder, and the fresh charge expands into the cylinder, displacing the burnt charge. The upward stroke of the piston then performs the usual compression of the charge preliminary to ignition.

The builders and buyers of gas-engines draw considerable distinction between the so-called " Otto-cycle " and " two-cycle " engines. It is true that, although much skill has been expended in markedly improving the two-cycle type, yet there will always be an imperfection in its action and efficiency due to the impossibility of displacing instantaneously the burnt charge by the fresh one without either losing some of the latter through the exhaust-valve or retaining some of the former within the cylinder.[1] It is therefore proper to draw a distinction between the two. Yet it is obvious that the difference is mechanical, not thermodynamic, in character. The preliminary " compression " is merely a means of displacing the burnt charge. Since the exhaust-valve is wide open at the instant that

[1] In the large Körting engines, in the region of 1000 horse-power each, which operate upon this cycle, this difficulty is obviated by having two displacer-cylinders. One of them pumps fresh air through the main cylinder immediately after exhaust, cleansing or "scavenging " the cylinder of all burnt gases. The other follows with the fresh charge of explosive mixture, small enough in volume to ensure that none of it shall pass out through the exhaust-port, and rich enough to stand dilution with the remnant of fresh air remaining after the scavenging.

the charge enters the main cylinder, the pressure must be atmospheric. Whatever preliminary compression there was has been lost. Thermodynamically, the Otto-cycle and the two-cycle engines operate upon exactly the same cycle.

Problem 28. What are the conditions of operation and the efficiency of an Otto engine working under the same
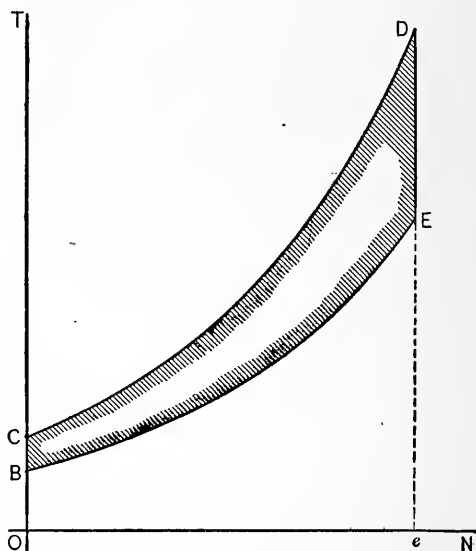
FIG. 21

premises as the Lenoir of Problem 27, except that the charge is compressed to an absolute pressure of 75 pounds per square inch before explosion?

The cycle is depicted in Figs. 21 and 22. *AB* is the suction of the charge, not appearing in the *NT*-diagram. *BC* is the adiabatic compression, *CD* is the explosion, *DE* the adiabatic expansion, and *EB* the exhaust at constant volume at the end of the stroke. The starting-point of the calculation of the cycle may now be taken as based

upon the fact that $V_{AB} = 1$ cubic foot. If P_C is to be 75 pounds per square inch, then

$$\left(\frac{V_B}{V_C}\right)^{1.408} = \frac{P_C}{P_B} = \frac{75}{14.5}.$$

$$V_B = 1 + V_C. \quad \text{Therefore,}$$

$$V_C = V_A = 0.452 \quad \text{and} \quad V_B = 1.452.$$

$$T_C = 837. \qquad N_B = N_C = O.$$

To the mass of the 1 cubic foot of explosive mixture of Problem 27 must now be added the 0.452 cubic foot of burnt
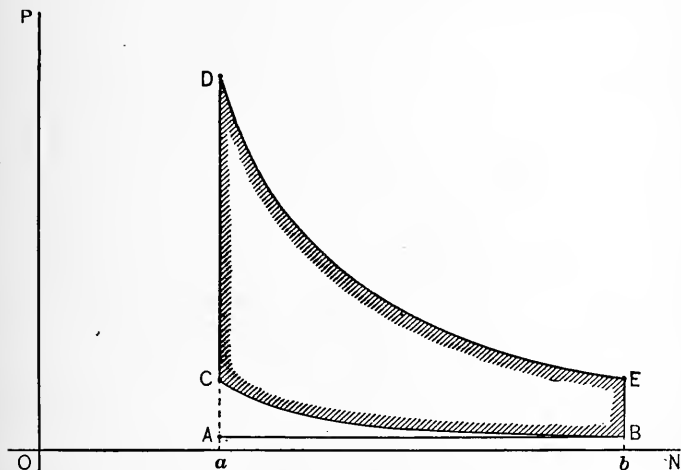
FIG. 22

gases in the clearance-space, or $M = 0.1057$. Proceeding as before, $T_D = 4498$, $N_D = 0.02972$, and $P_D = 403$. The expansion DE cannot be carried down to atmospheric pressure, because the engine's cycle limits the final volume after expansion to the initial volume before compression.[1]

[1] See page 287, Part II, *et seq.*

Hence, $V_E = V_B = 1.452.$ $\dfrac{P_D}{P_E} = \dfrac{P_C}{P_B}.$ $P_E = 78,$ and $T_E = 2792.$

The two isomorphic curves CD and BE of Fig. 21 have the same equation, except that the coefficients of specific heat differ; the curves DE and CB of Fig. 22 bear a similar relation. Q_1 is the area $OCDe$ (Fig. 21), and Q_2 is the area $OBEe$. But geometrical inspection will prove that these areas must bear the same proportion to one another as T_C to T_B, or as T_D to T_E.[1] Therefore $Q_w = Q_1 - Q_2$ must bear the same proportion to Q_1 or Q_2 as $T_C - T_B$ does to T_C or T_B, respectively;

or
$$\frac{Q_w}{Q_1} = \frac{T_C - T_B}{T_C} = \frac{T_D - T_E}{T_D}, \qquad (38)$$

from which the efficiency of the cycle is found to be 37.9%.

It is obvious from this that it is the vertical separation of the curves CD and BE of Fig. 21, or the proportion of the total available fall in temperature which is utilized, which determines both the efficiency and the power of the engine. This explains why the early Otto engines, with their slight degree of compression, were so slightly more economical than the Lenoir. It also explains why the chief line of progress in gas-engine design between the years 1875 and 1895 has consisted in increasing the degree of compression. Progress in this direction has now almost

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[1] The area beneath CD, Fig. 21, is equal to an integration of the expression TdN. The area beneath EF is measured by the same expression. For any given value of dN or of N, however, the value of these two expressions will be the proportional to the values of T. Since the equation of CD is $N - N_0 = S \log_e \dfrac{T}{T_C}$ and that of EB is $N - N_0 = S_v \log_e \dfrac{T}{T_B}$, for any given value of N the corresponding values of T for the two curves must be proportional to $\dfrac{T_C}{T_B}$. Hence, the areas beneath CD and EF, between any given pair of ordinates, are proportional to $\dfrac{T_C}{T_B}$.

ceased, (1) because of the rapidly increasing mechanical objections due to the development of such high momentary pressures at the point D without proportionate increase in work done : the engine must be strong enough, in castings and bearings, to stand this maximum pressure, and yet, because the pressure is so evanescent, the engine derives little good from it ; (2) because of the danger of preignition when the degree of compression is high, particularly in oil-engines.

The Joule Cycle. — The Joule cycle was proposed by him who discovered the mechanical equivalent of heat. It has found its embodiment in several designs of engine, none of which have survived the test of competition with the Otto type. Some of them have been gas-engines, as have the Gardie and the Bergher, some oil-engines, as the Priestman, and some almost purely hot-air engines, as the Cayley-Buckett. The Joule type is nevertheless of the greatest importance because, while no one has ever yet put it into profitable practice, it offers possibilities of superiority over the Otto which have attracted to it a long line of inventors. Their failures may in each case be easily explained as inevitable under the mistaken line of design which they adopted. It is the personal opinion of the author that the large gas-engine of the future will operate on no other than the Joule cycle. His reasons, however, are not solely, nor even largely, based upon those thermodynamic points of superiority which are developed in the following pages. They are based chiefly upon purely mechanical considerations which lie outside the limits of the present discussion. It is worthy of note, however, that the purely thermodynamic aspect of the question shows that under any chosen limits of temperature and pressure, the theoretic efficiency of the Joule cycle is not only greater than that of the Beau de Rochas cycle, but it is greater than that of any other cycles known

to have been proposed or put into practice except the Carnot, the Stirling, and the Ericsson. Of these three, all are known by long trial to be impracticable.

The essential features of an engine operating on this cycle are shown in Fig. 23. They comprise two cylinders, C and W, the latter larger than the other, the two being combined in any way so that power may be transmitted from one to the other; a source of heat at a higher temperature T_1; a refrigerator at a lower one T_2. Supposing
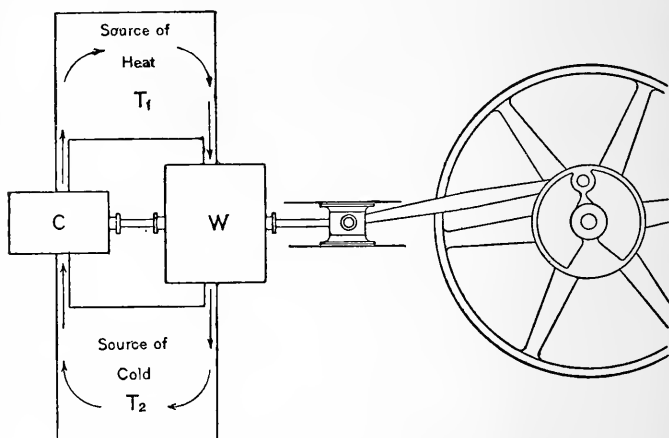
FIG. 23

the entire system to be filled with the chosen permanent-gas working-substance, air for example, the operation is as follows, the processes being traced in the diagrams of Figs. 24 and 25 : —

(1) There is drawn into cylinder C a charge of working-substance. This is shown by AB of Fig. 25; it does not appear in Fig. 24.

(2) The adiabatic compression of the charge to the maximum working-pressure of the engine. This is shown by BC in both diagrams.

(3) The addition of heat under constant pressure; CD in both diagrams.

(This process really involves the triple process of (a) the expulsion of the compressed charge from the cylinder into the chamber T_1, shown by Cc of Fig. 25; (b) its increase of volume while there; (c) the taking of the increased volume into the cylinder W, which is shown by cD of Fig. 25. But as this combination consists entirely of changes of locality, it need not be expressed upon the thermodynamic diagrams.)

(4) The adiabatic expansion of the heated gases to atmospheric pressure. This is shown by DE in both diagrams.

(5) Their exhaust at constant (atmospheric) pressure. This is shown by EB in both diagrams.

It is immediately obvious that the process of heating the gases in the chamber T_1 may be done either by the transmission of heat to them through the surrounding walls or by their own internal combustion. If the latter be the case, the combustion must take on the characteristics of a flame, increasing in volume under constant pressure, rather than those of an explosion, which implies an increase of pressure under constant volume. In the Cayley-Buckett engine this chamber was a furnace enclosing an anthracite fire; the furnace was ordinary in every respect except that it was capable of withstanding internal pressure. In the Priestman oil-engine the working-substance was a mixture of oil and air; this chamber T_1 was a little combustion-chamber fit to contain an oil-flame and separated from the cylinder C by a shield of wire gauze to prevent the flame from travelling back into the compressed charge which was being driven out of C. In the Gardie and Bergher engines the combustion takes place in the cylinder W as the compressed charge enters. In all

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except the first the combustion is intermittent, ignition being provided every time that the cycle reaches the point C and the flame being extinguished at the point D; in the first the flame burns more or less continuously, the chamber T_1 being sufficiently large to store the extra energy evolved during the time when the cylinder W is not taking its charge.
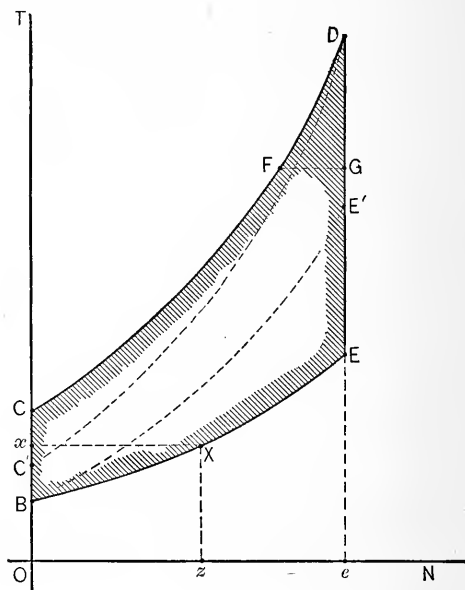
FIG. 24

Problem 29. What are the conditions of operation and the efficiency of a Joule engine working under the same premises as the engines of the two preceding problems, except that the charge is compressed to a pressure of 365 pounds per square inch absolute before ignition?

Here the original cubic foot of explosive mixture appears

as the volume AB. Calculating the values for the several points as before, it appears that $V_C = 0.1011$, $T_C = 1323$, $V_D = 0.3995$, $T_D = 5095$, $N_D = 0.02438$, $T_E = 2002$, and $V_E = 3.844$. As in the case of the Otto cycle, the curves CD and BE of Fig. 24 have the same equation and differ only in the point of origin; therefore the area beneath each curve is proportional to the ordinates at any abscissa, or $Q_w : Q_1$ as area $BCDE$: area $OCDe = BC : OC = 803 : 1323 = 60.7\%$ efficiency.
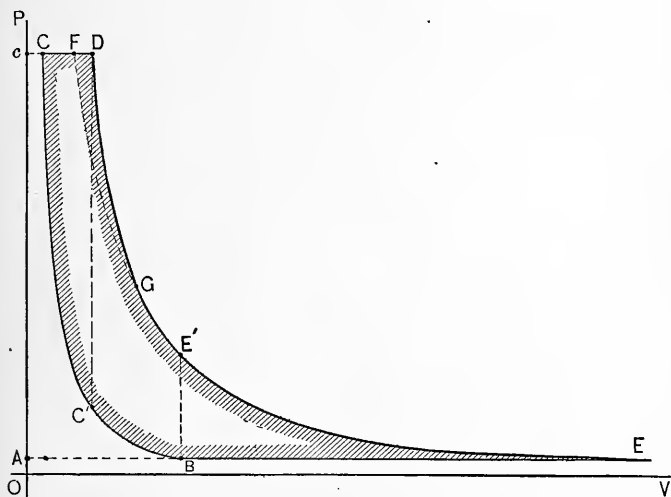
FIG. 25

Although it is obvious from these figures that the efficiency of the Joule cycle is much superior to either the Lenoir or the Otto under the conditions chosen, it is not equally clear that it would be under any conditions. The diagrams are not drawn to the same scale nor, even if so, could any set of conditions be chosen which would be equally fair for all three. But an excellent general comparison may be had by adding to Figs. 24 and 25 the

constant-volume lines $C'D$ and BE'. Then between the one pair of adiabatics, BC and DE, are shown :

(1) A Lenoir cycle, $BE'E$;

(2) An Otto cycle, $BC'DE'$; and

(3) A Joule cycle, $BCDE$. The comparison is not fair to the Lenoir cycle, because the efficiency of the latter increases with its width ; hence, in the Lenoir, the richer the gas or the oil used, the better will be the efficiency. But with both the Otto and the Joule cycles the efficiency is independent of their width in the NT-diagram, being solely dependent upon their height, or upon the degree of compression. Therefore, their limitation between a given pair of adiabatics is equally fair to both. Since the Lenoir cycle is out of the race in any event, the injustice done to it may be overlooked.

It is further obvious that the territories of the Lenoir and of the Otto cycles are mutually safe from trespass. The Lenoir type cannot get into the territory of the Otto, to the left or above BE' ; for it may not embody initial compression. The Otto cannot cross the same line in the opposite direction, into the territory of the Lenoir, without final expansion to a volume greater than its initial one, which is impossible.

The avoidance of this inherent limitation of the Otto cycle has been sought, in a single-cylinder type, by Mr. Atkinson in his "Differential" and "Cycle" engines, and by various designers in the multiple-cylinder type by adding larger low-pressure cylinders in which the expansion $E'E$ might be performed ; but all have failed, the first for mechanical and the second for thermal reasons.

It is also sought in some of the largest and best engines of the present day, without added mechanism, by restricting the volume taken in at atmospheric pressure to less than the full cylinder-volume. This also fails to attain

the desired end, for reasons which are explained on page 287.

So, with this limitation accepted as inevitable, and remembering that the chief limitations in constructive practicability are *pressure* and *temperature*, the broad conclusion may be safely drawn that *under the same maximum limits of pressure and temperature, the Joule cycle has inevitably the greatest efficiency of the three, its area not only completely covering the Otto and the bulk of the Lenoir, but having some territory of its own unattainable by either of the others.*

The universal failure of the Joule-cycle engines in the past can be explained, in the face of this statement, by the fact that in no case was a working-pressure adopted which would permit any possibility of success. Always arbitrarily limited by their designers to about 100 pounds as their maximum pressure, the Joule-cycle engines hitherto constructed could have no hope of successful competition with the Otto-cycle engines, when the latter regularly attained pressures of from 300 to 400 pounds as maxima.

The Diesel Cycle. — In the year 1897 Mr. Rudolph Diesel announced to the world the successful operation of an engine operating under a cycle which he had previously advocated from purely thermodynamic considerations.[1] In the argument which led him to his result Mr. Diesel started in search of the nearest possible approach to a Carnot cycle. Compression was made as purely adiabatic as possible, instead of as nearly isothermal as possible, as had usually been done in the embodiment of previous cycles. High efficiency being the main object in view, a wide range of temperature was needed, and this called for a similar range in pressure. The working pressure was therefore fixed as 500 pounds per square inch. As adiabatic compression to

[1] "A Rational Heat-motor," by Rudolph Diesel.

this pressure resulted in a temperature above that sufficient for the ignition of the fuel finally chosen, which was oil, igniting devices were omitted. The oil was simply injected into the cylinder when compression was complete, into an atmosphere above the temperature of ignition and in which, therefore, it could not help burning. The first result was a still further increase in temperature; but as a Carnot cycle demands isothermal addition of heat, Mr. Diesel invented his famous *isothermal combustion*, in which merely enough fuel was admitted to maintain a constant temperature as the gases expanded. This process replaced the ordinary temperature-raising combustion soon after the stroke commenced, and continued until the point in the stroke where the governor dictated a cut-off of fuel-supply. After this adiabatic expansion set in.

The general arrangement of the engine was similar to that of a vertical Otto-type engine, except that the cylinder was longer and narrower, and there was added an air-pump for creating a pressure on the surface of the oil sufficient to drive it into the compressed air within the cylinder when the valve opened. One impulse was received every two complete revolutions, as in the Otto engine.

The Diesel cycle is portrayed in Figs. 24 and 25 by the area $BCFGE'B$, though a much higher pressure-limit than that drawn must be imagined in comparing it with the other cycles. The compression to 500 pounds is shown by BC, the primary combustion by CF, the isothermal combustion by FG, the adiabatic expansion by GE', and the exhaust at lower dead-centre by $E'B$. It is obvious that the cycle is much nearer to a Joule than to a rectangular Carnot. It is further obvious that Mr. Diesel, in remembering the statement that the Carnot cycle was the one of maximum efficiency, forgot under what limitation that statement was made: "Under any given temperature-limits." For if

some other cycle depart from the Carnot rectangle but at the same time attain a greater temperature-range, it may have an efficiency greater than that to which the original Carnot cycle was restricted. Thus, here, the isothermal *FG*, which would impart greater efficiency than the iso-morphic *CD*, *if the upper temperature-limit were FG*, plainly decreases the efficiency when compared with that attained by admitting the heat beneath *FG* along the isomorphic *FD*, to some point near *D*, before expansion sets in.

From these considerations it may be concluded that the maximum efficiency available *under equal pressure-range* is always greater in the Joule than in the Diesel cycle. Under equal temperature-limits either may be the greater; but to attain equal temperature-limits the Diesel cycle de-mands an egregiously superior maximum pressure.

CHAPTER VII

THE HOT–AIR ENGINES

THE history of the hot-air engines, if the efforts of Hiero of Alexandria, who worked in the second century before the Christian era, and of a few unsuccessful inventors be excepted, is comprised between the years 1827 and 1854. On the former date **Robert Stirling** took out his patent for a regenerative air-engine; on the latter date **Ericsson's** hot-air propelled ship, with its enormous engines, sank in New York harbor, never to rise again into a world of "caloric" engines. During this period the energy of many of the best minds of the mechanical world were devoted to its perfection. It passed from the status of a toy to a prime mover of hundreds of horse-power, and back again. It experienced all the glamour and romance to be had from youthful promise of the most fascinating sort. It grew to manhood under the guidance of master minds. It was tried and found wanting. It is now relegated almost solely to the task of supplying occasional country-residences and small hotels with water. The question of its worth for any larger task than this has been effectually sealed by one of the most valuable failures on the part of one of the ablest engineers that the world has ever seen, — that of **Ericsson,** who, after twenty years of effort, failed to produce a machine which could compete with the steam-engine in the world's field.

From the very start the hot-air engine offered an irresistible fascination for the inventor, because even the crudest attempts with it always resulted in an efficiency far superior

to that of the steam-engine. This is readily explained as being due to the far greater temperature-range possible when working with a perfect gas than with steam, where the attempt to get high temperatures is always met by a need of impossibly high pressures. This contrast was of course more marked in those days, when 30 to 50 pounds per square inch was regarded as the highest steam-pressure practicable in stationary engines. Its final failure was due to phenomena which manifested themselves from the start and which are now known to be ineradicable. These phenomena are:

(1) The rapid deterioration of metallic surfaces when used to transmit high-temperature heat from an atmosphere of carbonic acid to a hot dry gas, such as air;

(2) The large amount of such surface required to transmit a given amount of heat when compared with like transmission to water or to saturated steam; and

(3) The great bulk of the weight of air requisite for handling a proper amount of energy.

The first two obstacles have been obviated in the natural offspring of the hot-air engine, the internal-combustion gas- and oil-engines. Incidentally the last has also been partly eliminated in the same way; though it is imaginable that the application to the hot-air engine of as great a working-pressure as is now common in the gas-engine might have been very beneficial to it in respect to bulk.

The Stirling Air-engine. — The utter impossibility of a practicable Carnot-cycle air-engine was pointed out at the end of Chapter V. The attainment of any practicable efficiency by means of any other cycle, with air as a working-substance, became only possible with the invention of the regenerator, by Stirling, in 1827 as a substitute for adiabatic compression and expansion for the transfer of the air from one temperature-level to another. The regenerator consists

of a chamber containing a mass of heat-absorbing material arranged so as to present as large a surface as possible to a current of air passed through it. In practice this material was usually iron wire, arranged in a close network. Through this network the air was passed back and forth, giving up to the wire its heat as it entered from the hotter end and picking it up again as it passed back again from the colder refrigerative chamber of the engine. The only forms in which the regenerator survives in modern industry are the hot-blast "stoves" of the blast-furnace, and the air-heating surfaces of the regenerative gas- and oil-stoves and lamps. With all of the work in this line the name of **Siemens** is chiefly and actively connected.

The arrangement of Stirling's regenerative engine is shown in Fig. 26, in which the regenerator occupies the annular space RR about the larger cylinder. When in operation the temperature of this generator at the lower end is T_1 and at the upper end T_2, being graded continuously from one to the other. Constructively, the engine consists of a crank-shaft and fly-wheel to which are coupled by connecting-rods the large "displacer piston" D and the "working-piston" W, but in such a way that their strokes take place alternately. (Theoretically, the stroke of each should be complete before that of the other begins. In practice, each stroke laps over the other somewhat; but in analyzing the engine's action the first will be assumed to be the case.) At the lower end of the cylinder D is placed the fire, in the furnace F, usually maintaining the iron at a temperature approaching a visible red, and at the upper end is the coil of condenser-pipes CC, filled with a current of cold water. The action is traced mathematically in the NT and PV diagrams of Figs. 27 and 28.

It were best noted at the start that the volume of air within the engine depends solely upon the position of the

piston W. The motion of the piston D, which fits loosely
in its cylinder, meets with no possible resistance and causes
no change of volume, because the two ends of its cylinder
are always open to one another.

The engine is shown in a position represented as midway
between the points A and B in the diagrams, containing a
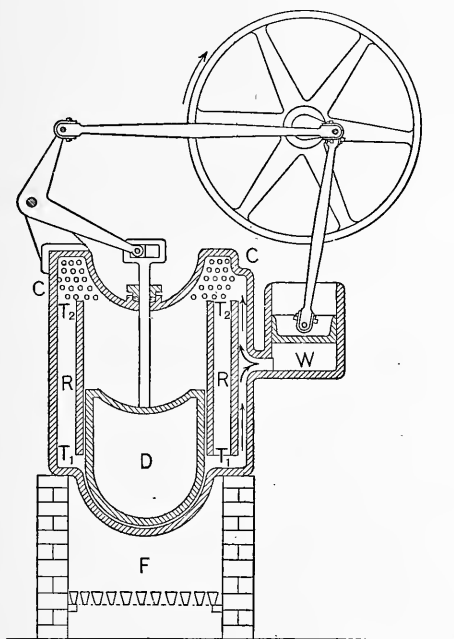
FIG. 26

quantity of air at approximately atmospheric pressure and
minimum temperature. The air had been at atmospheric
pressure at the beginning of W's downward stroke, at the
point A. It has now been somewhat compressed, by the
energy of the fly-wheel, isothermally because in contact
with the cold pipes CC. This process is shown by the line
AB, in both diagrams. At the end of W's stroke the

volume is a minimum, and D now moves upward, displacing
the cold air from the space CC through the regenerator RR
into the lower end of the cylinder. (The by-passing of the
air around the regenerator through the ports leading to W
is properly prevented, sometimes by valves, sometimes by
the small size of W as compared with R and D, and some-
times by making use of only one port.) This raises the
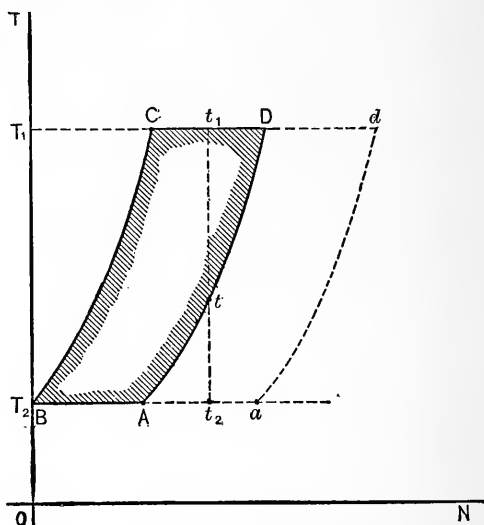
FIG. 27

air, at constant volume, from the pressure at B to that at C
by reason of the change of temperature from T_2 to T_1.
This is shown by BC in both diagrams.

While D is now quite at its upper dead-centre, W moves
up under the influence of the increased pressure and makes
its working-stroke, the air expanding isothermally instead of
falling in temperature adiabatically, because in contact with
the fire-surface at the temperature T'. This is shown by
the line CD in both diagrams.

Then, while W is quiet at its dead-centre, D comes down, displacing the air again to the top of the cylinder through the regenerator, cooling the air under constant volume from T_1 to T_2 and reducing the pressure from P_D to P_A. This is shown by the line DA in both diagrams. The cycle is then complete.

Thus the displacer-piston D, with the aid of the regenerator, changes the *pressure ;* the working-piston W, with
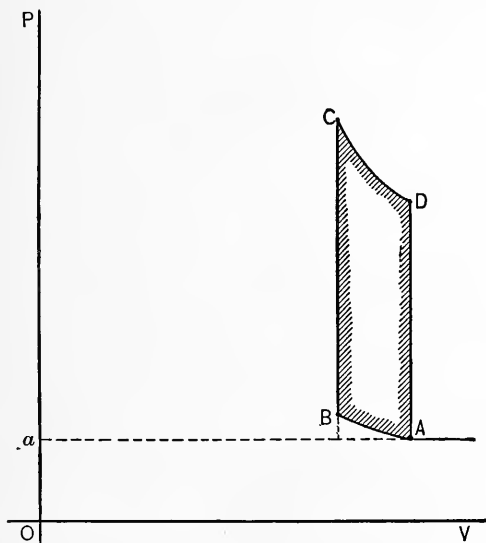
FIG. 28

the aid of fire and condenser, changes the *volume*, isothermally. This cycle may be summarized as follows :

I. D at bottom ; W moving down ; line AB in diagrams ; negative work done (area under AB, Fig. 27).

II. W at bottom ; D moving up ; line BC in diagrams.

III. D at top ; W moving up (working-stroke) ; line CD.

IV. W at top ; D moving down ; line DA in diagrams.

Problem 30.   What are the conditions of operation and the efficiency of a Stirling engine operating with one cubic foot of atmospheric air, the temperature of the fire-surface being 1539° F. and the volume of the rest of the engine being four times that of the cylinder W?

This gives the volume at $A = 1.00$ cubic foot, that at $B = 0.80$ cubic foot.   The temperature at A and B is 520° F. absolute, and that at C and D is 2000° absolute.   The pressure at A is 14.5 pounds per square inch.

AB being an isothermal, $P_B = \dfrac{1.00}{0.80} \times 14.50 = 18.12$ pounds per square inch.   In the heating at constant volume the pressures must change proportionally with the temperatures; therefore $P_C = \dfrac{2000}{520} \times 18.12 = 69.25$ pounds. CD being an isothermal again, $P_D = \dfrac{0.80}{1.00} \times 69.25 = 55.40$ pounds. The mass involved being 0.0758 pound and the heating being at constant volume, $N_C = 0.001301$.   The heat abstracted along AB must be equal to the (negative) external work done, or

$$Q_2 = \frac{1}{778} \cdot P_A V_A \log_e r = \frac{C_A T_A}{778} \cdot \log_e \frac{V_A}{V_B}$$

$$= \frac{53.18 \times 0.0758 \times 520}{778} \times \log_e \frac{1.00}{0.80} = 0.6011 \text{ B.t.u.}$$

The entropy absorbed along AB must be this divided by 520, or 0.001156.   Substituting T_D for T_A in the equation for Q_2 gives

$$Q_1 = 2.312 \text{ B.t.u.,}$$

$$Q_w = Q_1 - Q_2 = 1.711 \text{ B.t.u.,}$$

$$F = \frac{Q_w}{Q_1} = 74.0\%.$$

As to efficiency, it might seem from the diagram that the heat supplied by the fire should be the area beneath BCD,

Fig. 27, and that that absorbed by the refrigerator should be the area beneath DAB. But of these quantities the heat beneath BC enters the air, not from the fire, but from the regenerator, where it was stored up by the DA-process of the preceding cycle. Thus, while it might seem from the form of the cycle that the heat fell into the BC-line from the temperature T_1, and out of the DA-line into T_2, this is not the case, as will be evident if two succeeding cycles, such as $ABCD$ and $aADd$, are considered. Each atom of entropy, such as that at t, which reaches the AD-line of the second cycle has done so by first falling from t_1 to t, *doing work* in the first cycle; it is deposited, without free fall, in the regenerator at this level. It is afterward picked up by the AD-process of the second cycle, in which it falls from t to t_2, again doing work. So the heat supplied, Q_1, is only that beneath the line CD, and Q_2 is only that beneath AB. As the entropies of these two processes are equal, $\dfrac{Q_1}{Q_2} = \dfrac{T_1}{T_2}$, and the efficiency is that of a Carnot cycle, or, in this case, 74.0%.

Engines operating upon this cycle were built as large as 50 to 75 horse-power before their bulk and the lack of durability of the heating-surfaces threw them out of competition with the steam-engine.

The Ericsson Hot-air Engine. — The Ericsson engine differed from the Stirling constructively in that, instead of the working-piston also performing the compression, while the displacer-piston meets with no resistance, both pistons are under pressure, the smaller one doing the compression and the larger performing the work. The displacement through the regenerator is performed by the pistons passing the compressed charge from the smaller to the larger. Thermodynamically the Ericsson differs from the Stirling in that the regenerative processes joining the two isotherms

are under constant pressure instead of under constant volume. The cycle is that portrayed in Figs. 29 and 30. The intake of fresh cold air into the smaller compressing cylinder is shown at aA, Fig. 30, and does not appear in Fig. 29. It is then compressed isothermally to B in contact with the refrigerating surface. It is discharged along Bb, Fig. 30, into the regenerator, through which it passes on its way to the larger working-cylinder. There it ap-
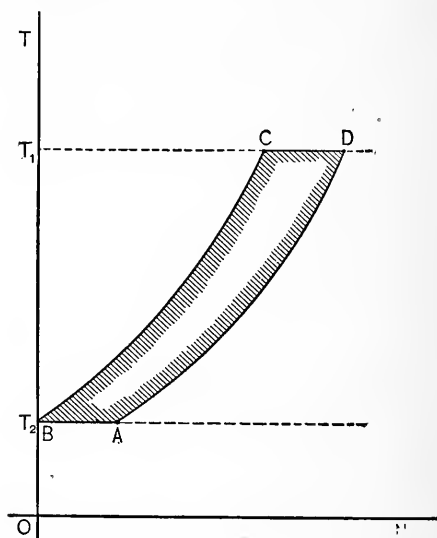
FIG. 29

pears as the volume bC, Fig. 30, having in the interim been heated along the constant-pressure isomorph BC of Fig. 29. At C the working-cylinder experiences a cut-off of supply exactly as in the steam-engine, after which isothermal expansion takes place in contact with the fire-surface until atmospheric pressure is reached.

Constructively and thermodynamically the Ericsson bears somewhat the same relation to the Stirling that the Joule

does to the Otto, except that the efficiencies of the two hot-air engines are the same, whereas those of the two gas-engines are not. Indeed, the diagram of the Joule engine, Fig. 23, will serve very well as an illustration of the general arrangement of the Ericsson engine if the chambers T_1 and T_2 be imagined as regenerators, each having its end toward C at the temperature T_2 and the end
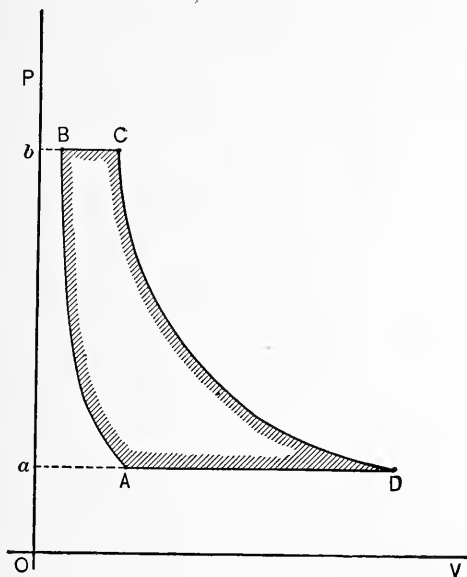
FIG. 30

toward W at the temperature T_1, and if the cylinder C be imagined as jacketed with cold water and the cylinder W as surrounded with fire; for the working-cylinders of the Ericsson engines were actually hung over furnaces, much in similitude to huge camp-kettles. Another difference between the two hot-air engines was that in the Stirling the motion of the two pistons had to be properly timed, one with the other, to produce the proper effect, whereas

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in the Ericsson they were, within certain limitations, independent.

Problem 31. What are the conditions of operation and the efficiency of an Ericsson engine working under the same temperature and pressure ranges as the Stirling engine of Problem 30?

Here $V_A = 1.000$. Since P_B is to be 69.25, $V_B = 0.2097$. Since $T_C = 2000$ and the heating is under constant pressure, $V_C = \dfrac{2000}{520} \times 0.2097 = 0.8073$, and $N_C = 0.02417$. The heat supplied along $CD = C_C T_C \log_e \dfrac{P_C}{P_D} \cdot \dfrac{1}{778}$, from which $Q_1 = 16.18$ B.t.u. and $N_{CD} = 0.00808$. $N_D = 0.03225$. $\dfrac{V_C}{V_B} = \dfrac{T_C}{T_B}$, whence $V_C = 0.8073$. $\dfrac{V_D}{V_C} = \dfrac{P_C}{P_D}$, whence $V_D = 3.847$. From the same argument as in Problem 30, the efficiency must be given by $\dfrac{T_1 - T_2}{T_1} = 74.0\%$. The amount of heat converted into work, or Q_w, $= 0.74 \times 16.18 = 11.97$ B.t.u.

The engines of the vessel *Ericsson*, the culmination of that engineer's efforts, well illustrate the reasons for the failure of the hot-air engine. Though designed to propel a vessel of 260 feet in length at the modest speed of seven miles per hour, the bulk and weight of the engine was enormous. It had four compressing-cylinders, each 11.4 feet in diameter, and four working-cylinders, each 14 feet in diameter; the stroke of each was 6 feet. The vessel sank before accurate tests of efficiency or capacity could be made, but as the maximum working temperature is mentioned as being about 443°, the theoretical efficiency could not be above 42.6%, and the actual efficiency must have been less than half of this, or not over 20%. As this is very much better than the best marine engines even of to-day can do, there is little wonder that the new "caloric" engine won tremendous applause as the superior

of the steam-engine. But the difficulty of floating engines powerful enough to give the hull a practicable speed, even with the weight saved in coal-bunkers and boilers and the constant trouble and rapid deterioration due to attempting to lubricate the pistons under such (then) unusually high temperatures, were so obvious that the *Ericsson*, though promptly raised to the surface, was reëngined as a steamer, and the attempt has never been repeated.

CHAPTER VIII

SUMMARY OF THEORETIC POSSIBILITIES OF VARIOUS TYPES

The efficiency of a heat-engine is only one, and often a minor one, of the many factors which determine its value. The one always of great, and often of the first, importance is the capacity for developing power within a given volume, weight, or cost of machinery. Usually the qualities coming next in the order of importance are simplicity, reliability, and adaptability to varied service. But, confining the attention entirely to mere thermodynamic considerations, it is interesting to note the maximum possible efficiency of the several types when compared with what may be called the "builder's factor," viz.: the volume of cylinder (in cubic feet) times the maximum pressure upon the piston (in pounds per square inch), divided by the work developed (measured in heat-units), which is as accurate an indication of the cost per horse-power as can be based upon purely thermodynamic conditions.

The higher the builder's ratio, the greater will be the cost of the engine per horse-power.

This table shows clearly, if coupled with the statements of the obstacles arising in attempting to utilize hot air, how those engines of the highest efficiency pay for it in bulk and cost, and, *vice versa*, how those of high capacity (or low cost) pay for it in poor efficiency. It will be noticed that all of the engines using permanent gases for working-

Engine	Efficiency	Builder's Ratio
Steam-engine, non-condensing (Problem 13) .	14.9%	16.0
Steam-engine, non-condensing, modified by incomplete expansion	13.3%	8.6
Steam-engine, condensing (Problem 15). .	23.6%	54.0
Steam-engine, condensing, modified by incomplete expansion	21.2%	15.2
Steam-engine, non-expansive (Problem 19) .	7.1%	5.5
Gas-engine, Lenoir (Problem 27) . . .	28.8%	44.4
Gas-engine, Otto (Problem 28) . . .	37.9%	31.5
Gas-engine, Joule (Problem 29) . . .	60.7%	47.9
Gas-engine, Joule, modified by incomplete expansion	57.7%	35.1
Hot-air engine, Stirling (not including displacer-piston and regenerator) . . .	74.0%	6.4
Hot-air engine, Ericsson (not including regenerator)	74.0%	22.2

substance exceed the non-condensing steam-engine in cost as well as in efficiency.

It should be noted that all of the engines which expand to atmospheric pressure end the indicator-diagram with a sharp toe; this calls for an exaggerated maximum volume without sufficient pressure, toward the end, to develop commensurate power. This trouble is always obviated in practice by ending the stroke with an appreciable pressure still in the cylinder, which allows the engine to make a better showing in regard to builder's ratio; this stands quite aside from those considerations which affect all of the engines in passing from theory to practice. Thus, by ending the expansion in Problem 13 at 15 pounds per square inch of net pressure the efficiency is reduced only from 14.9 to 13.35%, while the builder's ratio falls from 16.0 to 8.6. Similarly, ending the expansion in Problem 15 at 7 pounds per square

inch net pressure reduces the efficiency only from 23.6 to
21.2%, while the builder's ratio falls from 54.0 to 15.2%.
Shortening the maximum volume of the Joule cycle to
2.5 feet decreases the efficiency only from 60.7 to 57.7%,
while the builder's ratio falls from 47.9 to quite near that
of the Otto.

CHAPTER IX

THE REFRIGERATING MACHINES [1]

As in the case of the clockwise cycle, the reversed thermodynamic or refrigerating cycle does not exist in practice in the rectangular or perfect form. Practical considerations make it more profitable to accept a lessened thermodynamic efficiency for the sake of a saving in complexity, cost, bulk, etc.

The processes in actual use for the development of artificial cold may be divided broadly into two classes : —

I. Those depending upon the capacity of water for absorbing certain vapors at lower temperatures, which vapors may be driven off again by the addition of heat. They are called *absorption-machines*. They demand no supply of mechanical power, the heat-energy which keeps them in operation being supplied in the form of heat directly to the water, and they involve no more moving machinery than one or two circulating-pumps. They therefore exhibit no purely thermodynamic phenomena, and will be given no further consideration in these pages.

II. Those operating on some thermodynamic cycle such as has just been discussed, and always involving a supply of power from without to keep them in operation. They are known as *compression-machines*. They may be subdivided into two sub-classes : —

(*a*) Those using a permanent gas, usually air, as a working-substance ; and

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[1] For the general laws applying to the reversed cycle the student is referred to pages 55 to 62.

(*b*) Those using a saturated vapor, usually that of anhydrous ammonia, ether, carbonic acid, etc.

Of these two sub-classes the air-machine is restricted almost wholly to use on shipboard. Of the vapor-machines, probably 95 % rely on anhydrous ammonia as their working-substance.

The air-machines used purely for refrigeration always work on a reversed Joule cycle. Referring to Figs. 23, 24, and 25, suppose that the shaft of the engine receives power from without which suffices to rotate it in a reversed direction. The direction of flow of the air, as indicated by the arrows, will also be reversed. In Fig. 25 let AE represent a volume of atmospheric air taken in by the larger cylinder at a temperature of 520° absolute. Its thermodynamic condition is shown at E, Fig. 24. Let it be compressed adiabatically to D. Its temperature will be 1342° absolute, or 881° F. Let the air now be passed through the chamber T_1, which has now become a "refrigerator" by being equipped with coils of pipe containing a circulation of cold water, until its temperature has been reduced to 59° F. again, or 520° absolute. Its volume at E being unity, that at D is 0.1039 and that at C is 0.0591. Let this compressed air under ordinary temperature now enter the smaller cylinder and reëxpand there to atmospheric pressure. Its volume will now be 0.584 cubic foot and its absolute temperature 204°, or 257° below zero Fahrenheit. It may now be exhausted into the room which is to be cooled at atmospheric temperature.

Of course, in actual practice no such low temperature as the one just calculated is attained. The pressure-limits which were chosen as suitable for the heat-engine of Problem 29 are far too great for the refrigerating-machines; the cold produced is far greater than that desired for the preservation of meat, etc., and the machine itself would

freeze up in attempting to produce it.   But the figures will illustrate how a Joule engine can be reversed for the production of cold.

It is obvious from Fig. 25 that power must be supplied to keep the cycle in operation.   The positive work developed, the area $cCBA$, is much smaller than the negative work absorbed in compression, the area $AEDc$.   As to efficiency, that depends upon the temperature of the refrigerated room itself, which is represented in Fig. 24 by the xX-level.   The heat taken from the room is that beneath the curve BX.   That beneath XE represents leakage of heat from atmospheric temperature into the working-substance.   Again, the heat is discharged, along DC, into water which cannot have a temperature higher than the C-level; therefore all of the area above that level represents wasted work, — similar to that of the man who supposedly lifts a barrel on his shoulder in order to raise it to a platform situated a foot above the ground.   As $BEDC$ represents the work done, the efficiency must be the area $OBXz$ divided by the area $BEDC$, obviously a much poorer efficiency than the maximum possible, or that of a rectangle lying between the temperature-levels of C and X.

The Ammonia Compression Machine. — The class of refrigerating machines to which the ammonia compression machine belongs comprises the great majority of those used in industrial enterprise.   All of the largest machines are of this type.   Its action also typifies that of all other machines making use of a liquid refrigerant which is evaporated into a saturated vapor and then returned to liquid form by compression and condensation.

The operation of the machine is illustrated diagrammatically in Fig. 31.   At the left is seen a "liquid-reservoir," which holds the stock of refrigerant (in this case liquid anhydrous ammonia) under a pressure sufficiently heavy to

eliminate all danger of vaporization under all ordinary temperatures. The liquid exists, in fact, at atmospheric temperature, and may be carried in this condition in un-lagged pipes to any locality, however distant, without any loss of its refrigerative capacity. Leakage, of even the most minute character, must always be guarded against, on account of the penetrating and unpleasant character of the pungent odor of ammonia, and also on account of the
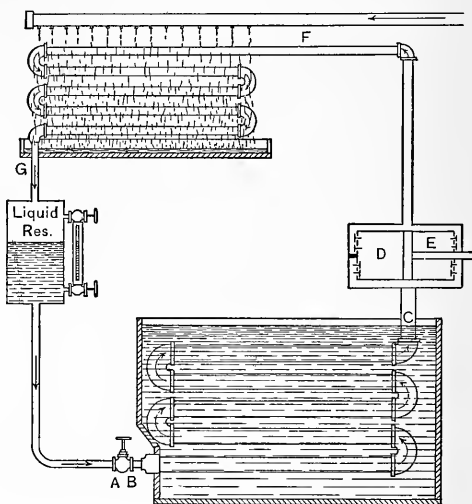
FIG. 31

expense involved in maintaining the stock of ammonia in the face of leakage. The liquid-reservoir is shown equipped with a gauge-glass whereby the condition of the stock of liquid may be seen at a glance.

From the liquid-reservoir the ammonia passes, through a pipe known as the "liquid-line," to one or any number of expansion-valves, situated wherever refrigeration is desired. One of these expansion-valves is shown at *AB*. Upon escaping through this valve, which is usually a needle-valve

of quite fine construction, the liquid finds itself freed from the so-called " head-pressure" of something like 160 to 200 pounds per square inch which kept it in its liquid condition. Beyond this valve it is under the comparatively light "back-pressure" of about 15 pounds by gage which prevails in the coil BC. The boiling-point of ammonia under this pressure is about zero Fahrenheit. In consequence, so long as the ammonia finds itself in contact with anything warmer than this temperature it will extract heat from it which is absorbed in the form of latent heat of vaporization. In exactly similar fashion a tea-kettle filled with water, finding itself in contact with a surface hotter than 212° F., will abstract heat from it with which to vaporize the water; nor can the metal of the tea-kettle be brought to a higher temperature than 212°, by the application of ever so much heat, so long as there still be water in it.

In this way the ammonia cools the pipes BC so nearly as it can to zero Fahrenheit, and with them anything in contact with their outer surface. The coil BC may be situated in the room to be refrigerated, but more commonly it is placed in a tank of strong brine, as shown in Fig. 31, which is chilled to a temperature of 10° to 20° F., and is then circulated through pipes in the rooms to be refrigerated, or around cans containing pure water to be frozen into ice. The object in interposing the brine between the real refrigerating process and the object to be refrigerated is to obtain stability of temperature. The slightest touch of the valve AB will produce considerable variation in the refrigerative activity of the coil BC. With the great mass of brine present all such variations are absorbed without producing a correspondingly fluctuating effect upon the object to be refrigerated.

So long as any liquid ammonia remains unevaporated in the coil BC it maintains its temperature at zero Fahrenheit.

Immediately upon complete evaporation, however, the higher temperature of the brine produces superheat, and this is increased by the heat of the atmosphere as the ammonia-gas passes from C to the compressor at D, a transit which may cover many feet of pipe.

In the compressor DE the pressure of the ammonia-gas is raised from the back-pressure to the head-pressure again,
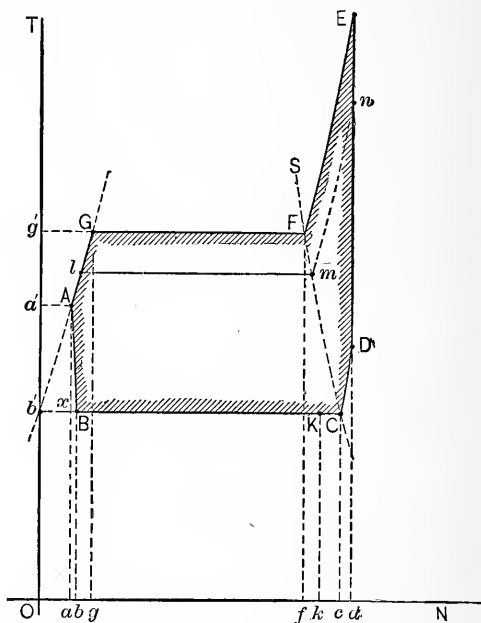
FIG. 32

and under this heavy pressure the hot, dry gas is passed through the condenser-coils FG, over which is running a circulation of cold water. The chill of the water first reduces the temperature of the gas to the boiling-point under the head-pressure, and then, since this is always hotter than the water, condenses it back into its liquid form. From the condenser FG the liquid trickles down into the

liquid-reservoir, or receiver, as it is sometimes called, where it awaits use over again.

It is to be noted that, going in the direction of the flow of the ammonia, the back-pressure prevails from the expansion-valve AB to the compressor DE, while the head-pressure prevails from the compressor DE to the expansion-valve again. This divides the system between two distinct pressures. The first, or lower, is so regulated as to maintain the proper temperature in the coil BC; this is done by adjustment of the expansion-valve and sometimes by vary-
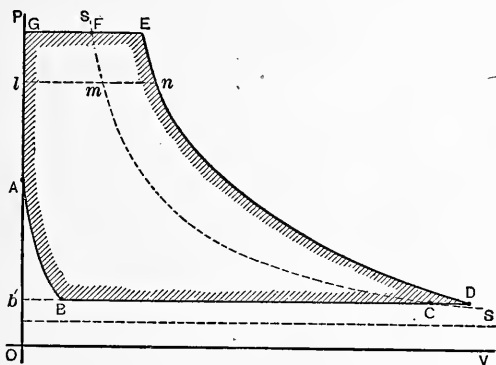
FIG. 33

ing the speed of the compressor. The second, or higher, pressure depends upon the temperature and amount of cooling-water supplied; it regulates itself more or less automatically, rising, under the influence of the discharge from the compressor, until the increased temperature makes the transfer of heat to the water sufficiently rapid to maintain thermal equilibrium.

The thermodynamics of the process is shown in Figs. 32 and 33. The point A represents the primary condition of the liquid as it approaches the expansion-valve AB. The passage through the valve, which consists of simple wire-

drawing or throttling, is shown by the constant-heat curve
AB. It is evident from this that a portion of the refrigera-
tive power of the ammonia is consumed in reducing its
own temperature to its boiling-point, before any can be
available for refrigerating other bodies. At the point B,
when the ammonia is first ready to chill the coil BC, the
proportion $\dfrac{b'B}{b'C}$ has already been vaporized and is incapable
of further refrigerative activity.

As the ammonia traverses the coil BC it is gradually
evaporated under the constant back-pressure, as shown by
the line BC. When vaporization is complete at C, which
is usually accomplished before the ammonia leaves the
coil, superheat begins. It ends as the gas reaches the
compressor in the condition D.

The next step consists of the adiabatic compression of
the gas in the compressor, as shown by the line DE, the
gas being discharged from the compressor in a condition
of high superheat.

As the gas flows from the compressor through the con-
denser-coils its temperature is first reduced from E along
the line EF, it is then condensed isothermally to liquid
form along the line FG, and finally it is cooled as a liquid,
during its stay in the receiver and on its way to the ex-
pansion-valves, along the curve GA. The cycle is now
complete.

The efficiency, capacity, etc., may be calculated as in the
other cycles. It is to be especially noted, however, that
whereas in the heat-engine cycles the thing supplied is
heat, or Q_1, and the thing wanted is work, or Q_w, and
hence the efficiency is $\dfrac{Q_w}{Q_1}$, now the thing supplied is work,
or Q_w, and the thing wanted is refrigeration, or heat-ab-
stracted, or Q_2, and hence the efficiency of a refrigerating-

machine is given by the expression $\dfrac{Q_2}{Q_w}$. But since the former is not a portion of the latter, the two being quite independent quantities of energy, the ratio just expressed is not properly an efficiency at all, but a sort of coefficient of efficiency. It is none the less useful, and custom has warranted its continuance.

To illustrate, let a set of conditions be assumed.

Problem 32. What will be the amount of refrigerative effect and the efficiency of operation if one pound of liquid anhydrous ammonia at a temperature of 69° F. and a pressure of 210.7 pounds absolute be evaporated under a back-pressure of 29.74 pounds absolute, the temperature of the gas reaching the compressor being 39° F. ?

Reference to Figs. 32 and 33 and to the ammonia-tables [1] places at hand all of the necessary guides and data for calculating all of the heat-quantities desired. The values of abscissæ and ordinates for the various points of the cycle are tabulated below. The manner of their calculation is as follows : —

Point	Pressure Lbs. p. sq. in. abs.	Volume Cubic feet	Temperature Fahr.	Temperature F. Abs.	Entropy
A	104.91	0.109	59	520	0.1325
B	29.74	0.835	0	461	0.1401
C	29.74	9.63	0	461	1.2277
D	29.74	10.44	39	500	1.2689
E	210.7	2.361	340	801	1.2689
F	210.7	1.59	100	561	1.0826
G	210.7	0.109	100	561	0.2161
K	29.74	8.877	0	461	1.1317

The cycle is most readily understood if assumed to start at *A*, where the pressure upon the liquid first becomes low

[1] See Peabody's "Steam-Tables," Table X, for instance.

enough to permit vaporization. The path AB is a constant-heat curve of wire-drawing, whence the heat at B, or the area $Ob'Bb$, must equal that at A, or the area $Ob'Aa$. From the ammonia-tables the latter is found to be 64.6 B.t.u., reckoning from an arbitrary zero (for this problem) of zero Fahrenheit, at b'. Therefore, in Fig. 32,

$$b'B = 64.6 \div 461 = 0.1401.$$

The path BC represents the isothermal vaporization in the coils BC. The area $Ob'Cc$ must represent the latent heat of vaporization, given as 566 B.t.u.; whence $b'C = 1.2277$. The path CD represents the superheat on the way to the compressor and has the equation

$$N - N_C = S \log_e \frac{T}{T_C}.$$

The path DE shows the adiabatic compression in the compressor, and $EFGA$ the cooling under constant pressure in the condensing-coils and in the liquid-lines. For these latter curves one must know the specific heats of ammonia under constant pressure: 1.095 for the liquid and 0.522 for the vapor, and the ratio of the two specific heats for the vapor: 1.31 or 1.317. Since all of the conditions of the vapor studied in the present problem lie close to the saturation-point, and consequently the specific heats are not constant, the laws governing these curves will not be found in exact agreement, in their mathematical results, with the ammonia-tables; but the variations will not be sufficient to affect the accuracy of the study from an engineering standpoint.

The determination of the heat-quantities handled, Q_1, Q_2, and Q_w, requires little discussion. Q_1, the area $aAGFEd$, is readily found to be the sum of the heat of the liquid ($aAGg = 45.4$), of the latent heat $gGFf (= 486.0$), and of the heat of superheat $fFEd (= 157.0$), or 688.4 B.t.u. Q_w,

the area $ABCDEFGA$, can only be found as the difference between Q_1 and Q_2, the area $aABCDd$. In Q_2 occurs the area $aABb$; reference to page 108 gives it as 4.3 B.t.u. The area beneath BC must be the latent heat of vaporization along $b'C$ minus the heat of the liquid between $b'A$; because the area $Ob'Bb$ must equal the area $Ob'Aa$. The heat of superheat beneath CD is readily found from the specific heat and the range in temperature.

From these data Q_2 is found to be equal to the sum of 4.3, 501.4, and 20.3 B.t.u., or 526.0 B.t.u. This makes $Q_w = 162.4$ B.t.u. But of this thermodynamic value of Q_2 only the area $bBCc$ is useful refrigeration; therefore the efficiency-coefficient of the cycle is

$$\frac{bBCc}{ABCDEFG} = \frac{501.4}{162.4} = 3.087.$$

It has been pointed out by Mr. William Lee Church that the efficiency of the cycle might be improved if the waste cold lost in the transfer of gas from refrigerating-coil to compressor, $cCDd$, might be used to chill the liquid to its boiling-point before it is released through the expansion-valve, by jacketing the pipe leading to the valve AB (Fig. 31) with the pipe CD. This would result in bringing the cycle down the path GAb' to the lower level instead of down the path GAB. Q_1 would now become 753 B.t.u., Q_2 586.4 B.t.u., and Q_w 166.6 B.t.u. But the cold $cCDd$ is not sufficient to absorb all of the heat $Ob'Aa$; in order to carry frost visibly through this jacket-modification some liquid must be allowed to boil over into it, sufficiently to add to the cold an amount, $kKCc$, large enough to make the area $kKCDd = Ob'Aa$. From these considerations KC is found to be 0.0960, making $b'K = 1.1317$ and the area of useful refrigeration, $Ob'Kk$, = 521.7 B.t.u. The efficiency-coefficient is therefore now equal to $521.7 \div 166.6 = 3.130$,

N

a result some 1.4% better theoretically than that deduced above. In practice the gain is greater than this.

The Series Refrigerating Process. — The ordinary industrial demand for artificial cold is for cold of a very moderate degree. The rooms of a cold-storage warehouse for the preservation of meats and other provisions which must not be frozen are kept as closely as possible at a temperature of 34°–36° F. Those where fish, butter, etc., are frozen need to be only slightly below 32° F. Either of these duties are usually performed by a circulation of brine not colder than 20° F. and the same medium at the same temperature serves to freeze artificial ice. The brine is maintained easily at this temperature by a refrigerant having no lower a temperature than 0° F.

But for scientific purposes the need of obtaining much lower temperatures than these has long been urgently felt, though the amount of heat to be handled is naturally small. For none of these purposes is the Joule air-cycle fit. The refrigerant is too diffuse. All of the successful methods which were formerly relied upon for very low temperatures made use of the ebullition under reduced pressure of some liquid whose boiling-point was unusually low even under atmospheric pressure. The locality to be refrigerated, usually a small piece of laboratory-apparatus, could be surrounded by the liquid and the precious cold conserved to the last degree.

The liquids commonly used were carbonic acid or ether, operated upon the cycle of the ammonia-machine. But even under vacua these liquids would not absorb heat at a sufficiently low temperature. Recourse was then had to a thermodynamic series. Liquid carbonic acid, boiling under reduced pressure, would produce a degree of cold which, aided by high pressure, would condense some more refractory vapor, its heat being absorbed by the boiling

carbonic acid.   This refractory liquid, when once the pressure upon its surface were relieved, would boil and absorb heat at a much lower temperature than would the carbonic acid.   This process is the thermodynamic obverse of the Series heat-engine.

By this means many of the so-called permanent gases were liquefied in scientific laboratories and much of inestimable value was accomplished.   But the limitations of the process were found, in complication, expense, and rapidly diminishing efficiency, and rapidly increasing labor with which even the smallest quantity of heat could be abstracted at the lowest temperatures, at a point which still fell far short of attaining degrees of cold for which scientific investigators felt urgent need.   The entire situation has now been solved by the Linde liquid-air process, which can of course be applied to the liquefaction of other "permanent" gases almost as readily as it can to that of atmospheric air.

Lord Kelvin's Warming-machine. — It has been pointed out by Lord Kelvin that whereas, in the ordinary refrigerating-machine, the absorption of the heat Q_2 is the thing desired, the development of the discharged heat Q_1 being an undesired incident in the process, this aspect of the cycle might be reversed by applying Q_1 to the heating of buildings.   Indeed, the machine might be wholly devoted to this purpose, in which case Q_1 becomes the end in view and Q_2 becomes the incidental factor.   Thus, instead of applying the heat developed from burning coal directly to the rooms to be heated, this heat might be used in a heat-engine to develop power, the exhaust from the heat-engine being utilized for heating the building; the power so developed might then be utilized in driving a refrigerating-machine which absorbed low-temperature heat from out-of-doors and discharged its high-temperature heat into the

building.   The proposition might be paralleled, for eluci-
dation, to that of a man wishing to fill a land-locked basin
lying some few feet above the sea-level with water and
having, as a means with which to work, a small stream
falling from a mill-pond situated ten times as high as the
basin above the sea.   He might, in the first place, simply
let the stream flow into the basin, which it would eventually
fill.   This would correspond to the ordinary method of
heating buildings.   But if his small water-power were
extremely valuable, or if time were valuable also, he might
hasten matters by letting the small stream, in its fall into
the basin, drive a water-wheel.   By attaching to this water-
wheel a pump piped to lift water from the sea and to dis-
charge it into the basin, it is plain that the latter would be
filled much more quickly and at a much less cost of mill-
pond water.   If the basin were 10 feet and the small mill-
pond 100 feet above the sea-level, respectively, the basin
would be filled in the second case in one-tenth of the time,
or with one-tenth as much water drawn from the mill-pond,
as compared with the first case.   To illustrate the proposi-
tion thermodynamically the following problem will be
useful : —

Problem 33.   What will be the comparative amount of
heating accomplished (1) by applying a given amount
of heat directly to a building, and (2) by making use of a
Kelvin warming-machine made up of the steam-engine of
Problem 14 driving the ammonia compression machine of
Problem 32 ?   (In this, the temperature out-of-doors is sup-
posed to be that of the *BC*-level of Fig. 32.)

According to Problem 14, for every 100 B.t.u. supplied
to the engine 23.6 B.t.u. are transformed into work.   The
remaining 76.4 B.t.u. may be imagined as exhausted into
the building at a temperature of 131° F.   According to
Problem 32 (2d Case) the supply of 23.6 B.t.u. of work

will absorb from the lower level of zero Fahrenheit and discharge at the higher level of 100° F.: $3.13 \times 23.6 = 73.9$ B.t.u. of low-temperature out-door heat. This, added to the 23.6 B.t.u. of work which is converted into heat within the refrigerating-machine and to the 76.4 B.t.u. of exhaust-heat from the engine, makes a total of 173.9 B.t.u., which reach the building from the original supply of 100 B.t.u. The efficiency of the fuel has been increased by 73.9 %.

A little consideration will show that this effect does not stand as an infraction of the law of the conservation of energy, as may at first appear, but as a combination of the operation of that law within the heat-engine with the operation of the second law of thermodynamics in the refrigerating-machine. No heat has been created. Heat has been merely lifted up-temperature from out-of-doors into the building.

This plan may be best imagined as put into practice by heating the building with hot-water pipes filled with the circulation coming from both the steam and the ammonia condensers. It is, of course, impracticable at present by reason of the great cost of apparatus and of attendance. But it is by no means improbable that at some time during the present century, when coal shall have become much dearer and machinery much cheaper and more simple than now, the plan may be relied upon for the heating in winter (and the cooling in summer) of our larger buildings.
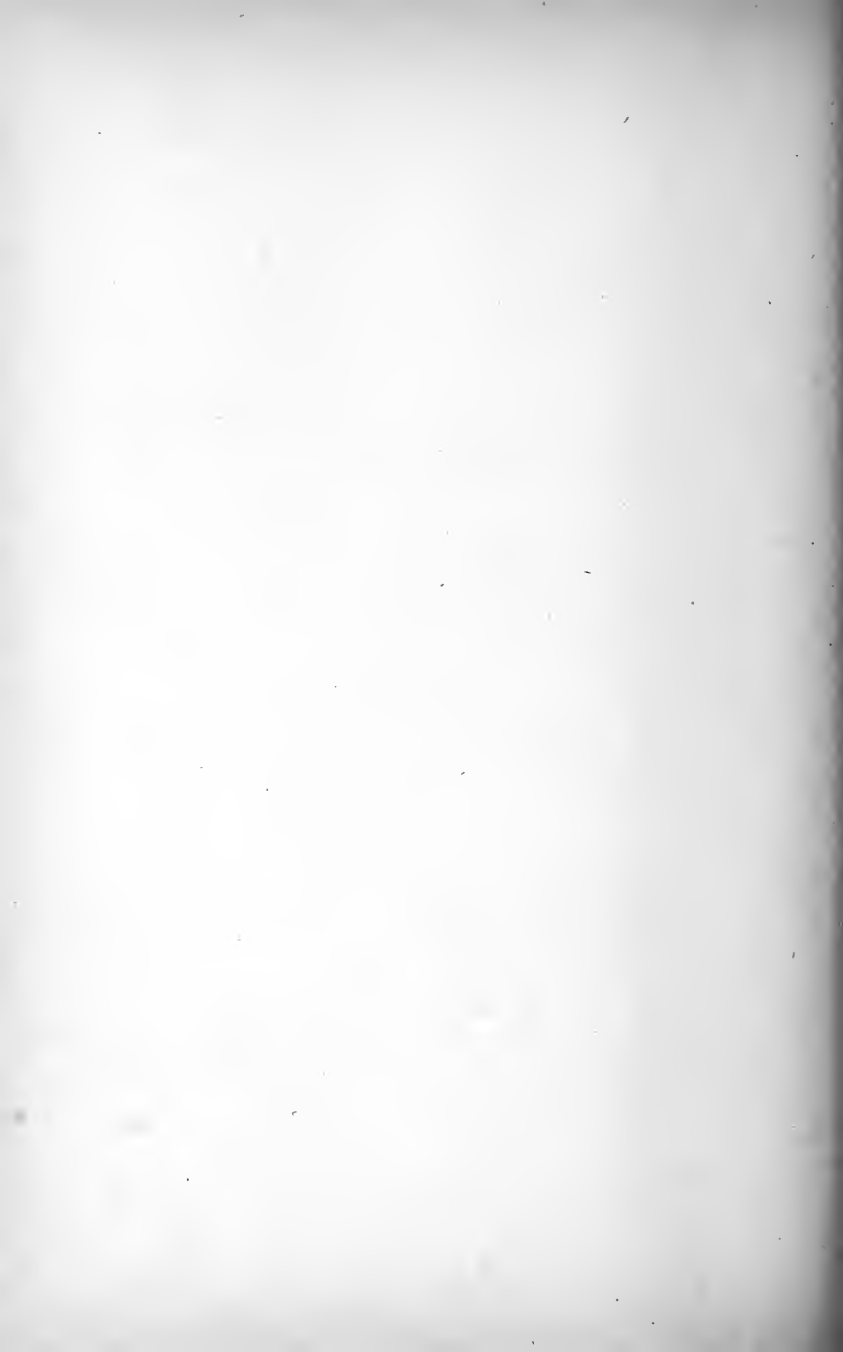
PART II

APPLICATION OF THEORY TO PRACTICE

CHAPTER I

THE SIMPLE STEAM-ENGINE

WITH a full understanding of the theoretical laws which limit and control the development of power from heat in the various types of engines in use, it is next in order to discuss in what points the operation of these theoretically perfect engines are modified by the limitations of actual construction and by those thermal phenomena, such as the conductivity of the cylinder-walls, etc., which have been carefully excluded from the foregoing analyses.

The theoretical cycles are modified by these factors in two distinct ways, either of which may be of prime importance in any given case : —

(1) *In Efficiency*, or in the proportion of the heat theoretically available for work which is actually developed ; and

(2) *In Capacity*, or in the amount of power available from a given size or weight of engine.

The first characteristic is customarily expressed, in regard to any given engine, by a ratio called its *cylinder-efficiency*. Following the notation of the first portion of the book, this *cylinder-efficiency* may be defined as follows : Let the heat supplied to the engine be represented by Q_1, that abstracted by the refrigerator by Q_2, that theoretically available for conversion into work by Q_w, and that actually converted into work by q_w; then the expression for the cylinder-efficiency would be —

$$F_c = \frac{q_w}{Q_w} = \frac{q_w}{Q_1 - Q_2}.$$

This expression is a valuable one for the indication of the proper place of the engine upon the efficiency-records. It covers the skill with which the designer and the builder together have been able to develop within the cylinder a proper portion of the work which the thermodynamic limitations permitted them to expect. It sweeps away all the complexity and ambiguity involved in attempts at deduction from the varied conditions of actual operation to a common basis of comparison for engines of any sort. Not only do the most diverse types of steam-engine fall into line for such equity of comparison, but gas-engines or hot-air engines lend themselves to the same process with equal docility and permit themselves to be placed thereby upon the same platform with the steam-engines without injustice to either.

For instance, it happens that a common value of cylinder-efficiency for simple single-cylinder steam-engines and for Otto gas-engines is 65%. That is, of the work theoretically available in each case the designer is able to develop about two-thirds. This proportion sometimes rises above 70%; it more frequently falls to 60%, or even below. It is also an invariable rule that this proportion varies inversely with the engine's complexity. That is, it is highest in simple engines, lower in double compound, and lowest in triple compound engines.

The only other factor needed to express the value of the engine as a means of development of power is the efficiency with which the work developed within the cylinder is transmitted to some useful destination, usually the belt. The loss in transmission is, of course, due solely to engine-friction. The ratio of work arriving at the belt to that developed upon the piston is referred to as the *mechanical efficiency* of the engine. If we represent the work thus transmitted to the belt by q_b, the several efficiencies might be expressed as follows : —

(1) *Cycle-efficiency* $= \dfrac{Q_w}{Q_1}$;

(2) *Cylinder-efficiency* $= \dfrac{q_w}{Q_w}$;

(3) *Thermodynamic efficiency* $= \dfrac{q_b}{Q_1}$ or $\dfrac{q_w}{Q_1}$;

(4) *Mechanical efficiency* $= \dfrac{q_b}{q_w}$.

Only the last three are ordinarily needed or used. As to the thermodynamic efficiency, there is a lack of unanimity as to whether the belt-work or the indicated work should form the basis for comparison; but the only grounds for adhering to the latter are a custom grown up from an unfortunate mistake in the past. All of the arguments are in favor of referring to belt-power only. If this is done, the cylinder-efficiency and the thermodynamic efficiency cover all needs for identifying the engine's rank as a transformer of heat into useful energy.

As to capacity, there is no accepted form for the expression of the engine's rank relatively to other engines. A common one is to state the number of horse-power developed per ton of machinery. This definition usually includes the boilers and all accessories, and is very useful in marine engineering. But for the study of the science of engine-design it is too comprehensive. The number of pounds weight of actual engine per horse-power is also widely used. The only objection open against it as a characteristic of capacity is that it gives no idea as to either space or cost involved. In the writer's study of the gas-engine problem he has needed to bring engines of the most diverse types into some comparison which would include all of these items in a way at least approximately just, and for that purpose has made use of what was defined as the *builder's ratio* in Chapter VIII, Part I. It

was found to be of great service for this purpose; but the practice is not yet a matter of general acceptance.

Before entering upon a general discussion of the modification of theory in practice, however, it is necessary to acquire some familiarity with the machinery, so to speak, of engine-design, of the graphical methods whereby the actual engine-cylinder is brought into close touch with the purely theoretical diagrams which have been discussed hitherto.   This can best be done by entering upon a given problem of engine-design.

Problem 34. *Wanted:* An engine to develop 100 horse-power under a boiler-pressure of 100 pounds by gage, when exhausting to the atmosphere and running at a speed of 100 revolutions per minute.

The choice of the items just specified might call for the exercise of judgment on the part of the designer or they might be determined for him by existing conditions.   In either case, immediately the problem is entered upon, his judgment is called upon to decide upon what *type* of engine the design is to be founded.

The wisdom of this method of procedure needs more emphasis than mere passing mention.   The chief characteristics in operation which determine the worth or the unworth of an engine are determined, not by the detail of design and construction which the engine embodies, but to the general type or class to which it belongs, and to the correspondence between the type chosen and the service to be performed.   It is too widespread an idea that the successful design of a steam-plant consists in the discovery of the "best" engine, the "best" boiler, the "best" pump, etc.   This is far from right.   There is no literally best engine, etc.   There are many better ones and many worse. There are some so bad that they would be worthless any-

where; but there are none so good that they will not be worthless if located unwisely and set to perform work for which they were not designed. It is for the identification of the points in which engine and service should harmonize, and their recognition in practice, for which consideration must be turned to the question of the broad *types* of engine. If the wrong type be selected, for purchase or for design, no amount of skill in detail can produce good, or at any rate, the best, results.

The distinguishing characteristics between the several types of engine, excluding questions purely of form, such as vertical *vs.* horizontal, etc., depend almost entirely upon the class of valves and valve-gear with which each is provided. The study of the kinematics of valve-gears is no part of the work in hand; but the following analysis of their mechanical and thermodynamic effects is essential.

All cylinder-and-piston steam-engines come under classification into three distinct types, each of which has its plain characteristics : —

      I. The single-valve fixed-eccentric type ;
     II. The four-valve fixed-eccentric type ;
    III. The single-valve variable-eccentric type.

Type I. This finds its typical illustration in the plain slide-valve throttle-governed engine, now almost obsolete for general power purposes. Its typical indicator-cards are shown in Fig. 34, the full line representing normal load, the dotted line overload, and the broken line light load. The resultant characteristics of the engine, when compared with those of the other types, may be stated relatively as follows :

1. *Steam-distribution :* All events fixed.
2. *Efficiency :* The poorest.
3. *Capacity by unit of space or weight :* The greatest.
4. *Speed :* Not limited by valve-gear ; usually medium.

5. *Standard Ratio of expansion:* 1.2 to 1.4.
6. *Clearance:* Maximum.[1]
7. *Regulation:* Poor.
8. *Complexity:* The least.
9. *Cost:* The least.
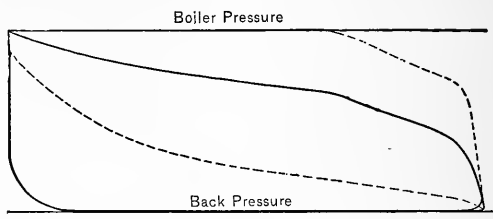
FIG. 34

It may sometimes be found modified in the following particulars, without effect upon its characteristics:—

(1) Piston-valve instead of slide-valve;

(2) Hand-control of speed instead of governor-control. It is now used chiefly for portable work, such as hoisting, pile-driving, etc., or in very small powers. Even for small powers, however, if the service be a continuous one, engines of Type III may better be substituted.
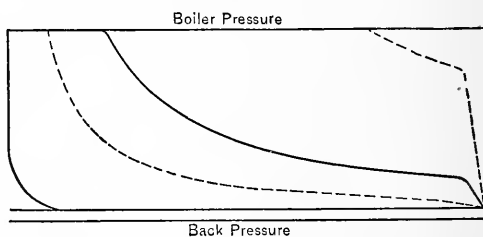
FIG. 35

Type II. This finds its typical illustration in the standard Corliss engine, with centrifugal governor and drop cut-off gear. Its typical indicator-cards are shown in Fig. 35.

[1] For definitions of "clearance" and "cylindrus" see page 205.

The resultant characteristics, compared with those of the other types, are : —

Steam-distribution : Cut-off variable by governor; exhaust, compression, and admission fixed.

Efficiency : Above about 100 horse-power, maximum; below about 100 horse-power, medium.

Capacity per unit of space or weight : Minimum.

Speed : Limited by valve-gear to about 100 r.p.m.; usually less.

Standard Ratio of expansion : 5 to 6.

Clearance : Minimum.

Regulation : Poor to medium.

Complexity : Maximum.

Cost : Maximum.

This type is found under very wide modifications without effect upon its characteristics. Any engine in which the steam-distribution is as stated belongs to this class, although the valves may be slide-valves, as in the Greene, or poppet-valves, as in the Putnam, or gridiron-valves, as in the Wheelock, or Myers cut-off double-slides, as in the Buckeye. It may have two fixed eccentrics instead of one.

If the steam-valves, although Corliss in form, are actuated by positive connection to a variable eccentric in a shaft-governor, as is now done in some designs of Corliss engines in order to permit high rotative speed, the engine belongs not to Type II but to

Type III, which finds its typical illustration in the high-speed, so-called " automatic," shaft-governor engine, usually fitted with a single piston-valve. Its typical diagrams are those of Fig. 36. The resultant characteristics, compared with those of the other types, are : —

Steam-distribution : All events varied with the cut-off by the governor.

Efficiency: Above about 100 horse-power, medium;
  below about 100 horse-power, maximum.

Capacity: Medium.

Speed: Not limited by valve-gear; usually high.

Standard Ratio of expansion: 4.

Clearance: Medium.

Regulation: The best.

Complexity: Medium.

Cost: Medium.

Amongst the possible modifications of this type the one
of the first importance is that involved in the substitution
of the Stephenson-link or similar reversing-gear in place of
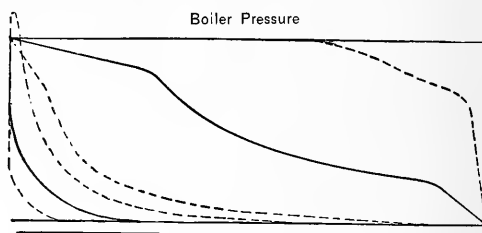
FIG. 36

the variable eccentric, for they are virtual equivalents.
This brings all locomotive and all marine engines, except
the old-fashioned walking-beam paddle-engine, into this
class.   Any engine having a slide-valve, balanced or un-
balanced, single or double ported, or divided into two or
four parts, if driven from a variable eccentric, comes within
the same classification.

To return to Problem 34, it may be assumed that the
engine is to belong to the Type II class, with a normal
cut-off at one-fifth stroke.   For while the text-book must
emphasize the importance of selection according to type-
characteristics, it can give no rules for making a wise

selection.   That must depend upon judgment cultivated by experience.

The first duty is to construct a preliminary hypothetical indicator-card upon which to base estimates.   This will, in the first instance, take the simple form $ABCDE$, Fig. 37. The pressure at AB is 100 pounds by gage, or 114.5 absolute; that at DE is atmospheric plus a proper allowance for back-pressure, say, 16 pounds absolute.   The volumes are unknown; but $ED = 5\,AB$.

In the expansion of saturated steam in an engine-cylinder the supposed adiabatic is always modified by heat-interchanges with the cylinder-walls.   These interchanges are quite complex, and vary widely in different cases.   They will be discussed more elaborately at a later point in the text.   Taken all together, however, under average conditions, they usually have the effect of bringing the expansion-curve away from a true adiabatic and into approximation with the equilateral hyperbola having the equation $\dfrac{P_1}{P_2} = \dfrac{V_2}{V_1}$. So, for the purposes in hand, it is always considered sufficiently exact (and it is certainly very convenient) to adopt this curve as being the same as BC of Fig. 37.

N.B.   This equation is identical with that for the *isothermal curve with air or other permanent gas as the working-fluid*.   But this fact must not lead to confusion with the conditions which prevail when steam is the working-fluid.   The above equation does *not* represent an isothermal with steam, as is often erroneously stated.   The isothermals in the steam-engine cycle are the lines AB and DE, and their equation is $P = $ a constant.

For the drawing of the equilateral hyperbola BC, Fig. 37, there is a very convenient graphical method.   Although the data of the problem under discussion do not give any direct information as to volumes, and it is therefore impossible to draw the curve to scale, at this point in the development of the problem, yet it will be convenient to have

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before the eye an illustrative curve of indefinite scale upon which to base argument. Taking any initial volume at the stated boiler-pressure, as at the point B, the remainder of the curve may be determined in this way :—
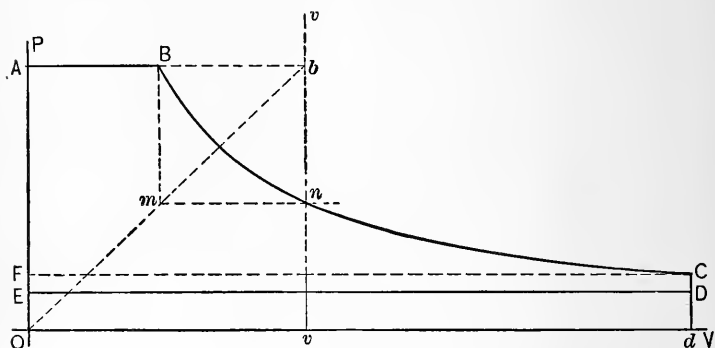
FIG. 37

Let it be desired to know the pressure which will prevail at some other *volume* than the initial, such as at vv.

(1) Draw the constant-*pressure* line Bb until it intersects vv at b;

(2) From b draw a diagonal bO to the absolute zero-point of pressure and volume ;

(3) From B draw a constant-*volume* line Bm until it intersects bO at m ;

(4) From m draw a constant-*pressure* line mn until it intersects vv at n. n is the desired point through which the curve is to be drawn.

For finding an unknown point at any desired *pressure* other than the original, interchange all the italicized words "pressure" and "volume" in the above. Volumes or pressures may be greater or less than the original without altering the rule to be followed.

The area of the cycle of Fig. 37 may be conveniently divided into two portions: $ABCF$ and $FCDE$. Let the

volume at $C = V_C$. Then area $ABCF = P_B V_B \log_e r = (114.5 \times 144) \times \dfrac{V_C}{5} \times \log_e 5$. The area $FCDE = FC \times CD = V_C[(\frac{1}{5} \times 114.5) - 16] \times 144 = 6.9 \times V_C \times 144$. Since the engine is to develop 100 horse-power at 100 revolutions, or 200 strokes, per minute the work necessarily developed per stroke, in foot-pounds, must be

$$W = \frac{100 \times 33000}{200} = 16,500 \text{ foot-pounds.}$$

Hence $(114.5 \times 144 \times \frac{1}{5} V_C \times 1.609) + (6.9 \times 144 \times V_C)$

$$= 6300\, V_C = 16,500,$$

or $\qquad V_C = 2.620$ cubic feet.

This result is of no immediate value, because no allowance has been made for the effect of clearance and compression. In professional engineering offices this allowance is usually made by judgment, as a percentage in addition to the calculated volume; and since the result desired is nothing more accurate than the general dimensions of a cylinder of a capacity which is entirely nominal, this is usually amply accurate. But where the necessary experience as a basis for estimate is lacking, clearance must be allowed for in the following way. In compound engines, too, where it is desired that a certain effective cylinder-volume be attained in order to establish a certain effective cylinder-ratio, it is important that the effect of clearance be carefully considered.

Let the probable clearance to be expected in the cylinder when completed be estimated at 2 per cent. Suppose that it be the designer's judgment that this clearance should be filled with steam of two-thirds boiler-pressure at the completion of compression. The situation is shown in Fig. 38.

For the moment call the volume of the cylindrus, or fC,

equal to unity. Then $Ff = 0.02$ and $FC = 1.02$. AB must equal 0.204, in order to maintain a ratio of expansion of 5. Since the V_C of Fig. 38 is equal to 1.02 times fC, it may be stated, from Fig. 37, that the area $aBCDE$ of Fig. 38 is
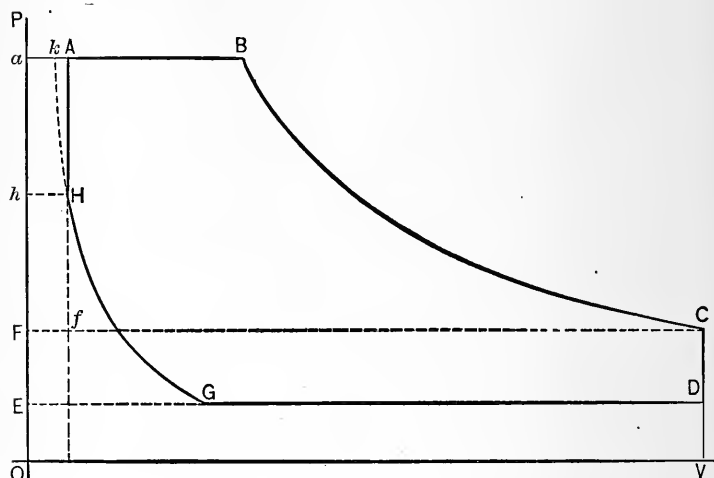
FIG. 38

equal to $1.02 \times 6300 fC = 6426 fC$. The area $hHGE$ must be equal to $P_H V_H \log_e (P_H \div P_G)$

$$= \left(\frac{2}{3} 114.5 \times 144\right) 0.02 \, fC \log_e \left(\frac{2}{3} \times \frac{114.5}{16}\right) = 343 \, fC.$$

The area $aAHh = \frac{1}{3} \times 114.5 \times 144 \times 0.02 \, fC = 110 \, fC$. Therefore the area $ABCDGHA = (6426 - 343 - 110) fC = 5973 \, fC$. Therefore, by the premises of the problem, $5973 \, fC = 16,500$ ft.-lbs.; whence $fC = 2.76$ cubic feet.

Piston-speed. — This volume can of course be arranged in any form that the designer prefers: a long narrow cylinder or a short and wide one. This would ordinarily be determined by the *piston-speed*, which is arbitrarily defined as the total distance travelled by the piston in feet per

minute. It is usually considered desirable to keep this speed between 400 and 800 feet per minute, the larger figure for large engines and the smaller one for the little ones. In the present problem a piston-speed of 450 feet per minute would give a stroke of 27 inches and a diameter of 15 inches, in order to displace 2.76 cubic feet per stroke. In practice the stroke would probably be made 28 inches, rather than 27.

Mean Effective Pressure. — The mean effective pressure in an engine-cylinder is defined as *that pressure which, if evenly maintained throughout the entire stroke, would accomplish the same work as the actual varying pressure.*

The writer's experience has gone to show that for the ordinary purposes of a preliminary estimate, lumping all the questions as to probable back-pressure, cushion, and clearance to be incurred under a single coefficient based upon the type of engine, the following formulæ are sufficiently accurate and are much quicker to handle than the plan just described. If P be the given boiler-pressure (by gage) then the mean effective pressure, p, can be had as follows:[1] —

For Type I: $p = 0.7 P - 6$; or $p = \frac{3}{4}(P + 15) - 20$.

For Type II: $p = 0.4 P - 2$; or $p = \frac{1}{2}(P + 15) - 17$.

For Type III: $p = 0.6 P - 8$; or $p = \frac{2}{3}(P + 15) - 23$.

The second set of formulæ are somewhat more accurate and also more cumbrous than the first set. Since accuracy in such a case is a purely comparative term it is proper to add that the first set becomes unsatisfactory only when extreme conditions of low boiler-pressure prevail or when a vacuum is used.

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[1] These formulæ are also useful for estimating the proper capacity of an engine the records of which are wanting.

Application of the first formula for Type II to the present problem gives $V_C = 3.015$, or a cylinder measuring $15'' \times 30''$, which is on the safe side of the more exact figure determined above. The second formula gives a cylinder $15'' \times 28''$, which agrees exactly with the dimensions calculated above. At pressures lower than 100 by gage, which is unusually high for a simple engine, the agreement between the results from the empirical and the exact formulæ would be closer.

With this preliminary outline of the relations between pressure-volume curves and the actual cylinder, attention may be intelligently returned to the thermodynamics of engine-action. The discussion of the steam-engine cycle, which was left in its purely theoretic aspect, may now be carried on into its actual aspect in practice.

The Effect of the Cylinder-walls upon Efficiency. — The Rankine cycle, the cycle of the perfect steam-engine, is impossible of reproduction in practice. The suppositions upon which its theory is founded are : —

(1) No heat-interchange between the steam and the surrounding walls ;

(2) Complete expansion to the lowest available pressure and temperature ;

(3) Instantaneous action of valves ;

(4) No leakage past valves or piston ; and

(5) No clearance.

None of these conditions is realizable in practice except the second, and that is seldom desirable for practical reasons. (See pages 234–235.)

Condensation and Reëvaporation. — Of the five sources of variation of the actual cycle from the theoretic, the first is by far the most important in its effects upon efficiency. It may be remembered that the work which placed the

name of Watt at the head of the list of steam-engineers was the study of this phenomenon. Impressed by the large amount of steam consumed by a small model engine submitted to him for repair, he attained its explanation by the following series of determinations: —

1. *The proof beyond doubt that a greater volume of steam entered the cylinder at each stroke than the cylinder could possibly contain.* Since the only possible method of measuring the quantity of steam entering the cylinder at each stroke was in terms of its *weight*, which could be measured before the water entered the boiler, Watt's task involved the determination of the specific volume of steam. His first approximation gave 1 : 1800 as the ratio between volumes of a given weight of water when liquid and when vapor. This led to the rule-of-thumb, current for long afterward among steam-engineers : "A cubic inch of water makes a cubic foot of steam," which implies a ratio of 1 : 1728. This, of course, was all based upon steam of atmospheric pressure. We now know that the true ratio is about 1 : 1660. Knowing the weight of water evaporated in the boiler for each stroke of the engine, Watt knew from this ratio its volume. He was surprised to find it some four to six times that of the cylinder itself. In other words, the cylinder-condensation amounted to from 75 to 83%.

2. *Watt proved that this condensation was not due to radiation of heat from the external surface of the cylinder.* More accurately, these proofs were reversed in order ; that is, suspecting condensation from general considerations, and radiation being the readiest explanation of condensation, Watt first proved that the prevention of radiation was no remedy for the trouble. As this result would lead the ordinary mind to the conclusion that there could not have been any condensation, it is significant of Watt's

genius that at this point in his investigations he persisted
in the difficult task of proving conclusively that condensa-
tion did take place, and to an enormous amount. If the
cylinder-walls did not take up the heat released by this
condensation and radiate it, what could pick it up and
carry it away? The only other thing in the cylinder
besides the metal was water, and this hint led him to his
next step.

3. *The determination of the latent and specific heats of
steam and water*, which showed that even a slight portion
of the water injected into the cylinder at each stroke for
the purpose of condensation was quite able to account for
the trouble. To prevent it, therefore, could obviously be
prescribed Watt's famous rule, easily deduced from what
had preceded : —

4. "*Keep the cylinder dry, and as hot as the entering
steam.*" To accomplish the first, one must use a separate
condenser. For the second was needed the steam-jacket.
Both, but especially the first, have survived until the present
day as the first essentials to steam-engine economy.

It was but a slight step in advance of this to recognize
that the cylinder-walls as well as the water upon them were
active in losing heat from the entering steam to the exhaust.
It had been proven beyond doubt that it was not loss of
heat *through* the walls to the air, but the alternating pas-
sage of heat from steam to metal and water, and then back
again from metal to steam, which lay back of the greatest
loss of efficiency in the steam-engine. This is still the fact
to-day, in spite of the century of effort at minimizing the
thermal action under discussion. Its determination at that
early date classed Watt as a scientist as well as a skilled
mechanic, and in both fields of activity a genius.

When boiler-steam enters an engine-cylinder, at or just
before dead-centre is reached, it finds itself within a disk-

shaped space enclosed by the cylinder-head, the piston and
a narrow strip of cylindrical barrel-surface.   This space is
also one with that of the ports and passages for admission
and exhaust of steam.   In both its portions this steam-filled
cavity presents a very great proportion of surface of sur-
rounding wall relatively to its volume.   This surface, too,
for reasons which have already been outlined and which
are to be elaborated later in this discussion, is always at a
temperature much below that of the entering steam.   The
transfer of heat from a moist vapor to iron is very rapid.
Therefore considerable steam condenses upon the surface
of the metal, at the same time heating the latter to approxi-
mately its own temperature to a certain depth, usually a
very small fraction of the thickness of the walls.

The steam thus condensed into water deposits upon the
surface of the metal in bead-like or sweat-like drops.   Since
the transfer of heat between water and iron, or between
steam and water, is still more rapid than that between
steam and iron or between iron and iron, the presence of
this water upon the surface of the metal very much exag-
gerates the phenomenon just described.   In fact, in many
of these cylinder-phenomena the water adhering to the
cylinder-walls is much more active as a thermal agent than
is the iron, which is to be explained by the fact that the
specific heat of water is much higher than that of iron.

It is to be especially noted that the greater portion of
all of the surface which is at any time brought to steam-
temperature is exposed to the steam *before* the piston moves
out upon its stroke, and this for a comparatively long
period of time.   This means that the bulk of the condensa-
tion occurring in the cylinder does so while the engine
is on dead-centre.   It is this fact which primarily ac-
counts for the need of *lead* in a steam-engine valve-gear.
Something like 20 to 40% of all the steam used is wanted

before the stroke begins.   The one other basis for lead —
the need for having the valve appreciably open when the
piston starts on its stroke with accelerating velocity — is
entirely a secondary one; for in every case the valve-open-
ing would be accelerated more rapidly than, and would
attain a maximum before, the velocity of the piston does
the same, even if there were no lead.

As the piston moves out upon its stroke more and more
cylinder-surface is exposed to boiler-temperature.   Con-
densation continues, but at a much reduced rate.   When
cut-off takes place the piston is usually moving quite rapidly
and the exposed surface is increasing correspondingly.

As soon as cut-off is effected, however, expansion ensues
and the temperature of the steam-and-water mixture within
the cylinder necessarily drops with the pressure.   The bulk
of the metal in the cylinder-walls, too, is not at the tempera-
ture of the atmosphere, but at some mean between that and
boiler-temperature, determined by the equilibrium which
is established between the various gains and losses of heat
without and within.   The temperature-difference between
the steam within the cylinder and the fresh surface un-
covered by the motion of the piston rapidly decreases,
therefore, after cut-off takes place, and with it decreases
the transfer of heat between the two.   The rate of con-
densation, which had steadily decreased as the piston ad-
vanced, soon becomes zero.   At this point in the stroke
the proportion of water to total weight of steam and water
within the cylinder may be anywhere from 10 to 60%,
according to the size of cylinder and conditions of opera-
tion.   At cut-off it is seldom more than 5 or 10% less
than this.

The temperature of the expanding steam has no sooner
fallen to the mean temperature of the iron than further ex-
pansion, or the opening of the exhaust-valve, carries it be-

low.   The temperature-difference between the two becomes
reversed, the direction of heat-transfer is reversed, and a
reëvaporation of the moisture deposited upon the cylinder-
walls takes the place of the previous condensation.   This
continues, being very much accelerated by the opening of
the exhaust-valve and the consequent reduction of pressure
upon the hot water, until the latter is completely evaporated
and the walls are dry.   After that the rate of heat-transfer
between metal and steam is reduced to a very small fraction
— something like one two-hundredth, according to some
authorities — of what it was while the walls were wet.

The reëvaporation of the water from the cylinder-walls
of course demands heat for its performance, and the ab-
straction of this heat from the metal lowers the temperature
of its inner surface practically to that of the exhaust-steam.
It is this abstraction of heat by reëvaporation, and not the
loss of heat to the outer air by external radiation or conduc-
tion, which makes the demand for initial condensation
upon the next admission of steam.   Comparing these two
sources of dissipation of heat : radiation and reëvaporation,
the former amounts to not over 1 to 10% of the latter.   It
is usually about 3% in single-cylinder non-jacketed engines.

The Entropy-temperature Analysis

This double process, *condensation* and *reëvaporation*, is
the first and greatest cause of the departure of actual steam-
engine efficiency from the maximum possible determined
by the external conditions (such as boiler-pressure, vacuum,
etc.) under which the engine is forced to operate.   The
reduction of this loss has been the chief anxiety of the
steam-engine designer since the days of Watt, and the
amount of its reduction has been one of the best measures
of progress accomplished.   The effect of purely external

conditions upon engine-efficiency is so well understood, the comparative cost and gain to be associated with a given rise of steam-pressure or decrease of back-pressure are so patent to the ordinary non-scientific designer, that their discussion, albeit important and demanding experience and judgment, is quite devoid of mystery.   With the internal factors of engine-efficiency, on the other hand, although identified in part as early as the time of Watt and pursued ever since, the difficulty of prediction before construction, or of measurement afterward, has prevented the profession from yet accumulating sufficient data for a satisfactory science of steam-engine design.   The amount of condensation and reëvaporation occurring in an engine is the resultant of a great many variables, all of which differ in each engine from those of any previous one with which experience has been had.   The building of a series of engines, in each successive one of which only one variable shall have been altered in comparison with the preceding one, is a task utterly beyond the limitations of engineering possibility, chiefly by reason of cost, but also on account of the indefinite length of the series needed for accomplishing a definite commercial result.   It is therefore of the utmost importance to reduce the study of condensation and reëvaporation, and of the other points in which the actual cycle departs from the theoretic, to as exact an analysis of cause and effect as possible.   It is equally important that the adopted method of doing this be applicable to every single engine-test, without necessity for using two tests in order to get one reading, and that a merely comparative one.

Such an analysis is quite possible, and demands for its performance, besides the modicum of patient labor always essential to an analysis of anything, only the following data : —

(1) *The dimensions of the cylinders*, including percentages of clearance and diameters of rods ;

(2) *The speed*, which should be known to a fraction of a revolution per minute, by counting over a period of several minutes ;

(3) *The indicated horse-power*, from cards taken under conditions, as to boiler-pressure, cut-off, etc., as nearly constant as possible ;

(4) *The amount of water (steam) passing through the cylinder per horse-power or per unit of time ;* and

(5) *That the leakage of valves and pistons has been brought to the minimum possible.*

These points are all known as the result of an ordinary engine-test.

At the start the following distinctions should be gotten clearly in mind.

Cylindrus and Clearance. — At every stroke of the piston a certain volume is displaced, equal to the effective area of the piston times the length of the stroke. This effective area may be the area of the piston itself, or there may be the cross-sectional area of a piston-rod, a tail-rod or a trunk to be deducted. The volume thus displaced is known as the *cylindrus*.

At the completion of each stroke there remains in the cylinder, beyond the piston in the direction of its late motion and between the piston and cylinder-head, a certain undisplaced volume, which includes all ports and passages back to the lips of the valves which enclose this space. This volume is known as the *clearance*. It is always measured and expressed as a percentage of the cylindrus.

The total volume in the cylinder at any point of the stroke always consists of the sum of the clearance and a portion of the cylindrus.

Cylinder-feed and Cushion-steam. — At each stroke of the engine there enters the cylinder a certain quantity of steam from the boiler. This steam may go to fill the clearance-space, it may condense in warming the cylinder-walls, it may go to propel the piston. Whatever its destination it has to be supplied by the boiler. It is known as the *cylinder-feed*. It is always measured in pounds avoirdupois.

At the beginning of each stroke, when the admission-valve opens, the entering steam finds the cylinder already occupied by a certain quantity of steam. This is the steam which has been imprisoned in the end of the cylinder by the closing of the exhaust-valve before the previous exhaust-stroke was completed. This steam is known as the *cushion-steam*, from the frequent reliance upon its elastic compressibility for arresting the motion of the piston at the end of the stroke without undue shock upon the crank and connecting-rod. It is always measured in terms of the weight of cylinder-feed passing through the engine at each stroke, and is stated as a proportion of one pound of the latter, as, for instance: "Twenty-three hundredths of a pound per pound of cylinder-feed." This ratio may be determined on the basis of a single stroke, or of a unit of time. It is usually easiest to measure the cushion-steam for a single stroke and the cylinder-feed for an hour; the two are then easily reduced to the same terms by means of the known engine-speed.

This cushion-steam should always be considered as a part of the engine's mechanism. If the cylinder-walls and the cylinder-feed were to refrain from thermal interference with it, it would give out in expansion during the working-stroke of the engine just as much work as it absorbed during compression. In this respect it would be the exact equivalent of a steel spring of perfect resilience. Nor does

this statement ignore the widely discussed question of the value of clearance and cushioning. The truth of it is not affected by the fact that the piston may *not* receive the work thus returned during expansion, because the exhaust-valve may be open or because the cylinder-walls never do refrain from thermal interference. Nor by the fact that, in some types of engines, the point in the stroke at which the exhaust-valve closes varies under varying load, and so varies the amount of steam trapped in the cylinder from moment to moment. All of these questions are quite secondary and incidental to the main one, viz.: That in every given stroke, or complete cycle, of an engine there is a portion of the steam present which does not enter with the entering steam, which does not leave with the exhaust-steam, and which cannot be regarded as a means of heat-supply to the engine. Moreover, this cushion-steam, unless it can be imagined as capable of storing up or supplying heat indefinitely, must have had balanced up to a net zero for it by the cylinder-feed, by the end of each cycle, all the losses or gains of heat whatever which it may have made at earlier points of the cycle. This must hold true whether the gains or losses went to the cylinder-feed itself, to the outside air, to work, or to aught else. Such is the clearing-house character of the cushion-steam.

For the cylinder-feed alone can bring heat into the engine. It alone, therefore, is chargeable with all losses of heat from the engine, whether they took place from it directly or indirectly, whether they be desirable losses (as in the form of work) or undesirable ones.

It is to be repeated, then, that the first object of a rational analysis of efficiency is to charge up all losses against the cylinder-feed. This can be done only through a knowledge of its volume and pressure at each instant of its existence in the engine. If, however, we turn to the steam-engine

indicator as a source of light upon this question, we are confronted with the immediate difficulty : *The indicator reveals volumes and pressures of cylindrus, not of cylinder-feed.* It is the first duty, then, to translate one into the other.

Let v equal the total volume existing within the cylinder at any point of the piston's motion. It is given by the abscissæ of the indicator-card, when measured from a vertical axis placed at the proper distance from the end to represent the percentage of clearance.

Let c equal the clearance-volume, a constant.

Let z equal the portion of the cylindrus which has been developed at the same point of the piston's motion. It is measured by the abscissæ of the indicator-card.

Let y equal the volume of cushion-steam at the same instant.

Let x equal the volume of cylinder-feed at the same instant.

Then $$v = c + z, \qquad (101)$$

and $$v = y + x. \qquad (102)$$

Whence $$x = z + c - y. \qquad (103)$$

In other words, from the known variable volume of cylindrus, z, the unknown variable volume of cylinder-feed, x, may be known by first adding the constant volume of clearance, c, and then subtracting the variable volume of cushion-steam, y.

Pressures, since all the volumes under discussion are freely open to another, must be the same in all the portions of steam at any given instant, and may be read from the indicator without translation.

Further consideration of the above proposition regarding volumes raises the following question : Since the proportion of cushion-steam to cylinder-feed is the same through-

out a given stroke, and since these two quantities of steam expand and compress together, in intimate mixture one with the other, why must not the volume of each vary in equal proportion as the pressure changes? Physically this must be the case. The entire body of steam within the cylinder must be in practically the same condition, except under unusual disturbances. But analytically and mathematically a marked distinction must be drawn. Since the cushion-steam cannot of itself constitute a source of heat, any loss of heat which it experiences at any given moment must be imagined as immediately made good by an equivalent supply of heat to it from the cylinder-feed. Conversely, any gain of heat which it may experience must be regarded as immediately turned over to the cylinder-feed, for the latter alone can carry heat out of the engine. In other words, the cushion-steam must always be regarded as expanding and compressing *adiabatically*. All losses and gains of heat, although actually experienced by either or both of the two steam-quantities, are to be charged against the cylinder-feed alone. This gives the volume of the cushion-steam as a known variable at any point, deducible from the known pressure at that point. This makes all of the terms of the right-hand member of Equation 103 known quantities.

The process indicated in Equation 103 is most easily performed graphically, especially as the quantities determined by the indicator come to us as graphical measurements. To facilitate this work, together with some other mathematical deductions needing to be drawn, the author has projected a large blank diagram fit for rapid work upon the drafting-board upon a scale so large as to minimize both graphical error and strain upon the eye.[1] There is nothing about it, however, which cannot be readily produced by

[1] Published by Progressive Age Pub. Co., 280 Broadway, New York City.

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oneself with the aid of a steam-table, though at considerable cost of time and trouble.   Its essentials are represented diagrámmatically in Fig. 39.

The sheet is divided into four portions, or quadrants, by one vertical and one horizontal axis meeting in the centre of the sheet at O.   The right-hand horizontal axis, from O as a zero-point, measures *volumes*, the scale being plainly
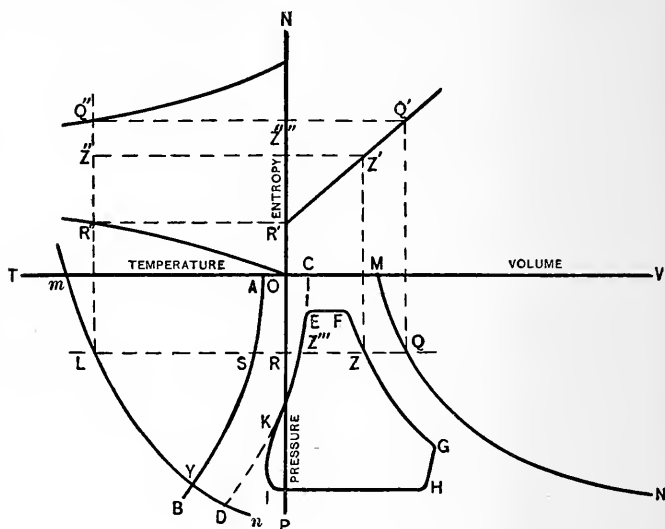
FIG. 39

indicated.   The lower vertical axis similarly measures *pressures*, except that the zero-point is at the edge of the paper.   The left-hand horizontal axis measures *temperatures*, the zero-point being at O, which represents the arbitrary zero of the steam-tables, the melting-point of ice, or 32° F.   The upper vertical axis measures *entropies* from a zero-point at O, which also assumes the arbitrary zero of the steam-tables.

From this arrangement it is obvious that the lower right-

hand quadrant may be used to indicate the relations between pressure and volume, as upon the ordinary indicator-card. The lower left-hand quadrant will display relations between temperature and pressure; it is also used, however, as a prolongation of the pressure-volume quadrant into a field of *negative volumes*, which are measured horizontally to the left from O on the same scale as positive volumes are measured from O to the right. The upper right-hand quadrant will display relations between volume and entropy. The upper left-hand quadrant will display relations between temperature and entropy, but in a position in reference to the eye rotated ninety degrees from that adopted in the previous chapters.

It is in the premises of the diagram that these several axes and fields of coördinates are to be used to reveal the condition of the cylinder-feed within the engine during any given stroke. It is supposed that from the data listed on page 205 (this chapter) the cylindrus and the clearance have been calculated in cubic feet, that the weight of steam passing through the engine in pounds per hour is known, and that the engine has run under conditions so nearly constant that either a selected actual indicator-card, or an artificial one constructed from the actual ones on a wise system of averages, may be taken as accurately representing the change through which the cylinder-steam passes as the stroke progresses. Further, as a matter of convenience solely, the unit of time upon which all calculations are to be based is *that required for the passage of one pound of cylinder-feed* through the engine. But this unit of time is most conveniently expressed in number of engine-strokes rather than in number of seconds, minutes, etc. Any other unit of time, such as the hour, minute, second, or the single stroke, might be used with equal propriety, but at much greater labor of calculation. To determine this unit

of time the number of working-strokes per hour need merely be divided by the number of pounds of cylinder-feed per hour. It is obvious that the result will depend largely upon the size of the engine, amongst other factors. It may be almost any figure, from a quite large number for very small engines to a fraction of unity for very large ones.

The volume-scale of the indicator-card which fits properly with the above assumptions is given by the fact that the total length of the card must represent the cylindrus-volume evolved during the chosen unit of time. This is plainly the actual cylindrus multiplied by the basic number of strokes. This is the basic volumetric scale of the indicator-card, and *all other measurements taken from the card must be read to this same scale.*[1]

Next, let the clearance, known as a percentage of the cylindrus, be laid off from the admission-end of the indicator-card as this same percentage of its total length, and through the point thus obtained let a vertical zero-axis of volumes be raised normal to the atmospheric line. If now any point of the card be measured as to horizontal distance from this axis, on the proper scale already obtained, the reading will be *cubic feet of total cylinder-volume per pound of cylinder-feed* passing the engine.

In order to know the volumetric condition of the cylinder-feed, it now remains to subtract the variable volume of the cushion-steam. One of the values of this variable may be known from the indicator-card; the others must be deduced from the first.

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[1] This ratio or proportion — for it could hardly be called a scale — may be reduced to a comparatively convenient scale by measuring the length of the indicator-card in sixteenths, thirty-seconds, or hundredths of an inch, or in millimeters, according to what scales are at hand, and then dividing the above total cylindrus-volume per unit of time by the same. This gives a scale reading "so many cubic feet per sixteenth inch, or per millimeter," etc.

Thus, it is known empirically from steam-engine practice that, except in extreme cases only, reëvaporation is always completed early in the exhaust-stroke. This insures that the steam in the cylinder at the beginning of compression is dry saturated steam, and hence its volume and pressure determine its quantity. Even in steam-jacketed cylinders, where the tendency would be to superheat the steam in the cylinder immediately reëvaporation were complete, the heat-transfer to the dry steam is so slow that the departure of the truth from this assumption is insignificant.

Immediately the exhaust-valve is closed, however, two forces begin to alter the condition of the steam:

(1) The thermodynamic supply of energy to the steam by compression tends to superheat it (see page 92, point e).

(2) The rise of temperature due to compression tends to conduct heat from steam to cylinder-walls, which now begin to present a very large surface to the comparatively small volume of steam.

In nearly every actual case the second process is much the stronger of the two, and the cushion-steam steadily becomes more moist as compression proceeds. It is not
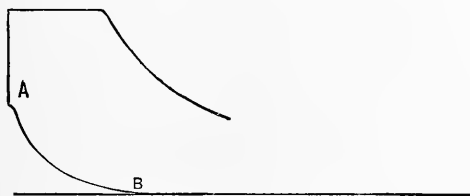
FIG. 40

uncommon to find an engine in which this is so far true that the compression finally attains a degree where the small plate-like volume of cushion-steam loses its heat so rapidly to the enveloping piston-face and cylinder-cover

that it condenses into water isothermally as the piston completes the last portion of its stroke. The indicator-card in such cases shows a compression-curve similar to Fig. 40, the isothermal condensation showing at A. It is therefore probable that all the steam in the cylinder is dry saturated immediately after exhaust-closure, and even in those cases where a light superheat is suspected the second of the above two forces operates to negative this source of error.[1] If, therefore, a point on the card, such

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[1] As this assumption of a condition of dry saturation at this point is a basic one, affecting all the subsequent work, it is proper to investigate not only its truth, which is difficult to do, but also the proportionate effect of its error upon the final results. To do this let it be assumed that the pressure-range of compression in the actual engine seldom exceeds that from 16 to 100 pounds absolute in a simple non-condensing engine, or an equivalent ratio for condensing engines or for cylinders exhausting into receivers. Under such conditions, therefore, let the resultant final volume be compared under four suppositions as to initial condition: (1) Dry saturation; (2) 10 % wet; (3) 20 % wet; (4) 50° of superheat. Any engine which did not fall within these limits would be so abnormal in all of its activities as to either be unworthy of a test at all or else worthy of especial investigation.

The initial volumes of these four conditions may be stated relatively by calling the first 100. Then (2) must be 90.68; (3) 81.36; and (4) about 107.39. After adiabatic compression to 100 pounds absolute the final volumes of the three would be: (1) 24.02; (2) 18.04; (3) 15.67; (4) 25.78.

If it be assumed that the reduction of volume for all conditions is the same as that for dry saturation at the start, the four results are: (1) 24.02; (2) 21.78; (3) 19.53; (4) 25.80. Numbers (2) and (3) are over 20 % too large; (4) is practically correct. If a similar assumption be applied to steam of original condition of (2), the four results are: (1) 19.85; (2) 18.04; (3) 16.14; (4) 21.34. Now (1) and (4) are some 17 % too small; (3) is within 3 % of correct.

When it is considered that these assumed conditions are extreme and that the weight of cushion-steam rarely exceeds one-fourth of the cylinder-feed (which would reduce these percentages of error, when compared with the whole quantity of steam in the cylinder, to one-fifth of the figures stated), good basis is seen for the conclusions: (1) An assumption of dry saturation at the beginning of compression leads to no appreciable error in results if the steam be actually dry or superheated at start, and to an error of from 1 % to 5 % if

as B, be chosen, which represents an instant when the exhaust-valve is known to have been only recently completely closed, and if the volume and pressure at B be measured by the scales already described, they will reveal the quantity of cushion-steam present at the instant *per one pound of cylinder-feed*.

Let these measurements be laid off on the diagram at the point Y, Fig. 39, the volume being regarded as a negative one. Through Y let an adiabatic AB be constructed. It will represent the volumes which the cushion-steam ought to occupy for every given pressure in the cylinder.

In the published blank diagram will be found, in this portion of lower left-hand quadrant, a number of such adiabatics. With them as guides, when once the point Y is determined, the desired adiabatic may be laid off by eye. These adiabatics are the pressure-volume ones, for the several weights noted on the curves themselves, corresponding to the single entropy-temperature adiabatic shown in the entropy-temperature quadrant. All agree in being based upon an original dry-steam point at an absolute pressure of 41.56 pounds per square inch, at which pressure the volume of one pound is 10 cubic feet. Should the original observed point Y happen to fall at this pressure, the weight of cushion-steam present may be read directly from the weights pointed on the several curves. If not, however, as would usually be the case, the weight as given by the curves need only be multiplied by the percentage indicated in the list of them to be found in the extreme lower left-hand corner of the sheet, the proper one

the steam be quite wet at the start; (2) Assumption of moisture at the start leads to no appreciable error if the steam actually be as moist as assumed or even much more so, and to an error of from 1 % to 3 % if it actually be drier than assumed. The actual error, under average conditions, will not exceed one-third of these figures.

being chosen according to the pressure-level of the point Y, in order to give the actual weight of cushion-steam present per pound of cylinder-feed. A knowledge of this weight is not needed, however, for any further process of this analysis, but may be desired for other reasons.

In the straight vertical axis which was drawn outside the clearance on the indicator-card is to be had a *zero-axis of total cylinder-volumes*, as already noted. The object in view is to deduct from those volumes the variable volumes of cushion-steam in order to find the variable volumes of cylinder-feed. If, now, the line AB, Fig. 39, be used as a *curvilinear zero-axis of total cylinder-volumes*, and if from it any measurement taken from the indicator-card, such as SZ (translated into the proper scale), be laid off to the right, then the axis OP must automatically cut off from it the volume SR which should be properly occupied by the cushion-steam at this pressure. The remainder RZ measures the volume which the cylinder-feed would have were all the heat-losses and gains charged against it. OP therefore becomes the *zero-axis of cylinder-feed volumes*, and the lower right-hand quadrant becomes a rectangular coördinate field revealing the pressure-volume conditions of one pound of cylinder-feed.   Q.E.F.

The indicator-card should now be reproduced, point by point, in the above manner. It will appear, distorted in form, something like the curve $EFZGHIK$, Fig. 39. (By handling the points in horizontal pairs, as Z and Z''', much labor can be saved.) In this form it is of no immediate value, but from it can easily be deduced the entropy-temperature changes incurred by the cylinder-feed, and they, in turn, will show plainly just when and in what amount heat was lost or gained.

Let attention be confined to the illustrative point Z.

So long as the distorted indicator-card lies between the

curves AB and MN the assurance is had that the cylinder is filled with saturated steam more or less moist; for the curve MN is drawn as a "saturation-curve," representing the relation between volume and pressure of dry saturated steam; or, in other words, MN is one of the columns of the steam-table represented graphically. In the temperature-pressure quadrant is a similar curve, mn, showing the relation between the temperature and pressure of dry saturated steam. Therefore if a line be run horizontally across from Z until it meets mn, at L, and thence a vertical line be run to R'' and Q'', it is certain that the entropy-temperature condition of the steam at the point Z must be shown at some point, such as Z'', on the line $R''Q''$. Which is the proper point, however, can be known only through information as to volumes.

If a horizontal be run from R'' to the vertical axis at R', the latter point must represent the relation of entropy to volume for *water* at the pressure shown by Z; for the volume of one pound of water is so small as to be invisible on the scale chosen. The volume of one pound of *dry steam*, however, of the pressure shown at Z is shown by the point Q, and may be carried up into the entropy-volume quadrant by the ordinate QQ'. The entropy of one pound of dry steam of the pressure and temperature shown by Z is shown at Q'', and may be carried over into the entropy-volume quadrant by the abscissa $Q''Q'$. The intersection of QQ' and $Q''Q'$ at Q' gives the latter point as a measure of the relation of entropy to volume for one pound of *dry steam* at the pressure shown by the point Z.

Referring to the Equation 21, page 87, it will be remembered that when a given weight of water changes to steam by the addition of heat under constant pressure, the increments of heat, entropy, volume, and weight of vapor formed were all proportioned. Hence the relation of

entropy to volume for a steam-and-water mixture of a given pressure must be represented by a straight line. In the case under discussion the points R' and Q' must be the extremities of such a line. Joining them gives a line to which the *volume* of any such mixture as RZ may be referred, by means of the ordinate ZZ', to find its *entropy* OZ''''. Carrying across the horizontal $Z'Z''$ until it intersects $R''Q''$, must determine the point on the entropy-temperature quadrant which illustrates the thermal condition of the cylinder-feed at the point Z.

It will be noted that while there is only one temperature-pressure saturation-curve, *mn*, for all weights of steam and all conditions, and while there is only one pressure-volume saturation-curve, *MN*, for a given weight of steam, there are any number of entropy-volume curves similar to $R'Q'$ for a given weight of steam-and-water mixture, each serving for one particular pressure or temperature of mixture only.

The successive transfer of all points of the distorted indicator-card in the manner just described results in an entropy-temperature diagram of the actual cycle undergone by the cylinder-feed in the actual engine. This diagram, rotated through ninety degrees to bring it into the position in which these diagrams were always depicted in the first part of the book, would have something the appearance of Fig. 41, although wide variations are to be expected from engines of different types and sizes.

This diagram depicts the actual thermodynamic processes of the cycle performed within the cylinder. In it may be seen just when, how, and to what extent heat-energy reaches the steam and work departs from it. It shows the advent of heat by the increase in entropy wherever the curve takes a right-hand departure. It

shows, in literal fashion, just when this entropy falls down-temperature, doing work, the path by which it falls, and the amount of work done in the fall. It shows when and how the fallen entropy leaves the engine and by what path the unexpelled remnant is again raised to the head-level of temperature. It shows how much temperature-fall is wasted in getting the heat into the engine and how much in getting it out. It shows the leaks of heat into and out of the steam as it rises and falls, doing work, and just how much energy-waste is represented in these leaks.
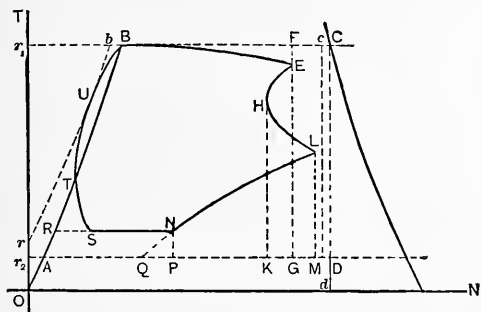
FIG. 41

It is important to study this photograph of the cycle carefully and to separate by accurate analysis these various factors.

It is first necessary to refer this actual cycle to the one which would have been followed by a perfect engine working within the same opportunities and limitations. Since the engine is a steam-engine, the utmost that can be expected of it is the Rankine cycle. If the engine were supplied with dry saturated steam, this cycle would be shown by the lines $ABCD$; if with wet or superheated steam, this cycle would be modified as illustrated in Problems 14 and 17.

It is a matter of less certainty, however, as to just what should be the level of the limits at top and bottom. If the engine be a simple, non-condensing one, situated near its boiler and with ample pipe-connection between, BC would clearly represent boiler-temperature and AD that corresponding to the barometric pressure of the atmosphere. Should there be a long or insufficient pipe-line feeding the engine, however, it is important to distinguish between boiler-pressure and throttle-pressure. If it is desired to include and examine the evil effect of the pipe-line upon engine-efficiency, both temperature-levels may be drawn in ; and while the question of piping-losses involves many other factors than the mere thermodynamic effect upon the engine, this last would be at least clearly and accurately represented, both absolutely and as to proportion to other losses, by this graphic comparison.

If the engine be a condensing one, the problem is still more open to discussion. The simplest and, when the engine is considered alone, the most accurate plan is to draw AD at the temperature-level corresponding to the actual vacuum. When the efficiency of the condenser and air-pump is to be included, however, as if this apparatus were a part of the engine whose efficiency is being analyzed, AD should be drawn at the maximum, or even the mean, temperature of the condensing water.

When the engine is a multicylinder compound, the result will appear as a vertical series of such cycle-diagrams, one over the other. The gaps between represent receiver-losses. In such cases it is usually sufficient to draw between the cycles horizontal lines at the temperatures of saturation at the recorded receiver-pressures, trusting to engineering judgment to draw the proper deductions from the case without attempt at greater refinement of analysis on the graphical diagram. The question as to where the

adiabatic CD should be drawn is less simple, however, and will be taken up later as a separate topic.

Assuming that we are concerned with a single cylinder only, supplied with dry steam, the following conclusions are safe.

Wire-drawing. — When the engine took steam, it should have done so along BC, cut-off occurring at C. Instead, it does so along BE, cut-off occurring at E. The energy available for doing work at E is less than that at C by two quantities, viz.: (1) The triangle BFE, (2) the rectangle $FCDG$. The second quantity of heat lost is measured by the rectangle between FC and the *axis of absolute zero of temperature*, not shown on the diagram but easily drawn in, or it may be obtained as the result of a single multiplication; but only $FCGD$ is available for doing work.

The first of these is plainly due to wire-drawing, for wire-drawing cannot rob steam of heat; it merely reduces the temperature. The loss of availability for work due to loss of temperature without loss of heat is shown by the area BFE, and this area is therefore chargeable against wire-drawing, as a loss due to it alone.

Initial Condensation. — The second loss, however, is due to loss of entropy, and not of temperature, and must therefore be due to an abstraction of heat. As the only known process present which can abstract heat is condensation on the cold cylinder-walls, this rectangular loss is chargeable against initial condensation.

Condensation during Expansion. — The energy available at cut-off at E would indicate that a perfect engine, by following the adiabatic EC, might develop the cycle $ABEGA$. But instead of doing this the actual engine follows the path EH. Its departure to the left indicates abstraction of heat by continued condensation. Some of this heat, however, by falling from later points of the path

BE to points vertically below in EH, has performed some work. The loss due to this further condensation is therefore only the area between EH and KG.

Reëvaporation. — Again, the engine, instead of expanding the steam adiabatically after condensation is complete, in which case it would follow the path HK, shows a deviation to the right along HL. This shows the addition of heat to the steam. This heat can come from no other source than the cylinder-walls, and must be due to the fact that the temperature of the steam has fallen by expansion below that of the walls, whereas originally it was above it. The addition of heat must result in the reëvaporation of some of the moisture on the cylinder-walls. The steam-heat thus brought in is of course available for doing work, but by no means to the same extent as the original heat, which came in at maximum temperature. The work which it can do must be measured by the area beneath HL, or $HLMK$, and this area may therefore be credited as a gain due to reëvaporation.

Incomplete Expansion follows next in the list of deviations from the adiabatic path to the lower level. Instead of following the path LMA back to A the temperature-fall next departs to the left, when the exhaust-valve opens, along LN, and, if it were not arrested by a still further impediment to perfect action, would reach the A-level at Q. The triangular area LQM is therefore chargeable to exhaust before expansion is complete. It is always to be remembered, however, that for practical reasons which will be explained later complete expansion is never profitable.

This line LQ, when compared with a theoretic constant-volume curve through L, gives the exaggeration of the loss due to incomplete expansion by clearance. This consideration, with those discussed on pages 223–225, gives an absolute measure of the deleterious effect of clearance.

Back-pressure. — It next develops that the cycle never attains the lowest possible temperature-limit, DA. It never falls below NR. The area representing the resultant loss, $NQAR$, usually measures a nominal rather than a real loss. Some back-pressure is inevitable. To supply ports and valves of the enormous size necessary for eliminating it would be very foolish. As with the loss due to incomplete expansion, the only question is whether it be excessive or not.

Cushion and Clearance. — With the next deviation from the theoretic, that of the path $STUB$ from the water-curve RB, arises a much more pregnant question of steam-engine design. This question lies between two broad types of steam-engine: (1) Those designed to have a minimum possible clearance, with little or no cushion, and (2) those designed to have a liberal clearance, which is to be filled with cushion-steam at each stroke compressed to as nearly boiler-pressure as is practicable. The first is naturally the drop-cut-off, slow-speed type; the second is the positive-gear, high-speed type. Before discussing the question, however, for it is a lengthy one, it were well to decide just how the amount of heat-work lost during compression is to be measured from the actual cycle before us.

It is obvious that at the point S the cylinder-feed possesses more heat than it theoretically should, and that during compression heat to the amount STR is abstracted. This little triangular area reveals the departure from the cylinder of exhaust-heat along the line ST which should have gone out by the path SR.

At some point in the compression, however, as at U, the cycle shows the cylinder-feed as having less heat than it would if it were water of the given temperature, which is manifestly impossible. What has happened is plainly that the cushion-steam, which alone occupies the cylinder in the actual engine, although theoretically all of the cylinder-feed

is present in the form of water, has lost some heat to the walls during its compression. The mathematical elimination of the cushion-steam, however, causes this loss to appear as one drawn from the cylinder-feed.

At first sight it might appear that what was wanted was an analysis of what happened to the cushion-steam during its compression. In reality, however, such determinations are misleading. For instance, it might thus be found that the cushion-steam at some point in its compression consisted of 50% water. This would seem a tremendous loss of heat, which should be remedied at any cost. If, however, the cushion-steam amounted to only 10% of the cylinder-feed, the lost heat reduces to 5% in terms of cylinder-feed, or one of the smallest in the engine. And since it is cylinder-feed alone which is a source of expense to the engine-user, all losses must be reduced to that basis of comparison.

It is obvious, too, that all heat lost by the cushion-steam during compression must be made good by the cylinder-feed entering at the next stroke before the latter can begin to accomplish any useful result. It is therefore quite proper to consider the point where the heat-abstraction from the cushion-steam reaches the extreme as the zero of all heat-gains and losses by the cylinder-feed. This is easily done graphically by drawing through the point U a curve parallel to BR, such as bUr, tangent to BTR. This arbitrarily moves all reference-points to the left by the distance Bb. A new limiting adiabatic must therefore be drawn parallel to CD and at the same distance to the left, as at cM. This alters the apparent value of heat lost in initial condensation from that prescribed on page 221. The other losses and gains are unaltered. What is in reality true is that a large portion of the initial condensation occurring in every cylinder is due to heat-losses

incurred before any steam was admitted, which losses are stored up by the cushion-steam for draft upon the cylinder-feed when the latter shall next appear in the cylinder. The process just defined separates these two responsibilities for condensation. The area $UrSUBbU$ is chargeable against clearance and cushion-steam. The area $FcMG$ is due to condensation which occurs after the clearance has been filled with boiler-steam and which would take place were the clearance zero. It is this preliminary demand for boiler-steam to reimburse the cushion-steam and to carry on the preliminary condensation which occurs before the stroke begins which constitutes the real reason for the lead of the valve. When it is remembered that the greatest part of the surface exposed to boiler-steam by the time cut-off takes place is already exposed to it when admission takes place into the clearance-space at dead-centre, it is obvious that there is good basis for expecting this preliminary demand for steam to constitute a goodly proportion of the total supply.   .

It is also obvious, from the foregoing argument, that it is the clearance-*surface*, rather than the clearance-*volume*, which is so deleterious to steam-engine efficiency.

As the assumption of initial dryness on the part of the cushion-steam is the only assumption entering the entire analysis, it may be insisted that the conclusions reached by means of the entropy method are exact to an unusual degree. If, therefore, the entropy-analysis should reveal that the area chargeable to cushion-condensation was so great, in any given instance, as to overbalance any gain from diminished first cost, space, etc., which the cushion, through the medium of the high rotative speed which it permits, has developed, then the minimum-clearance design should obviously be chosen in preference for this particular service.

Q

With the foregoing means of obtaining an exact knowledge of existing facts as a basis, some very plain and simple conclusions may be drawn concerning several features of steam-engine design which frequently find themselves under discussion.

SIZE AND SPEED

If by the size of an engine is meant the dimensions of its cylinder, then it may be stated as a well-known fact that the larger the cylinder, the less is the condensation, because the less the surface exposed per pound of cylinder-feed. The internal surface of similarly shaped vessels, such as engine-cylinders, must plainly increase directly with the square of any one dimension.   But as the cubical contents at the same time increase directly with the cube of the same dimensions, it is plain that the ratio between cylinder-surface and the amount of steam contained must vary inversely as the first power of the same dimension; whence, the larger the cylinder, the less will be the percentage of condensation.

More commonly and more accurately, however, the size of an engine is understood to refer to its power.   This being so, is it equally true to say that the larger the engine, the less the condensation?   If the variation in power between two engines subject to comparison is gotten by varying the cylinder-dimensions, while speed and pressure are kept constant, then it is plain that the proposition still holds true.   If, however, the increased power be gotten by increased speed, the cylinder-dimensions being kept constant, the problem is altered.   Here the same surface is exposed to the same fluctuations of temperature a greater number of times per minute.   As the amount of cylinder-feed is also roughly proportioned to the speed, the proportion of condensation could only remain the same, as the

speed increased, if the absolute quantity during each cycle of temperature remained constant. But as it takes time to transfer heat from steam to cylinder-wall and back again, and as the time occupied by each cycle diminishes as the speed increases, the absolute condensation per cycle will plainly *not* remain the same; instead, it will decrease, and hence the proportion of condensation suffered by the cylinder-feed must be said to decrease as the speed increases. But, whereas it was quite safe to assume that in alterations of cylinder-dimension the amount of condensation varied in true and simple proportion to the surface, it is not by any means probable that the condensation upon any given surface is proportional to the time of exposure. For the heat is not carried away by the metal, but is merely stored up within itself, and it is easy to imagine that the first heat imparted to the walls might raise its temperature sufficiently toward that of the steam so that the rate of heat-transmission would be substantially altered. Indeed, it is safe to say that the condensation must increase with the time of exposure in some ratio which is some fractional power of the time. If this should be true, then it would appear that it were more profitable, so far as condensation is concerned, to increase an engine's power by increasing the size of cylinder rather than by speeding up, and as purely mechanical argument also advises the running of larger cylinders at slower speeds, it is plain how thermodynamics and mechanics coincide in supporting the natural tendency visible in actual designs.

But it is also to be noted that since increase in speed is actually conducive to thermodynamic efficiency, but only less so than increase in dimension, the running of large-powered engines at high speed finds no censure in thermo-dynamic argument, when it is warranted by mechanical considerations.

Steam-pressure

As the range of pressure, and therefore of temperature, in an engine increases, the theoretic efficiency plainly increases. At the same time, by reason of this same temperature-range, the condensation must increase. At first, as one starts from a very low pressure-range, the theoretic gain will be very great because it is almost fully proportional to the temperature-range and because the latter increases, in the lower field of pressures, more rapidly than the pressure. But as higher pressures are reached, all of these conditions are first annulled and then reversed, and the increase in condensation is found to more than offset the gain in heat-available-for-work. The point where balance is established between the two gives the maximum pressure-range which it is profitable to impose upon the cylinder for purely thermodynamic considerations; though for other reasons higher pressures are often found profitable in special services, such as locomotives or torpedo-boats, where it is all-important to get a maximum of power out of each inch or pound of engine.

This limit may be determined only by experiment upon each type of engine considered. For reasons stated in the preceding article, it is plain that the limit would be lower in small engines than in large. It is generally understood to lie between 90° and 120° F. of temperature-range within a single cylinder, the lower figure for small cylinders and the higher for large ones. It should also be understood, however that when applied to unjacketed cylinders these figures give rather the maximum range obtainable without real detriment from condensation than the minimum range permissible without net loss from thermodynamic reasons; for the natural tendency is always to take the greatest permissible range, for the sake of capacity or power, rather than the least.

The Steam-jacket

It is the rôle of the steam-jacket to prevent or minimize condensation in the cylinder. Since the heat supplied to the steam through the medium of the steam-jacket is good boiler-heat, obtained at as great an expense as steam-heat entering by way of the throttle, some explanation is necessary as to why any gain in efficiency is possible by its aid. Indeed, since the steam-jacket is free to radiate heat into the exhaust during half the time, and since, even during the working-stroke, a large part of the heat supplied by it to the working-steam is imparted after cut-off, when the availability for doing work is much less than is the case with boiler-steam, explanation is also necessary why the steam-jacket should not constitute a distinct source of loss of efficiency. The answer to both points is found in the fact that the one thing in an engine-cylinder which is able to very much exaggerate the detrimental influence of the cylinder-walls is moisture. Able to exchange heat with the steam much more rapidly than can the iron of the walls, and having, too, a much higher specific heat, water is very much more active than metal in carrying on that condensation and reëvaporation which has always been the chief bar to the attainment of maximum theoretical efficiency.

It is not a sufficient guard against the phenomenon to provide the engine with perfectly dry steam, although the presence of wet steam makes the situation very much worse than if it were dry; for even with dry steam the initial condensation which must take place upon the walls as the clearance fills with boiler-steam forms a nucleus of moisture which invites additions in a geometrical ratio.

The steam-jacket prevents the appreciable accumulation of moisture upon the internal surface of the cylinder-walls

by warming up the cylinder with steam-heat, the resultant moisture from which *remains outside the cylinder* and is drained away. That is the real and essential difference between a jacketed and an unjacketed cylinder. The heat supplied from the boiler for radiation is just as great and just as costly in either case; but in the former the resultant moisture does not remain in the cylinder as an active carrier of heat into the exhaust, whereas in the latter it does.

There is an additional gain, too, due to the fact that the mean temperature of the cylinder-walls is kept nearer to boiler-temperature. Because the walls are open to the exhaust during the back-stroke, the steam-jacket cannot quite bring them to boiler-temperature. But the initial condensation is much decreased in amount, and its reëvaporation, instead of awaiting the fall of temperature within the body of expanding steam below a mean between boiler and exhaust-temperatures for its inception, begins as soon as cut-off takes place and usually finishes before the working-stroke is complete. This being so, the steam-jacket can lose very little heat to the comparatively dry exhaust-steam, and by the time cushion takes place can get the walls well back toward boiler-temperature. The result of its action therefore is: —

(1) To much decrease the amount of boiler-heat undergoing free fall within the cylinder by keeping out of the cylinder the moisture developed in heating it;

(2) To impart the bulk of what does undergo this free fall of temperature to the working-steam at an earlier point in the cycle, before the latter's expansion is complete, whereby a portion of its temperature-fall to exhaust-temperature is utilized.

Since the prime office of the steam-jacket is to diminish condensation, it must be of the most value where conden-

sation is excessive; that is, where a high thermodynamic efficiency is, or is thought to be, valuable, and is sought through the medium of (1) a large ratio of expansion or (2) slow speed of rotation. Under these conditions a good jacket will show an increase of efficiency varying from 5 to 15%, with probably 7% or 8% as a fair average.

It is to be especially noted, however, that under wrong conditions the steam-jacket may become a source of expense instead of economy. The majority of engine-builders, rating small and large together, omit it from their designs, and in this the majority of them are right. In debating the question as to when jackets are desirable and when they are not, the first fact to be noted is that a steam-jacket is of itself a very expensive and undesirable feature in an engine, and the saving which it develops must be a substantial and a certain one if the net result of its addition to the engine is not to be a distinct loss. A jacket is difficult and expensive to construct. The core for it is large and thin. Its presence usually entails coring several other portions of the casting where cores would otherwise be unnecessary. The percentage of cylinder-castings lost in the foundry is very much higher with jackets than without. The joints between barrel and head of cylinder at either end are much complicated by its presence. If it is remembered that the great bulk of the condensation of the working-steam takes place in the clearance while the piston is at, or very near to, the end of its stroke, it will be plain that the portions of the engine which especially need to be jacketed are the cylinder-heads, and to connect these jackets with those around the barrel and to avoid too close contact with the stuffing-box, all involves difficulty and expense in construction and maintenance. Therefore, while it is undoubtedly true that a steam-jacket would somewhat increase the thermodynamic efficiency of any cylinder whatever, it

is only when this increase is substantial that it may coun-
terbalance the commercial losses involved in its adoption.

The settlement of this question necessarily involves an
understanding of the relation existing between thermo-
dynamic and commercial arguments in any given case.
But its discussion applies with equal weight to all of the
factors which enter into engine-efficiency which have
already been discussed: size, speed, ratio of expansion,
clearance, and cushion. It is therefore proper to recon-
sider them all in the light of this one argument.

The design of a steam-engine, as is true of any merchant-
able article, properly follows the lines upon which demand
is expressed. The demand coming from the buyers of
steam-engines naturally divides itself into two broad sorts,
if minor differences due to special conditions be ignored.
Of these, the first comes from him to whom the item of
power is a large and important one in his business; the
second comes from him to whom power is a minor factor.
Examples of the first are electric lighting, electric genera-
tion in general, large textile mills, paper mills, marine pro-
pulsion, etc. Examples of the second are machine-shops
and the general run of factories of moderate size wherein,
as in the machine-shop, the pay-roll or the cost of raw
material is a very much larger item than is the cost of
power. All of the former class have presumably plenty
of capital to invest in steam-plant, for that is their business,
and return upon that capital will be a maximum when the
coal-consumption is the least. Of the second class, those
who have plenty of capital invest it in acquiring the best
plant possible of the sort which really does the earning
of the returns from their business, and which consists of
machine-tools, overhead cranes, and similar facilities for
doing the best possible work in the minimum of time and

with the least waste of raw material or labor.   The power plant of such a factory is properly a secondary consideration.   To it, therefore, is accorded only sufficient capital to prevent its being actually bad.

In addition to this latter class, too, should be considered those to whom the cost of power is a prominent item in the cost of doing work, but who have to approach every department of their business, owing to insufficient capital, in the attitude of cutting down investment to the lowest possible point without regard, always, to whether further investment might not be profitable.

These two sorts of buyers therefore demand, the one an engine which will operate upon the highest fuel-efficiency without regard to first cost, the other an engine which will bring the first cost for power and its incidentals to the lowest possible figure without regard to more than a moderate degree of efficiency.   It so happens that these two attitudes lead to decisions in regard to the several points of design just discussed which are singularly harmonious in their combination into a complete type of engine.   These points are : —

1. Degree of expansion, including the question of compounding.

2. Valve-motion.   5. Speed.

3. Clearance.   6. Steam-jacketing.

4. Cushion.   7. Condensation.

1. *Degree of expansion.* — It is plain that the greater the ratio of expansion, the greater is the ideal efficiency.   It is also obvious that the greater the degree of expansion, the greater are, not only the thermodynamic losses due to cylinder-condensation, etc., but also all those expenses due to larger cylinders and earlier cut-offs as well; for the entire cost of an engine-plant is closely proportional to the size of

cylinder. A comparatively late cut-off may be easily gotten
by means of a single valve and a very simple valve-motion;
but to attain an early cut-off, and at the same time preserve
the proper distribution of exhaust and compression, de-
mands the use of from two to four separate valves, and
often more than one eccentric. Therefore it can be safely
said that there is one broad type of engine which carries
the attainment of efficiency by means of higher ratios of
expansion at the expense of larger cylinders and more
complicated valve-gear; there is another which foregoes
this efficiency to an appreciable degree, to the end that
cylinders may be small and the valve-gear simple. In this
connection it ought also to be remembered that large cyl-
inders mean larger engines in every respect, and larger
engines mean heavier and more costly foundations, more
care, more oil, more space about them, heavier cranes over-
head, etc., etc. In fact, it may be safely said that the
entire cost of construction of engine-room and its mainte-
nance, aside from the cost of fuel, bears a rough but fairly
accurate proportion to the size of cylinder employed.

The foregoing remarks apply equally to the question of
the degree of completeness of expansion, or of terminal
pressure, which is somewhat independent of the question
of ratio of expansion. It is a very common fallacy that
the indicator-card with the sharpest toe, other things being
equal, shows the best efficiency. So false is this that it
may be broadly and responsibly stated that any indicator-
card with a sharp toe is inherently bad. The justification
of this statement lies in the following argument: It is
obvious that of the pressure in the cylinder at any moment
only that exerting a pull upon the belt is of any value to
the engine-owner; the rest is expended in merely keeping
the engine in motion. In other words, if we translate
engine-friction into its equivalent in mean pressure upon

the piston, we shall find the equivalent to amount to from about 4 pounds per square inch for large low-pressure cylinders to about 15 pounds for high-pressure cylinders. Whenever the pressure upon the piston is less than this, the belt must be dragging the piston after it, and the net result is a distinct loss. Moreover, since every inch of cylinder costs money to build, to erect, to maintain, and to operate it, the engine is a distinct source of loss until it is developing enough power of sufficient net value to make good this loss. If this power be also translated into mean effective pressure and added to the former figure, the results will vary between the extreme limits of about 5 and 25 pounds, or ordinarily between 7 and 20 pounds, per square inch of mean effective pressure. Of these figures the smaller apply to low-pressure, the larger to high-pressure cylinders. If the terminal pressure in the cylinder be allowed to fall below these figures, it is proof that the engine is not developing and delivering to the belt enough money-making power to fully pay for its keep. The plant would be more efficient under a later cut-off, either in a smaller cylinder or in the same cylinder under lower initial pressure.

The wide prevalence of the fallacy referred to in this connection is almost wholly due to the absurd practice of uniformly making guarantees and reports of engine-efficiencies upon the basis of the *indicated* instead of upon the *brake* horse-power. Deception as to what is a profitable degree of expansion is only one of the many ways in which the engineering public loses from this habit.

The question of the profit of compounding or not comes under exactly the same argument. The details of the science of compounding will be treated later under the proper heading; but it is important to note that the commercial efficiency of a steam-plant is not necessarily improved by

compounding, and that many existing compound engines are operated at a loss compared with what might be accomplished by a single cylinder.   But in the great majority of cases the mistake is in the other direction.   Single cylinders are adhered to for the sake of cheapness when the additional money needed for compounding would earn a far greater return than any other money invested in the business.

It might also be misapprehended from the above that compounding should only be regarded as an exaggeration of the policy of the short cut-off, four-valve type of engine, and to be adopted only after passing through this preliminary degree of search after efficiency.   This is not true, however.   There are many mechanical and commercial advantages to be conserved by retaining the compact, single-valve, high-speed type of machine, even when searching after greater efficiency by means of compounding. But the resultant type is necessarily an intermediate and more or less special one, limited properly to a comparatively narrow, although absolutely a very large, field of sizes and of diversity of employment.

2. *Valve-motion.* — This point has already been partly covered.   The valves and gear fit to produce early cut-offs are comparatively expensive to build and to maintain.   The question is also intimately connected with that of

3. *Clearance.* — It happens that the valves fit for producing a very short and sharp cut-off also permit the attainment of a very small clearance in the cylinder.   As clearance has been shown to be always somewhat detrimental to efficiency, here again the coincidence is harmonious with the other points in dictating large cylinders, four valves, and small clearance as characteristic of the more efficient type.   It also happens that the best forms of single-valve engines always involve large cylinder-clearance.   This brings into the discussion the next point : —

4. *Cushion.* — The detrimental effects of clearance can be very largely, although not entirely, counteracted by the use of sufficient cushion to bring the pressure in the clearance approximately to boiler-pressure by the time dead-centre is reached. It also happens that when a single valve and eccentric are used, the attainment of a cut-off earlier than about three-quarters stroke is impossible without making the exhaust-closure also correspondingly early. So that it appears that in the type of engine which needs cushion to make it efficient considerable cushion is mechanically inevitable. It is a fortunate coincidence that in this type of engine a great deal of cushion may be had very easily.

5. *Speed.* — It was pointed out in the analysis of the causes of cylinder-condensation that, for an engine of any given power, that phenomenon was probably not at all affected by speed. But the many items of cost of operation of an engine due to causes other than cylinder-condensation are all very sharply affected by speed. The higher the speed, the smaller and cheaper are the cylinder, the foundations, and the engine-room, the lighter are the belts and shafting, and the better the regulation. That there are any mechanical disadvantages or costs to be incurred, under proper design, from high engine-speed is not to be urged to-day. Twenty years ago high speed had to apologize and fight for itself, as a mechanical monstrosity. To-day even the advocates of the slowest speeds have so increased their rates of rotation that it is generally recognized that the arguments in favor of slow engines are based upon other points than purely mechanical ones. But one of the essential points in good design for high speed is plenty of cushion. It is therefore another one in the string of happy coincidences that the type of engine which naturally adopts the highest possible speed and the simplest

possible valve-gear is forced by the latter and permitted by its clearance to have just what is demanded by its speed, viz.: a high degree of cushion. On the other hand, the type of engine which naturally adopts a comparatively slow speed and a large cylinder is permitted, by the complex valves and gear which it needs and can pay for, to have very little clearance; consequently it may have little or no cushion. But cushion it does not need.

6. *Steam-jacketing.* — From the characteristic aims which have already been mentioned as outlining these two types of engine it is plain that, were the steam-jacket to develop under all conditions of operation the same average gain of efficiency, it would be quite likely that the large, slow-speed engine would find it profitable to adopt it, whereas the single-valve type would not. But if the conditions which exaggerate or minimize the effectiveness of jacketing be contrasted for the two types, it is seen that the four-valve engine possesses all of the former and the single-valve all of the latter. Large cylinders, slow speed, and short cut-off all increase the need and the effectiveness of a jacket; and the type which embodies these features can afford to pay for a jacket if it will even perceptibly aid efficiency. Small, compact cylinders, high speed, and moderate cut-off lead to moderate condensation, so that a jacket would have little chance of doing good; at the same time, the engine embodying these features cannot afford the complication of a jacket, nor does it care to go after increased cylinder-efficiency even were it more marked than under these conditions would be the case.

7. *Condensation or Non-condensation of Exhaust.* — This point in design is different from the others in that its feasibility depends very largely upon a purely external and largely fortuitous circumstance, viz.: the existence of a proper supply of cheap condensing water. The amount

of water needed is from one to two gallons per horse-power per minute. The amount of coal to be saved by its use is, to give a single figure which will roughly average all the varying conditions, not far from one pound per horse-power per hour. But even if it be true, in any particular case, that the water-supply be present and that the cost of condensation has been rightly interpreted, it is yet to be remembered, in close analogy to the preceding argument, that the conditions which speak for the adoption of condensation also dictate the use of the four-valve type of engine. On the other hand, conditions which would lead naturally to the choice of a single-cylinder type of engine would hardly warrant the adoption of the expensive complexities incidental to working under a vacuum. The crossing of these natural combinations — that is, of a single-valve engine with a condenser, or of a four-valve engine with atmospheric exhaust where condensation is possible — may be accepted upon its face as irrational and wasteful.

To summarize : There is no thermodynamic reason why engines of the highest speeds should not have the highest efficiencies. The work of Willans and Rites has proven this. The reasons why they usually do not are economic or commercial in character.

On the other hand, there are reasons both thermodynamic and economic in character why the high-speed type of engine excels the other in sizes below 75 to 100 horse-power. In the writer's opinion four-valve engines smaller than this should become as thoroughly obsolete as should the throttle-governed engine of any size whatever.

CHAPTER II

THE COMPOUND STEAM-ENGINE

THE study of the compound engine must be approached from the standpoints of two very distinct sets of considerations: (1) the **mechanical**, and (2) the **thermal**. In the development of the steam-engine, compounding was attempted by Woolf at a very early date, late in the eighteenth century, in fact, but was unsuccessful owing to tardiness of development of purely external considerations. More than half a century elapsed before McNaught introduced it again, and then it appeared chiefly as a modification of existing engines to permit the use of higher steam-pressure and the development of greater power. The objects sought were purely mechanical. It was quite as a secondary and incidental result that it appeared that the efficiency also was increased.

But McNaught's method of compounding applied only to the heavy stationary beam-engines of the first half of the last century, and found a very limited adoption. By 1860, however, it had come to be a pretty well established fact that the larger marine engines must include more than one working-cylinder and crank, for purely mechanical reasons. Weights of individual parts were thereby reduced, evenness of turning-moment on the shaft was much enhanced, the higher rotative speeds demanded by the substitution of the screw for the paddle-wheel were facilitated, and the danger of complete failure of power from the failure of any one small piece in the engine was considerably decreased. Then, the gain in efficiency from

compounding having been recognized from McNaught's experience, the substitution of two cylinders of dissimilar dimensions using the steam in series for two cylinders of equal size using it in parallel was an exceedingly simple, easy, and inexpensive step. Nevertheless it was one of supreme importance for the development of the world's maritime trade and of the steam-engine in all services. Upon the increased efficiency due to the adoption of compounding in marine engines was based the tremendous expansion of the merchant marine in radius of steaming ability, in speed, and in cargo-carrying capacity. In the earlier days of simple engines the transatlantic voyage was apt to consume twenty days, and the daily consumption of coal was such that a vessel could scarcely carry her own fuel for a longer voyage; certainly not with a proper margin of freight-paying cargo. But as compounding made the engines more efficient more power could be put on board, shortening the time-length of a given voyage; the decreased time joined hands with the increased efficiency to reduce the demand upon cargo-space for fuel.

By 1875–80 the growth of the ocean liner had attained such a size that propelling engines upwards of ten to fifteen thousand horse-power had become common. All of these engines were compounded, of course; but the same considerations which had led to the splitting of the non-compound engine of 1850 into two cylinders now forced the adoption of three or more cylinders working upon three separate cranks in the design of these big compound engines. The high-pressure engine was usually placed in the centre, with two low-pressure cylinders forward and aft of it respectively. The cranks were usually set at an angle of 120° apart; the steam-pressure was ordinarily between 100 and 125 pounds.

When the triple compound was introduced (1880–85)

R

as a more efficient and no more expensive way of building the standard three-cylinder engine, it soon led to engines of still larger power, because under the improved efficiency the limited coal-bunkers and boiler-weights would permit it; and as the size of engine increased, four or more cylinders and cranks were adopted in triple-compounding just as three had been used in double-compounding.

To-day it is an accepted fact that the marine engine of any appreciable size must have at least four separate cylinders, and often more, working three to five separate cranks. The steam-distribution in these cylinders may be upon the double, triple, or quadruple plan of compounding, according to the special conditions of the service in view, and to the designer's estimate of the thermodynamic gain to be expected; but the multiplication of cylinders has been dictated by purely mechanical considerations and has lain back of and quite independent of all thermodynamic factors. For this reason the mechanical features of the question of compound *vs.* simple engines will be taken up first, although in mere outline, and the thermodynamic considerations left to a secondary consideration.

Mechanical Differences. — To gain a clear idea of what is meant by compounding reference must be had to Fig. 42.

Let BB' represent the given boiler-pressure, AA' the atmospheric line, and CC' the vacuum in the condenser. Then, with a given supply of boiler-steam BD, the power available is given by the area $BDC'C$, supposing the curve DRC'' to be extended to intersect CC'. This power would ordinarily be developed in a cylinder having the total volume CC' and cutting off at $\dfrac{BD}{CC'}$ of the stroke. If it be desired to use compounding, however, a high-pressure cylinder may be added to the engine and an intermediate receiver introduced between the two cylinders.

Let the pressure in the receiver be RR. This pressure is determined by the volume at cut-off in the low-pressure cylinder RR, which is, of course, capable of adjustment. The indicator-card of the high-pressure cylinder now becomes $BDRR$, and that of the low-pressure cylinder $RRC'C$. If it is desired to limit the effective terminal pressure to the profitable minimum defined on page 234, making both cylinders smaller will cut off the toes of the

FIG. 42
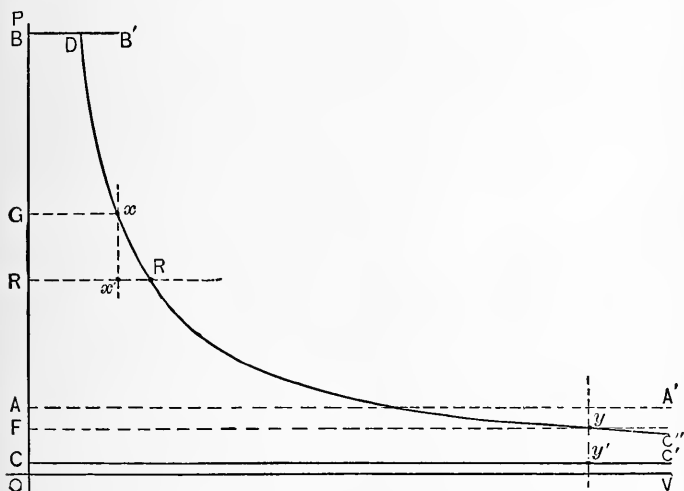
cards. If the volume of the high-pressure cylinder be reduced from RR to Gx, and that of the low-pressure cylinder be reduced from CC to Fy, the toes of the cards will be cut off at xx' and yy' respectively.

In triple compounding there would be two such dividing lines as RR, representing the pressures in the first and second receivers respectively; in quadruple compounding there would be three such horizontal lines.

The first and most obvious conclusion from this is that the total area of available work is no greater after com-

pounding than it was before. Indeed, it is somewhat
smaller by the loss of pressure inevitable in passing the
steam from one cylinder to another. From this broad
fact is deduced the rule that —

*In a given compound engine, under a given boiler-pressure
and a given ratio of expansion, the power developed is inde-
pendent of the number of cylinders through which the steam
is worked before reaching the low-pressure cylinder; it is
the same as if the same boiler-steam performed the same
number of expansions in the low-pressure cylinder alone.*

The simplest form of compound engine is the tandem,
where both pistons work upon the same piston-rod and
crank. With the exception of the compound beam-engine
of McNaught, which never found a wide acceptance,
although some very large engines have been built on his
plan, it was also the first. It is undoubtedly the cheapest
to build, and so finds wide acceptance yet, for stationary
engines of moderate power. Its latest field of adoption is
in locomotive work. Here it contests for the laurels with
the Vauclain arrangement of a pair of parallel cylinders,
high-pressure and low-pressure, on each side of the loco-
motive, each pair working on a single cross-head or with
the several European types of four-cylinder compounds.
The two-cylinder compound locomotive, with a high-press-
ure cylinder upon one side and a low-pressure cylinder on
the other, does not promise to be a permanent type.

The purely mechanical advantages of compounding have
been stated as consisting of lighter weights all around, a
more even turning-moment, better equalized strains upon
all working-parts, and greater ease in properly distributing
the steam. It would seem, at first sight, that in the tandem
compound none of these advantages could appear; for
since a single crank and set of connections is relied upon

to develop the same power in either case, the difference would appear to lie wholly within the cylinders, and that their complexity and weight must be doubled. Upon investigation, however, such does not appear to be the case.

Referring to Fig. 42, let a simple-cylinder engine be compared with a tandem-compound, supposing that in each case the boiler-pressure be 145 pounds by gage, or 160 pounds absolute, and that the final volume Fy, in order to develop the desired power, must be 10 cubic feet. This may be considered as supplied by a cylinder $27'' \times 30''$, which would constitute the only cylinder of the simple engine and the low-pressure cylinder of the compound. To the latter must be added the high-pressure cylinder upon the same axis with the low-pressure cylinder, with their pistons attached to a common piston-rod. If the proper ratio between the two be assumed to be four to one, this would call for a high-pressure cylinder $13\frac{1}{2}'' \times 30''$.

In the case of the simple engine the maximum pressure upon the piston, which would occur just at the beginning of the stroke, would be (neglecting back-pressure) 160 pounds over an area of 572 square inches, or a total of 91,520 pounds. The cylinder, piston, piston-rod, connecting-rod, crank, shaft, and engine-bed must all be designed to withstand this stress.

In the case of the compound (if the simple empirical equation of $pv = a$ constant be assumed as a basis for estimates) the receiver-pressure should be $\frac{1}{4}$ of $160 = 40$ pounds absolute. Then the high-pressure piston would be subjected to a maximum stress of 120 pounds over an area of 143 square inches, or a total of 17,160 pounds. The low-pressure piston would be subject to a maximum pressure of 40 pounds over an area of 572 square inches, or a total of 22,880 pounds.

It is immediately obvious that, in so far as cylinders and

pistons alone are concerned, they need to be built to with-
stand a stress only about one-quarter as large as before.
Not only will they be much cheaper to build and handle,
but all their appurtenances in the way of valves and gear
will be much lighter, cheaper, and more reliable, and engine-
friction will be much reduced.   Even when attention is
turned to cross-head, crank, and connections a great gain is
visible.   The total maximum stress upon them in the case
of the compound is 40,040 pounds as compared with 91,520
pounds for the simple engine, representing a gain of over
60%.

The explanation of the great difference, for the same
power developed on a single crank, lies in the fact that
whereas in the simple engine the maximum stress would
continue only throughout about one-sixteenth of the stroke
(if a terminal pressure of 10 pounds be assumed), in the
compound the cut-off would not take place until one-quarter
of the stroke had elapsed.   The pressure would be more
moderate in the case of the compound, but would be more
continuously sustained.

If attention be turned from the tandem-compound to the
cross-compound, where each cylinder has its own crank, or
to the marine-type of compound, where the work developed
in a single one of the stages is often distributed between
two cylinders and cranks, the degree to which this gain in
lightness and strength may be exaggerated can be well
imagined.   In the last-named type, too, an additional argu-
ment for multiplication of stages of compounding, as well
as of cylinders and cranks, lies in the fact that these engines
can carry no fly-wheel and possess light foundations, and
therefore require a very even turning-moment on the shaft.
A short cut-off under high pressure upon a single large
piston would start off the stroke with a disastrously violent
jump.   At the other end of the stroke, too, since the re-

versing valve-gear under which all marine engines must work introduces a heavy cushion as an inevitable accompaniment to an early cut-off, the piston would be arrested in its motion by an excessive negative pressure. The engine would either fail to pass dead-centre altogether, or would do so reluctantly and with noise or vibration. Compounding remedies all of these troubles. As even so moderate a cut-off as one-quarter stroke, however, would develop a troublesome cushion, the designer of a marine engine to work under a pressure as high as was assumed in the above illustration would choose between the alternatives of a less complete expansion on the one hand or of the extra cost and complexity of triple-compounding on the other. Thus, in the illustration, a terminal pressure of 10 pounds absolute was assumed in the low-pressure cylinder and complete expansion to receiver-pressure in the high-pressure cylinder. This would make the indicator-card of the former follow the diagram $RRyy'C$, Fig. 42, and the latter the diagram $BDRR$. If, however, the stroke of each piston were shortened from 30 to 24 inches, the terminal pressure in the high-pressure cylinder would be raised to

$$\frac{BD}{Gx} \cdot BO = \frac{7.5}{24} \times 160 = 50 \text{ pounds,}$$

and in the low-pressure cylinder to

$$\frac{RR}{Fy} \cdot RO = \frac{7.5}{24} \times 40 = 12.5 \text{ pounds}$$

absolute, or 10 pounds net; which is much better engineering practice. The cut-off would have been lengthened from 25 to 31% of the stroke and the excessive cushion have been moderated thereby.

Let it be supposed again, to take up the second alternative in comparison, that this same expansion were to be

carried out in a four-crank, four-cylinder, triple-compound engine, having one high-pressure, one intermediate, and two low-pressure cylinders. Since the last stage of the expansion is to be divided between two cranks, a little extra proportion of expansion and of power may be thrown into the low-pressure stage. This could be done, for instance, by giving the engine two low-pressure cylinders of 19 inches in diameter, one intermediate cylinder 15 inches in diameter, and one high-pressure cylinder 10 inches in diameter; the stroke of all of them may be considered as still 30 inches. It is not to be considered by the student that this necessarily represents the best engineering practice, but it enables a simple comparison to be drawn between engines of widely different type under the same steam-pressure and expansion, in order to bring out most plainly one or two particular points; viz., comparative maximum stresses and comparative cut-offs. The dimensions chosen give a ratio between high-pressure and intermediate of 2.31, between intermediate and low-pressure the same, and 3 as the ratio of expansion within the low-pressure cylinder. This places the first-receiver pressure at $160 \div 2.31 = 69.3$ pounds, and the second at $69.3 \div 2.31 = 30$ pounds absolute. The maximum pressure falling upon the high-pressure piston and crank is $(160 - 69.3) \times 78.5 = 7120$ pounds. That upon the intermediate piston and crank is $(69.3 - 30) \times 177 = 6956$ pounds. That upon each low-pressure piston and crank is $30 \times 283 = 8490$ pounds. If these figures be compared with the maximum stress on a single crank of 17,280 pounds for the two-crank cross-compound, 40,320 pounds for the tandem-compound, and 92,160 pounds for the simple engine, the gain is obvious. The cut-off in each of the two smaller cylinders of the four-cylinder triple-compound would be 43.3% and in the largest cylinders 33.3%.

Finally, were the strokes again reduced to 24 inches in

order to give a profitable terminal pressure and to minimize cushion, the cut-offs would lengthen from 43% to 54% and from 33% to 42% respectively.

Although intended for illustration only, these proportions, if modified so as to give a shorter stroke and larger diameter of cylinder and worked under a higher steam-pressure than 145 pounds by gage, would represent a close approach to good marine-engine practice.

Thermal Considerations

It will be remembered that in the discussion of cylinder-condensation and its kindred phenomena it was noted that both theoretic efficiency and condensation increase as the temperature-range within the cylinder increases. The former increases almost proportionately to the temperature-range; the latter, insignificant at first and increasing very slowly with the first growth of temperature-range, takes on an ever-increasing augmentation as the additions to the temperature-range accumulate, in somewhat geometric ratio, until finally cylinder-condensation becomes the all-important phenomenon, overbalancing and eclipsing all other considerations. In other words, as the theoretic efficiency increases, the cylinder-efficiency decreases, at accelerating rate, until the resultant thermodynamic efficiency, at first ascendant, becomes stationary and finally declines. A temperature-range a little beyond that at which the latter is a maximum dictates the limits for the proper use of simple expansion in a single cylinder.

If any higher efficiency than that just described is desired, compounding must be resorted to. But it is immediately evident that compounding under a total temperature-range and pressure-range (between boiler and exhaust) which gives a maximum efficiency in a single cylinder must

result in a loss rather than a gain; for compounding is expensive, not only in cost of construction but in incidental losses of heat as well. The steam cannot be transferred from one cylinder to another without distinct loss, in friction and in radiation. Even at a moderate advance of temperature-range over the point just mentioned these losses must overbalance the gains due to

(1) The increased theoretic efficiency, and
(2) The increased cylinder efficiency

due to dividing a greater total temperature-range into two portions, each of which is smaller than that of the simple engine. The first conclusion to be observed, therefore, is that compounding cannot be profitable except under a total temperature-range decidedly greater than that specified as profitable for a single cylinder on page 228. This is the same as saying that compounding cannot possibly be profitable under less than 125 pounds of boiler-pressure when non-condensing, nor under less than 90 pounds condensing.

That is the lower limit. The upper one is probably about 175 pounds when condensing, or about 250 pounds when non-condensing. For it is not until these figures are surpassed that the temperature-range in a single cylinder becomes great enough to warrant the introduction of a third stage of expansion. Where the latter policy has been adopted under pressures similar to the above it has either been for mechanical reasons, as in the marine-engine, or in error of judgment, as has come to be generally admitted in connection with triple-expansion stationary engines.

It is therefore obvious that in the design of a compound engine, in the choice of boiler-pressure under which it should work and of cylinder-ratio to be adopted, there is

the widest possible field for variation of individual judg-
ment.   There will be found a correspondingly wide varia-
tion in existing practice.   In general the problem of the
determination of exact cylinder-dimensions would be at-
tacked somewhat in this way.

Problem : To design a compound engine of 100 horse-
power normal load, to operate under a steam-pressure of
not over 150 pounds and to exhaust into a vacuum.   It is
assumed that the general type of engine, the speed, the
valve-gear, and all similar points outside of thermodynamic
ones are already determined; that the engine is to be of
the four-valve, receiver type, and to run at 100 revolutions
per minute.   What should be the cylinder-dimensions ?

According to the rule stated on page 244, the size of
low-pressure cylinder must first be determined as if there
were to be no high-pressure cylinder present, the full
degree of expansion taking place supposedly in the single
cylinder.   Since condensation and reëvaporation so modify
the true adiabatic process that its pressure-volume repre-
sentation closely agrees with the curve

$$P V = \text{a constant,}$$

it is common practice to adopt this as the basis for the
projection of indicator-cards in such work as this; partly
because the entire problem can never be exact, in either
statement or solution, and partly because the simplicity of
the curve lends itself to the rapid investigation of the sev-
eral supposed cases which usually need comparison.

It would first be decided that if 150 pounds by gage
were to be the normal boiler-pressure, 145 pounds by gage
or 160 pounds absolute are all that could be expected as
the initial pressure within the cylinder, considering varia-
tions in boiler-pressure and pipe-friction.   Next, the most
advantageous terminal pressure would be chosen; say at

8 pounds absolute. This establishes the total ratio of expansion at $\frac{160}{8} = 20$. Let it be assumed that the vacuum to be expected in the cylinder is 25 inches, or $12\frac{1}{4}$ pounds below atmospheric pressure, or $2\frac{1}{4}$ pounds absolute.

Referring to Fig. 42, the expansion to the final terminal pressure at y, when exhaust into the condenser takes place, would develop the work-diagram $CBDyy'$. Of this, the area $BDyF$ is given by the formula (deduced for this curve
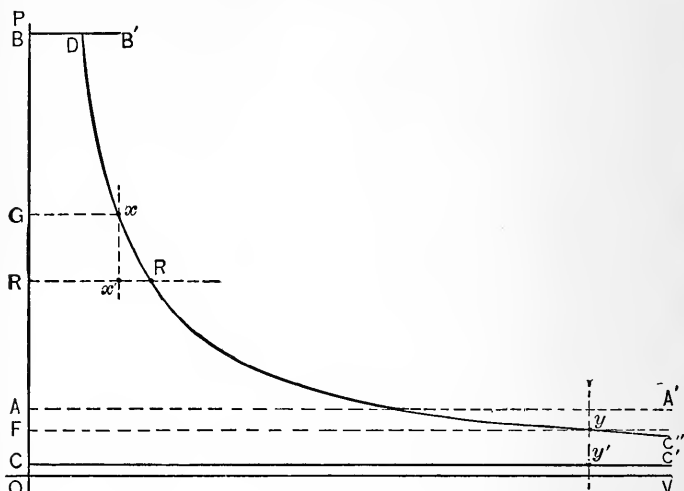
FIG. 42

on page 122): Work $= P_1 V_1 \log r$. The final volume attained by the steam, at the end of the stroke in the largest cylinder, will be represented by Fy. Let this be represented by V_1, in cubic feet. Then, since $P_1 = 8 \times 144 = 1152$ pounds per square foot and $r = 20$, the work represented by the area $BDyF$ must be $1152\, V_1 \log_e 20 = 3451\, V_1$ foot-pounds. The work represented by the area $CFyy'$ is $(8 - 2\frac{1}{4})\, 144\, V_1 = 828\, V_1$ foot-pounds. The total is

$$4279\, V_1 \text{ foot-pounds.}$$

Since the engine is to be of 100 horse-power, the work done per minute must be 3,300,000 foot-pounds. Placing this equal to the expression just found and solving for V_1, gives the value of the latter as 771 cubic feet. This must be the volume displaced by the piston each minute. If the engine is to make 100 revolutions, or 200 strokes, per minute, the displacement per stroke must be 3.85 cubic feet.

This volume may be attained by any proportion of diameter to stroke which suits the mechanical sense of the designer. The chief guide in the choice is the speed: the slower the speed, the longer the stroke; the higher the speed, the shorter the stroke. A good proportion for this case would seem to be given by a cylinder $17'' \times 30''$, which gives slightly more than the desired displacement.[1]

Receiver-pressure and Cylinder-ratio. — Next arises the question as to the size of the high-pressure cylinder. This is determined by the choice of receiver-pressure and of terminal pressure in the high-pressure cylinder.

The simplest solution of the case is to consider that the total ratio of expansion is to be divided equally between the two cylinders; that is, that the square root of the total ratio is to be the ratio in each cylinder. In Fig. 42 RR is the receiver-pressure and xx' the exhaust-line at the termination of the high-pressure stroke. By the above argument the pressure at x would be

$$OG = OF\sqrt{\frac{OB}{OF}} = \sqrt{OF \times OB} = \sqrt{8 \times 160} = 35.8 \text{ pounds.}$$

Similarly,

$$OR = OC\sqrt{\frac{OB}{OC}} = \sqrt{OC \times OB} = \sqrt{2.25 \times 160} = 19 \text{ pounds,}$$

[1] It is to be remembered that no consideration is being given here to the effects of clearance and cushion upon the ratio of expansion. Their inclusion would probably bring the figures up to about $18'' \times 30''$.

or the receiver-pressure would be 4.5 pounds by gage. The volume at x is

$$Gx = Fy \cdot \frac{OF}{OG} = Fy \cdot \frac{8}{35.8} = Fy \cdot \frac{1}{4.475}.$$

It is stated in this way because 4.475 is therefore the desired ratio between the volumes of the two cylinders; but as the strokes of the two are nearly always the same, the ratio of diameters may be taken as the square root of the ratio of volumes. This gives, for the diameter of the high-pressure cylinder,

$$17 \div \sqrt{4.475} = 8.036, \text{ or, say, 8 inches.}$$

This gives, as the proper cut-off in the low-pressure cylinder, $RR \div Fy = OF \div OR = 22\%$, and in the high-pressure cylinder $BD \div Gx = OG \div OB = 22\%$.

Another common rule for determining the receiver-pressure is to divide the work done equally between the two cylinders.

The total work was found to be $4279 V_1$ foot-pounds per minute. Half of this is $2140 V_1$ foot-pounds. Deducting the area $CFyy'$, the remainder of the low-pressure area, $FRRy$, is $1312 V_1$ foot-pounds per minute. Since this is equal to $P_1 V_1 \log r$, and since $P_1 = 8 \times 144$, r is easily found to be equal to 3.125. $OR = 8 \times 3.125 = 25$ pounds absolute, as compared with 19 pounds by the former method.

The area remaining for the high-pressure cylinder to develop would be $RBDR$. Some triangular portion of this, as xRx', would properly be cut-off in order to give that cylinder an appreciable and profitable terminal pressure, and would be lost; but as it is small compared to the whole, and as the rating of 100 horse-power taken as a basis for all of the calculations is a purely nominal one,

such incidental minor losses are usually neglected for the
time, and are allowed for in lump by increasing the calcu-
lated cylinder-dimensions by the fractions necessary to
bring them up to some convenient integral figure. Thus,
if 20 pounds be chosen as a proper high-pressure terminal,
the total pressure at x would become 45 pounds, and the
cylinder-ratio would be $Fy \div Gx = OG \div OF = 45 \div 8 = 5.625$,
which calls for a high-pressure cylinder about $7\frac{3}{16}$ inches
in diameter.

But any and all of these methods, of which there are
several, are rule-of-thumb substitutes for the only rational
thermodynamic one, viz.: the equal division of tempera-
ture between the cylinders. In the present case the total
temperature-range, from 160 to $2\frac{1}{4}$ pounds absolute, is
232.8°. Half of this is 116.4°, which, added to the back-
pressure temperature in the low-pressure cylinder, gives
247° F. as the proper receiver-temperature, or 28.3 as its
proper absolute pressure. A high-pressure net terminal
of 20 pounds again would make the gross high-pressure
terminal at x, 48.3. The cylinder-ratio would be $\frac{48.3}{8} = 6.04$.
The diameter of the high-pressure cylinder would be
$17 \div \sqrt{6.04} = 6\frac{15}{16}''$; say 7 inches.

It is obvious that all of these methods lead to very
similar conclusions. The second one is very commonly
resorted to, originally because it was very easily carried
out by means of the indicator. It rests upon good foun-
dation in those cases where mechanical considerations out-
weigh thermodynamic ones; but the false idea has grown
out of this practice, and been widely taught to operating
engineers, that the equal division of load between the
cylinders is thermodynamically proper. It is usually me-
chanically proper, though sometimes not. It is only a co-
incidence if the thermodynamic results happen to be good.

Clearance and Cushion. — The influence of clearance and cushion upon capacity, or upon ratio of expansion, has been openly excluded from the above calculation.   The method of allowing for it was explained in Chapter I, page 196. Suppose that in the low-pressure cylinder of the above engine it be known that the actual clearance (which would be estimated from a knowledge of the type of valve-gear) would be about 2%. The calculations on page 196 showed that such a clearance necessitated the increase in volume of the cylinder, in order to get the same results from the same quantity of steam, from 2.62 to 2.77 cubic feet, or by about 6%. Such an allowance for clearance as this would call for the increase of the 17-inch cylinder to $17\frac{3}{4}$ inches in diameter. The high-pressure cylinder should of course be similarly increased in diameter, to correspond.

The preceding problems cover the bulk of what may be stated with mathematical accuracy regarding the choice of cylinder-ratio in compounding. In the remaining portion of the field, which is open to variation of opinion, there is the widest divergence between the judgments of men of the highest standing. The Westinghouse compound stationary engine and the Vauclain compound locomotive, both of which have been exceptionally successful designs, use a ratio of about 2.7 to 1 under as high as 200 pounds boiler-pressure, but with no vacuum. Mr. George I. Rockwood advocates 7 to 1 as a proper ratio, under even lower boiler-pressures when exhausting into a vacuum, and his engines have given good results. Between these two extremes might probably be assorted all the different opinions regarding compound-cylinder ratios in the country. It would seem unquestionable, however, in spite of this wide divergence of opinion, that a broad survey of the progress of experience in engine-design reveals the following facts:

(1) That the steam-pressure which can be efficiently expanded in two stages is higher than was formerly supposed. The need for the intermediate cylinder in the triple compound is based upon purely mechanical, not thermodynamic, principles. The maximum cylinder-efficiency is attained in each cylinder of a compound engine under approximately the same conditions as in the cylinder of a simple engine, viz.: under from four to six expansions. These three statements are practically synonymous.

(2) That, as a result, cylinder-ratios are steadily upon the increase.

The favorite ratio in England was, for many years, for double compounds four to one and for triple compounds six to one, as between low- and high-pressure cylinders. This applied normally to large condensing compounds. Compounding was adopted more reluctantly in America and at first copied English practice in this respect. By 1895, however, the plan of carrying a standard line of compound designs and patterns for all services was well established. As the majority of these engines were sold to operate either under steam of abnormally low pressure from existing boilers, or under atmospheric exhaust, the ratio of four to one was found to be too large. General practice soon settled down to the region between 2.75 and 3.25 to one. Within the past few years, however, the trend is back toward the higher ratios, and values above four to one are now quite common in new engines from the best works.

To the marine engine, where mechanical considerations outweigh thermodynamic ones, these remarks, of course, do not apply. Cylinder-ratios are much lower there, in order to avoid too early cut-offs and too heavy cushion for engines without fly-wheels.

s

The Receiver-type and the Woolf-type of Compounds

It is almost universal practice, in compounding, to exhaust from the high-pressure cylinder into a chamber of considerable volume, called a *receiver*, and to draw from it the supply for the low-pressure cylinder as from a boiler. The advantages resulting from this plan are several. They may be lumped under two heads, as follows : —

(1) *Mechanical.* — The timing of the motions of the two pistons is quite independent one of the other. The cranks driven by the two cylinders may be set at any desired angle.

(2) *Thermodynamic.* — The pressure in the receiver remains nearly constant; hence the temperature-range in the high-pressure cylinder is only that from boiler to receiver temperature, and the temperature-range in the low-pressure cylinder is only that from receiver to exhaust temperature, Were this not so, were there a variation in the receiver-pressure, the high-pressure temperature-range would extend from boiler to the lowest receiver-temperature, while the low-pressure range would be from the highest receiver-temperature to exhaust. The two ranges would lap, their sum would exceed the total range from boiler to exhaust, and the resultant condensation would be greater than it otherwise might be.

This is the condition of affairs in the Woolf-type of compound. It has no receiver. The two cylinders are separated only by the exhaust and admission valves. This necessitates —

(1) That the pistons must move simultaneously. (They may move in opposite directions, however.)

(2) The intake of the larger cylinder can consist only of the exhaust from the smaller; the difference in size be-

tween the two pistons therefore insures that the "receiver-pressure" must steadily decrease, owing to expansion, during the exhaust-stroke of the high-pressure and the admission-stroke of the low-pressure cylinder. These phenomena are visible in Fig. 43, in which the dotted lines show the effect of shortening the cut-off from that shown by the full lines.[1]

The Woolf engine was one of the earliest compounds to

BOILER-PRESSURE.

HIGH-PRESSURE CYLINDER; CLEARANCE, 33%.

LOW-PRESSURE CYLINDER; CLEARANCE, 7%.

ATMOSPHERIC PRESSURE.
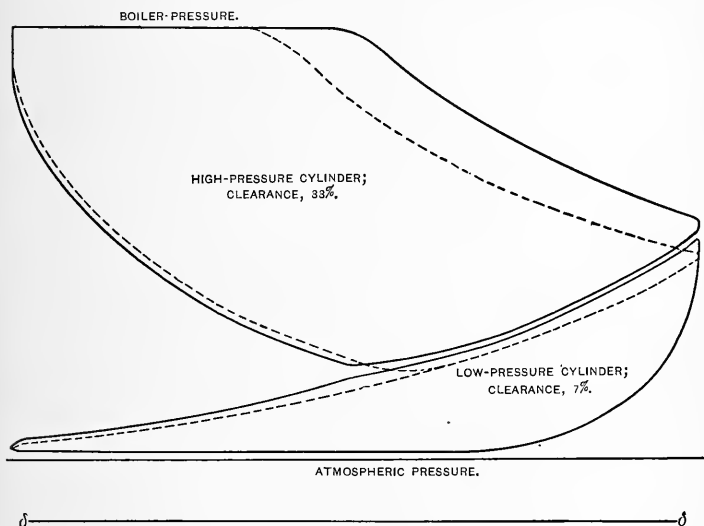
FIG. 43

appear, late in the eighteenth century. Unable to successfully compete with Watt's monopoly or to establish its superiority under the low steam-pressures then prevailing, it failed and disappeared. Within the last twenty years, however, its marked superiority under certain conditions has been appreciated and it has reappeared.

For the reasons just given, the receiver-type of com-

[1] See Trans. Am. Soc. M. E., Vol. XIII, "Steam-distribution in a Single-acting Engine," F. M. Rites.

pound is superior, both in mechanical adaptability to diverse arrangements of piston-motion and in thermal efficiency, to the Woolf-type, under nearly all conditions of service. It is therefore only necessary, in order to properly define the worth of the latter, to consider the conditions under which the receiver-type is *not* a success. These may be very easily and narrowly defined as a light or variable load combined with atmospheric exhaust. Under these conditions the efficiency of the receiver-type diminishes so very rapidly as to easily fall below that of the simple engine, and sometimes below the poorest which could ever be expected from a good simple engine.

The reason for this is that under light loads the governor admits to the engine so small a volume of boiler-steam that the terminal pressure in either or both cylinders falls below the exhaust-pressure. The result is a loop at the lower end of the card, the pressure in the cylinder rising when the exhaust-valve is opened, instead of falling, as it should. While a loop at the upper corner of the indicator-card signifies little loss of efficiency, if any, a loop at the lower end of the card is trebly bad.

(1) It much exaggerates the evils already described as arising from complete expansion to exhaust-pressure, on a sharp-toed diagram.

(2) It absorbs power excessively which is not returned to the engine in the slightest appreciable degree.

(3) It exaggerates cylinder-condensation by sucking in through the exhaust-port moisture which, much of it, must reëvaporate within the cylinder at the expense of heat made good from the next initial condensation.

Trouble (2) could theoretically be cured by equipping the cylinder with check-valves opening back from exhaust into the cylinder whenever the pressure in the latter fell too low; but they would be noisy, frail, and troublesome,

and would have no effect on the other evils. They have never been widely applied to mill-engines, though common in locomotives.

The effect of these losses is pernicious in the extreme. In one case it was reported, by a high authority, that a good specimen of Corliss compound engine, new and in good condition, which would ordinarily be expected to develop a horse-power upon about 14 to 15 pounds of steam per horse-power-hour when properly loaded and supplied with a vacuum, actually consumed, when lightly loaded and exhausting into the atmosphere, 72 pounds of steam per horse-power-hour. If the low-pressure cylinder had been disconnected, the other cylinder alone would have developed the power, under these conditions, at a rate not much exceeding one-third of this figure. It did develop about 180 horse-power at just about that rate; but the low-pressure cylinder, acting as a brake by reason of its having to drive its exhaust out against an atmospheric pressure which was higher than its own terminal pressure, absorbed over 100 horse-power, so that the net rate was that stated. The high-pressure cylinder alone would not, of course, have developed the 70 or 80 net horse-power as efficiently as it did the 180 horse-power; but the rate, even so, could hardly have risen above 30 pounds per horse-power-hour.

The superiority of the Woolf-type under these same conditions is due to the following circumstances: —

There being no volume between the two cylinders, any cut-off in the low-pressure cylinder must also act as the closing of the exhaust of the high-pressure cylinder, or as the point of compression in that cylinder. The fact that an ordinary cylinder cannot be given more than a certain degree of cushion (without exceeding boiler-pressure before admission takes place) has led to the oft-stated but false

idea that the low-pressure cylinder of a Woolf compound could have no cut-off. But any cylinder may be given as early a cushion as is desired, provided there be sufficient clearance to absorb the compression before too high a terminal pressure is reached. Therefore all modern designs of Woolf-type compounds are provided with an unusually large clearance in the high-pressure cylinder. This permits an early cut-off in the low-pressure cylinder and an early, slowly rising cushion in the high-pressure cylinder.

If this cushion were sufficient to engender boiler-pressure in the clearance space before admission took place, and if there were no thermal losses to cylinder walls, this large clearance would involve no loss of efficiency (see page 206; 236). In practice neither condition is supplied. Losses are involved from both sources. The large clearance must be partially refilled with boiler steam before each stroke commences; owing to thermal interchanges between the steam and walls the cushion-steam does not return as much work during expansion as it absorbed during compression. Therefore the Woolf-type never attains the maximum efficiency usual in receiver-compounds when run under proper conditions. Nevertheless, the sum of these two losses in the Woolf-type is not nearly so great as is that incurred by the over-expansion of the receiver-type under light loads and no vacuum. The latter loss can be entirely avoided in the Woolf-type by making the high-pressure clearance sufficiently large, so that under the lightest possible load, — zero load, say, for an extreme illustration, — there will always be steam enough in the engine to expand through both cylinders and still be above atmospheric pressure at the end of the low-pressure stroke. Since the cushions in both cylinders are supposedly so designed that the ordinary back-pressure, while the exhaust is open to the atmosphere, is compressed back to boiler-pressure in the high-pressure

clearance, there would be, theoretically, in the extreme case of no load just supposed, no intake of steam from the boiler at all. In practice this nice adjustment of valve-motions is found impracticable, and some steam is always consumed in merely refilling the high-pressure clearance at each stroke; but the approach to the theory is quite close.

The origination of this plan of operation the world owes to Mr. F. M. Rites, who incorporated it into the design of the Westinghouse compound engine, which is a pure Woolf-type engine. It is also embodied in modified form in the four-cylinder Baldwin compound locomotives of Mr. Vauclain's design. Since this beginning of its revival other designs of Woolf-type engines have been placed upon the market.

METHODS OF STEAM-DISTRIBUTION

Compound engines may be rigged with any of the several types of valve-gear used on simple engines: The single balanced-slide or piston-valve, driven either by a fixed eccentric (with throttle governor), by a shaft-governor, or by link-motion; the four valves of the Corliss engine or their equivalent, closed either by drop cut-off gear or positively; or the double slide or piston-valves of the Myers-type. But the differences between these gears are much the same in compound as in simple engines. Their discussion belongs to a study of valve-gears, not that of the thermodynamics of compounding.

There is one question calling for decision in such work, however, which has direct thermodynamic influence upon the efficiency. That is: Shall the cut-off in the low-pressure cylinder be varied by the governor as the load changes, or shall it remain fixed?

There is much needless discussion of the question, chiefly

because of doubt as to what result is desired, rather than as to what will ensue from a given course of action.   For it is plain that if the low-pressure cut-off remains fixed while the steam admitted to the high-pressure cylinder is varied with the varying load, the receiver-pressure must also vary with the load. When the latter is heavy the receiver-pressure will rise; when the load is light, it will fall.   If, on the other hand, the cut-offs of the two cylin-

FIG. 44

ders were varied together and equally by the governor as the load altered, the receiver-pressure would remain practically constant.

I. *Low-pressure Cut-off Fixed.*   This situation is shown by Fig. 44, wherein, under normal load, the governor admits the volume AB of boiler-steam to the engine. $ABCDE$ is the high-pressure indicator-card and $EFGHK$ is the low-pressure card.

When the load increases, the governor admits the volume

of boiler-steam *Ab*. Its curve of expansion is *bcfg*. The high-pressure card becomes *Abcde* and the low-pressure *efgHK*.

Under lighter loads the expansion-curve becomes $b'c'f'g'$ and the receiver-pressure falls to $e'f'$. The high-pressure card becomes $Ab'c'd'e'$ and the low-pressure card becomes $e'f'g'HK$.

II. *Low-pressure Cut-off always Equal to High-pressure Cut-off.* This situation is shown in Fig. 45. The normal-load
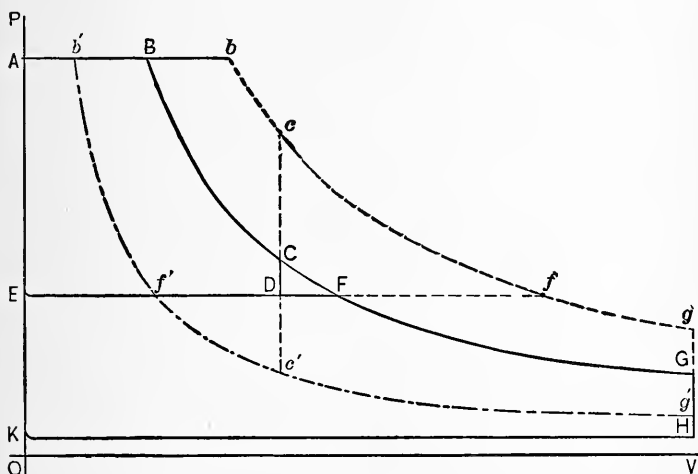
FIG. 45

diagrams are *ABCDE* for the high-pressure and *EFGHK* for the low-pressure, as before.

An increase in load moves the expansion-curve out to *bcfg*, as before; but because the low-pressure cut-off has lengthened to *f* the receiver-pressure does not rise. The high-pressure card becomes *AbcDE*. The low-pressure card becomes *EfgHK*.

Under lighter loads the expansion-curve becomes $b'f'c'g'$; but again, because the low-pressure cut-off has shortened

from F to f', the receiver-pressure remains the same. The high-pressure card becomes $Ab'f'c'DE$. The low-pressure card becomes $Ef'c'g'HK$.

Comparative Efficiency. — The chief differences between the two plans are obvious. The primary advantage of the first is that under all loads the small triangular area CDF, which represents receiver-loss, remains practically constant. It is entirely under control and is fixed in the original design of the engine and the set of the low-pressure cut-off. If the volume in the low-pressure cylinder at the instant of cut-off were equal to the cylindrus of the high-pressure cylinder, this loss-area would be zero under all loads; by making the high-pressure cylinder a little smaller than this, some cost and friction can be saved, at the expense of no appreciable loss of power or efficiency, and the receiver-drop CD can be made whatever is desired.

In the second plan, on the other hand, the receiver-loss varies widely. Under heavy loads it grows to cDf. As the load falls below normal it decreases first to zero and finally becomes *negative*, an extreme which is far more harmful to efficiency than its opposite. The terminal loss at GH or gH is alike in either plan. So that the thermodynamic advantages lie all with the first plan.

In respect to mechanical features, however, it is quite otherwise. Under the second plan the work is divided between the two cylinders quite evenly under all loads. Under the first, on the other hand, heavy loads are developed almost entirely in the low-pressure cylinder and light loads almost entirely in the high-pressure. This is mechanically bad. It could be allowed for, however, by proper design and construction. But this distribution of power has a secondary and very detrimental effect upon another important feature of engine-operation, viz. : —

Regulation. — Under the first plan the receiver-pressure

is varied up or down by the constant volume-pull of the low-
pressure cylinder upon a varying supply of steam to the
receiver from the high-pressure cylinder.   The receiver-
pressure will therefore be constant after any alteration of
load, only after these two factors have had time to attain a
condition of equilibrium.   In other words, when the load
decreases, the low-pressure cylinder finds itself supplied
with steam of a much higher pressure than that of equi-
librium.   It therefore develops more power than it would
after the steady conditions which are normal to this load
have been attained.   But, the load having fallen off, this is
just the time when *less* power, not *more*, is wanted from the
low-pressure cylinder.   The engine therefore jumps above
speed until the excessively short high-pressure cut-off in-
duced thereby, coupled with the heavy draft of the low-
pressure cylinder on the receiver, has brought the pressure
of the latter to the proper point.

But the process cannot stop there, for equilibrium is not
yet attained.   The engine-speed is above normal; the high-
pressure cut-off is shorter than that suited to the load.   The
receiver-pressure therefore continues to fall below that of
equilibrium until arrested and brought back by a reversal of
the entire above process, which will be instituted only when
the engine-speed has again fallen below normal, a thing
which must finally result from the too short cut-off in the
high-pressure cylinder.

This process of wavering above and below normal speed
at every change of load is called " hunting."   There are a
long list of causes of hunting which lie in the mechanics of
the governor.   But the effect of all such causes will be
much exaggerated by compounding with fixed low-pressure
cut-off.   Indeed, so extreme is this exaggeration in some
cases that there is no position of equilibrium possibly to be
found; the engine surges above and below normal speed

so violently that operation under loads which vary appreciably is quite impossible.

Extreme conditions may dictate the adoption of either plan unreservedly. That is, extreme demand for perfect regulation may enforce the full variation of low-pressure cut-off with that of the high-pressure cylinder, regardless of the effect upon the efficiency; or the need for the most refined efficiency may call for the abandonment of all hope of close regulation. Usually, however, a compromise is wise. The attachment of the low-pressure cut-off to the governor will insure moderately close regulation, but by diminishing the extent and rapidity with which it is varied by the governor a good grade of efficiency is maintained.

Reheaters. — It will be remembered, from the theory of thermodynamics, that the adiabatic expansion of saturated steam must always develop moisture. Since all the thermal effect of the cylinder-walls must be to produce considerable moisture before expansion begins, and since comparatively little reëvaporation takes place before the exhaust-valve opens, there is always a considerable percentage of water thrown into the receiver from the high-pressure cylinder.

In many engines, particularly where the importance of constructive simplicity outweighs thermodynamic considerations, the presence of this water is entirely neglected. In the greater number of large stationary engines, however, the receivers are drained of this water with more or less care, by traps or by hand. In this way the receiver acts as a separator upon the passing steam; but as scarcely any separator can be expected to deliver steam containing less than one or two per cent of moisture, on the average, there is still an appreciable amount of moisture which finds its way into the low-pressure cylinder, where it operates to exaggerate condensation in the manner already described.

The thermodynamic disadvantages of this condensation and the corresponding advantages of using perfectly dry, or even superheated, steam, coupled with the facility with which such moisture can be dried out by means of coils of pipe filled with boiler-steam inserted in the receiver-volume, has led to the wide adoption of such a plan in engines of considerable size which aim at the best thermodynamic efficiency. Since the temperature within the coils is much higher than that of receiver-saturation the conduction of heat to the steam is comparatively rapid; at any rate, until all moisture has disappeared. The cost of the engine is slightly greater with the added coils, but that of operation is decreased. The trap which would have been used to drain the receiver can be used to drain the coils. The receiver-steam always enters the low-pressure cylinder dry; sometimes it is considerably superheated. The gain in efficiency is always marked. It varies so widely, however, with variations of service, size of receiver and coils, etc., that no rule can be given, either for the volumes or heating-surface to be chosen or for results to be expected.

The Steam-turbine. — The applicability of thermodynamic theory to the action of steam-turbines is very limited. This action is, theoretically, upon the Rankine cycle. It is often, and most profitably, modified by superheating the steam. In this case, however, arises the difference between the turbine and the piston-engine that in the latter the superheat seldom succeeds in penetrating the cylinder until cut-off occurs, and in so influencing expansion; while in the former, since reciprocation and its corollary, condensation and reëvaporation, are wanting, the steam actually operative within the engine is superheated.

The essential process of the steam-turbine is the preliminary conversion of the steam-heat into kinetic energy of translation before its absorption by the machine. If

this process occupy the entire temperature-fall available, the resultant velocity of translation is excessively high; and since the velocity of the vane which is to absorb its energy should be a fair fraction thereof, this calls for excessively high speeds of rotation. To avoid this the temperature fall is usually split up, or compounded, into a great many steps, each of which develops only a moderate velocity, which velocity is absorbed by moving vanes before the next step is undertaken.

But velocity cannot be developed without concurrent friction. The adiabatic conversion of the heat into kinetic energy is therefore no sooner undertaken than frictional losses enter, tending to divert the path of fall from the vertical adiabatic toward the constant-heat curve (Fig. 14). The besetting sin of the piston-engine, on the other hand, it may be noted, is the departure of the path of fall from the adiabatic in the other direction, into the southwest quadrant, by condensation.

The questions arising as to the proper limits for the development of velocity before the frictional losses become too great, and as to the best form of vane, of wheel, etc., for absorbing the velocity developed, are problems in the mechanics of fluids rather than in thermodynamics.

CHAPTER III

THE OTTO GAS-ENGINE

The variation of the actual cycle of the Otto-type of gas-engine from the theoretic is much more difficult to trace than is that of the steam-engine. More observations of a much more difficult character go to the organization of a gas-engine test than to that of a steam-engine test. The data which are most essential to the complete thermodynamic analysis of the processes occurring within the cylinder cannot be gotten at all, by any means yet adopted by the profession; assumptions and estimates based upon what observations it is possible to make must be substituted therefor. For instance, one of the most important observations needed, that of the temperature of the uncompressed charge of explosive mixture in the cylinder at the beginning of the compression-stroke, has never been taken but once or twice and then at a great cost of money and effort.

The prime factors in the modification of the actual gas-engine cycle from the theoretic are the following. They are named about in the order of their importance.

(1) The abstraction of the heat from the burning gases by the water-jacket.

(2) Heat-interchanges with the cylinder-walls similar to the condensation and reëvaporation of the steam-engine.

(3) The influence of variations from the normal cycle introduced in order to modify the power developed.

(4) The slowness of inflammation.

(5) Heat-abstraction by the water-jacket during compression.

Of these, numbers (1), (3), and (4) only are visible in the indicator-card. The others become graphically visible only in the entropy-temperature diagram. They can all of them, of course, be calculated algebraically.

In Fig. 46 let the area $BCDE$ represent the theoretic possibilities of an Otto gas-engine in the same way as did

FIG. 46

Fig. 21, page 140. If the corresponding pressure-volume diagram, Fig. 22, were reproduced here and the indicator-card from the actual engine superimposed upon it, there would be found wide discrepancies between the two. It is not worth while to do this, however, because a comparison between theoretic and real pressure-volume diagrams does not illustrate differences in heat-energy, which is the thing needful, but only differences in their effects; and as analysis is undertaken only for the purpose of detecting causes, recourse must be had to the entropy-temperature diagram. The method of doing this mathematically will be explained on page 281. In the meantime let attention be turned to what happens within the cylinder.

During the process of compression, which theoretically passes adiabatically from B to C, heat is abstracted by the water-jacket. The actual path of compression therefore departs to the left and follows some course similar to Bc. Then ignition and inflammation take place and temperature

and entropy increase along *cd*. The heat supplied is sufficient to carry the explosive development of heat to *D*. While the process is in operation, however, the water-jacket steals some 35% to 50% of the total heat supplied, or the area *GdDH*; the isomorphic development of heat therefore cannot pass beyond *d*. By observation of the heat absorbed by the jacket-water during the test of the gas-engine in question the point *d* may be determined on the theoretic diagram. But the abstraction of heat by the water-jacket does not occur in such a way as to take the simple form *GdDH*, leaving the actual cycle to be *BCde*. Some of it occurs during compression, and is measured by *OBcL*.
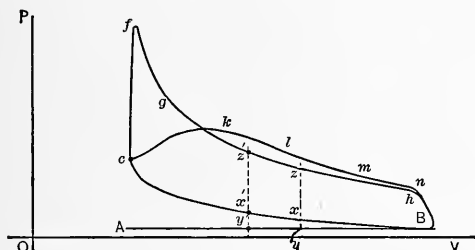
FIG. 47

But the greater portion occurs during explosion and the first portion of expansion, and is measured by *HDfgheG*, in which the line *eG* now takes such position as will make this area and *OBcL* together equal the measured loss of heat. For this reason the limit of explosive rise of temperature, *f*, never quite coincides with the point *d*.

Standard gas-engine diagrams may be divided into two sorts or classes, one from each of which is shown in Figs. 46 and 47. The first is the true standard card of maximum efficiency under a given degree of compression and quality of gas; it is shown by *BcfgheB*. The other is what very often appears in its place, but which is usually due to no more serious defect than a lack of proper adjustment. It is

T

shown by *BcklmnB*. In it inflammation takes place too late or too slowly. It is always due to one of two causes:

(1) Late ignition;

(2) Mixture of gas and air in improper proportions.

As to its effect upon the efficiency, the pressure-volume diagram reveals nothing certain; because it is not obvious, from a single diagram such as *cklmn*, Fig. 47, whether the area developed would be greater or less than that which would otherwise have been produced had inflammation been more prompt, such as *cfgh*. Or, if comparison by measurement be instituted, there is no argument whereby the result can be known to apply to other cases. But in Fig. 46 it is plain that any departure of the piston upon its course before inflammation is complete must result in work being done and hence in the bending of the curve *kf* around to the right toward first the horizontal isothermal and beyond that toward the vertical adiabatic, into some such path as *klm*. This is fatal to the attainment of the best efficiency. It is evident without further inspection that nothing which the cycle may do later in its course may hope to make good this loss of availability for work.

The transition from the isomorphic *cf* to the approximated vertical adiabatic of expansion (Fig. 46) is usually and properly in the form of a quite acute cusp, as at *f*. Otherwise it is more rounded, as along *kl*. Sooner or later, however, the transition is made. It is then to be particularly noted that the change of direction *always* swings beyond the vertically downward, or true southerly, direction into the southwesterly quadrant. This shows that the development of heat within the working-substance is replaced by an abstraction of heat. The only exceptions to this rule are in occasional cards in which the first portion of the expansion-line, *fg*, of a sharp-pointed card is truly vertical. So that even these exceptions show that

the development of heat has come sharply to an end and that for the moment no heat-interchanges are taking place whatever.

Considerable emphasis must be laid upon this point, because for many years much has been said and written about " delayed combustion " or " afterburning," and about dissociation as one of its causes. Since the basis for all such arguments has always been the form of the expansion-curve, it is important to note that the first thing proved by the form of that curve is that combustion is always complete by the time that maximum pressure is reached; or that if it be delayed it is only in those cases where the incipience of combustion has been delayed. In other words, explosive combustion in an Otto gas-engine, instead of being a dual or complex process, part of it occurring immediately after ignition and part of it delayed by some mysterious cause until late in the expansion-stroke, appears, when seen in the entropy-diagram, to be a simple continuous process occupying a fairly definite amount of time. If it be inaugurated well before the crank reaches dead-centre, it is usually complete by the time the piston has moved appreciably on its outward stroke. If it be not started until that stroke has begun, or if the mixture be a slow one, it may last continuously until the stroke is well toward completion. But there is no sign of its being interrupted, or of its being delayed or postponed by any other causes than those which always lead to the lapse of an appreciable period of time in explosive combustion.

It is next to be noticed that the portion of the cycle just under discussion, which is always convex toward the right, whether in the form of an apparently sharp point, as at f, or of a well-rounded curve, such as kl, is always followed by a process which is *concave* toward the right. The only exceptions to this rule which the writer has ever met are in

those cases where the exhaust-valve opens before combustion is, or could reasonably be expected to be, complete.

The significance of this change of curvature is marked. Convexity toward the right shows the rate of heat-supply becoming steadily less. Mathematically, it might also show rate of heat-abstraction steadily increasing; but consideration of conditions actually prevailing makes this proposition untenable.

Concavity toward the right, on the other hand, necessarily shows active development of heat. Where the concavity has not yet brought the expansion-curve again into the vertical adiabatic there may be some debate of this proposition. Where it has brought the curve into the southeasterly quadrant, however, — and this is nearly always the case, — there can be no question. The departure of the curve to the east shows the addition of heat.

It was this portion of the process which led to the theory of delayed combustion. Because the pressure-volume diagram is very meagre in its information, it was not observed at the same time that this addition of heat is separated from the original development of the fuel-heat of the gas in combustion by a distinct period when no production of heat took place. This intervening period being now known, what explanation of the situation is most probable?

Since the expansion-curve, whether of the form of *fgh* or of *klmn*, is strikingly similar to the expansion-curve of the simple, non-jacketed steam-engine,[1] and since in the latter case there is no possible source of heat except the cylinder-walls, it is only rational to assume that there takes place in the gas-engine cylinder a reversing transfer of heat between working-substance and walls which is in every way the parallel of the familiar condensation and reëvaporation of the steam-engine cylinder. Such a simple explanation

[1] See Fig. 41, page 219.

covers the demands of all the known facts. It eliminates the unscientific mystery of "delayed combustion." It may be assumed as demonstrated that nothing occurs within the gas-engine between the point of ignition and the opening of the exhaust-valve but the prompt, continuous combustion of the charge, its expansion due to the motion of the piston, and the varying action of the jacketed walls upon it in the ways naturally to be expected.

It is a widespread fault in the commercial and professional treatment of the gas-engine which has led to this doubt and confusion; and against it the student should be warned. I refer to the common practice of making repeated tests of gas-engines at great expense of time, labor, and money, without commensurate effort at an analysis of the results obtained. It is of little use to ascertain the facts unless we learn their causes. Such knowledge of causes cannot be had from the indicator-card direct. The compression and expansion curves of Fig. 49 are dumb as to all the thermal phenomena of which their translations into the curves of Fig. 48 become eloquent. Such translation is simple and fairly expeditious. It is not to be understood why it is not more frequently performed. Its omission is in line with the frequent elaborate reports of elaborate tests, with no mention made of the clearance of the cylinders or of the calorific power of the gas.

The Entropy-temperature Analysis. — In the translation of pressure-volume or indicator diagrams into entropy-temperature curves, the following argument is relied upon : —

Let Fig. 49 be one of the indicator-diagrams, supposedly an average one, of a set taken under constant conditions. Let Fig. 48 be an entropy-temperature coördinate field on which has been located only the initial point B.

In the location of this point B only the temperature need be considered. As to entropies, all that is usually desired

in the entire analysis is a knowledge of comparative altera-
tions, to know whether heat be entering or leaving the
working-substance.   But even where the quantities of heat
leaving or entering are desired, and where entropies are
therefore to be taken to exact scale, the zero-point may be
located horizontally arbitrarily.   As to temperature, how-
ever, there is considerable room for discussion.   It cannot
be known by observation, except at a cost which is nearly
always prohibitive.   Without it the absolute temperature
at any point in the cycle cannot be known.   But, on the
one hand, much can be learned from the entropy-analysis
with only knowledge as to *comparative* alterations of tem-
perature.   On the other, the temperature B cannot be out-
side of certain fairly definite limits.   It cannot be below
atmospheric temperature.   The engine cannot have raised
it above atmospheric temperature by more than a reason-
able amount.   Finally, since the temperatures after explo-
sion, as calculated from the assumed initial at B, will be in
error by four or five degrees for every one of error in the
assumption at B, too great error will be checked out by
improbability at either the top or the bottom of the cycle.
The writer usually assumes the temperature of B at the
convenient round number of 600° absolute, Fahrenheit.[1]

On the indicator-diagram first lay off the vertical axis
OP (Fig. 49) at the proper distance to the left to measure

[1] Professor Burstall's measurements show higher temperatures, at all por-
tions of the cycle, than would justify this assumption.   The mean of those made
for the Gas-engine Research Committee of the Brit. Inst. M. E., upwards of
250° F. during suction and 3600° F. during explosion, would point to an
assumed initial temperature properly as high as 700° absolute, instead of 600°.
But as these measurements were made with a very small engine, running below
speed and missing some five explosions out of six, the rigidity with which they
may be applied under more normal conditions must be questioned.   The
writer's figure, 600°, is undoubtedly as low as the truth.   (See Trans. Brit.
Inst. M. E., 1901.)

the clearance. Also draw the zero-axis of pressure, OV. Then erect any number of ordinates through any desired points whatever. It will save work to carry these ordinates completely across both curves. Then in order to locate upon Fig. 48 any desired point, such as x, of Fig. 49, lay off (in the imagination) from B, Fig. 48, a constant-pressure isomorphic By. This must represent geometrically the same process as By of Fig. 49. Whether any
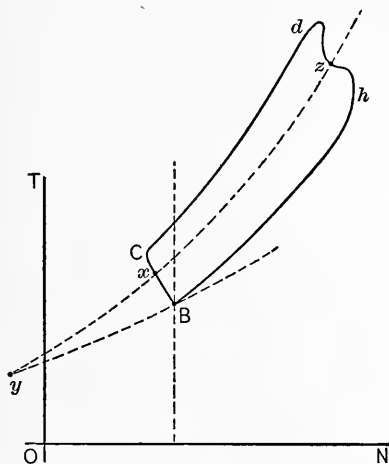
FIG. 48

such process actually took place or not does not affect the mathematical argument. From y, Fig. 48, lay off the constant-volume isomorphic yxz. This must correspond in the same way to the line yxz of Fig. 49, and upon it may be located both x and z.

The temperatures and entropies of the points y, x, and z may be had by calculation by remembering —

(1) That on the constant-pressure process By the temperatures must be proportionate to the volumes, which are measurable from OP;

(2) That on the constant-volume process yxz the temperatures must be proportionate to the absolute pressures, which are measurable from OV; and

(3) That in any case the equation $\dfrac{PV}{T} = \dfrac{P_1 V_1}{T_1}$ holds true.

Let the pressures and volumes at the points B, x, and z be measured in any convenient scale whatever; a decimal scale will be found the most convenient. Let the ratios $\dfrac{V_B}{V_x}, \dfrac{V_B}{V_{x'}}$, etc., the ratios $\dfrac{P_x}{P_B}, \dfrac{P_{x'}}{P_B}$, etc., and the ratios $\dfrac{P_z}{P_x}, \dfrac{P_{z'}}{P_x}$, etc., be found and tabulated. (By setting a slide-rule to, first, V_B, and then to P_B, and going over the entire diagram,
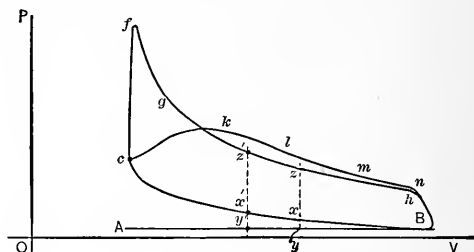
FIG. 49

first for volume-ratios and then for pressure-ratios, this can be done very expeditiously.)

Next, note for each desired point the logarithms of these ratios. If greater accuracy be desired, the logarithms of the original measurements might have been noted, the logarithms of P_B or V_B being subtracted from them; but this is seldom necessary.

From the equation noted in paragraph (3) above: —

$$\log \frac{T_x}{T_B} = \log \frac{P_x}{P_B} - \log \frac{V_B}{V_x}. \qquad \text{(See page 120.)}$$

From page 83,

$$N_x - N_y = S_v \log_e \frac{T_x}{T_y}, \quad \text{and} \quad N_B - N_y = S_p \log_e \frac{T_B}{T_y}.$$

Therefore,

$$N_x - N_B = S_v \log_e \frac{T_x}{T_y} - S_p \log_e \frac{T_B}{T_y}$$

$$= S_v\left(\log_e \frac{T_x}{T_y} - 1.408 \log_e \frac{T_B}{T_y}\right)$$

$$= 2.3026\ S_v\left(\log \frac{T_x}{T_y} - 1.408 \log \frac{T_B}{T_y}\right)$$

$$= 2.3026\ S_v\left(\log \frac{P_x}{P_B} - 1.408 \log \frac{V_B}{V_x}\right).$$

When the actual scale of heat-interchanges is desired, the value of the coefficient $2.3026\ S_v$ must be determined, the mass of the gas involved being carefully considered; or, more simply, the whole calculation may be considered as covering that number of working-strokes which will bring the value of $2.3026\ S_v$ times the mass of the gases passing through the engine in that period equal to unity. Usually, however, entropy-values to any scale will suffice, in which case the coefficient $2.3026\ S_v$ may be neglected.

In either of the latter two cases the calculation for any given point, such as x, reduces to this simple form:—

$$\frac{V_B}{V_x} = 1.490 \qquad\qquad \log \frac{V_B}{V_x} = 0.1732$$

Adding 0.4   1093

and then 0.008   22

                                       0.2847

$$\frac{P_x}{P_B} = 1.778 \qquad\qquad \log \frac{P_x}{P_B} = 0.2499$$

Subtracting, $N_x - N_B = -0.0348$

$$\frac{P_z}{P_x} = 3.602 \qquad\qquad \log \frac{P_z}{P_x} = 0.5565$$

Adding $\log \dfrac{P_z}{P_x}$ to $N_x - N_B$ above:—

$$N_z - N_B = 0.5217$$

Subtracting $\log \dfrac{V_B}{V_x}$ from $\log \dfrac{P_x}{P_B}$, $\log \dfrac{T_x}{T_B} = 0.0767$

$$\log T_B = \log 600° = 2.7781$$

Adding, $\log T_x = \overline{2.8548}$

$$T_x = \ \ 715.8$$

$$t_x = 255° \ \text{F.}$$

Adding $\log \dfrac{P_z}{P_x}$ to $\log T_x$: $\log T_z = 3.4113$

$$T_z = \ \ 2578$$

$$t_z = 2117° \ \text{F.}$$

This gives the entropies relatively to B and the absolute
temperatures of any desired pair of points. If entropies
be desired to true scale, they may now be multiplied by
$2.3026\ S_v$ times the mass of gas passing through the
engine per stroke or per unit of time.

The entropy-diagram of the actual cycle thus obtained
may then be analyzed in a manner very similar to that
given for the steam-engine. In such analysis, as in every
portion of the test of which it forms a part, the threefold
object of the work should be kept clearly in mind, viz.: —

- (1) The determination of the division of the heat sup-
     plied in the fuel between the three destinations: —

  - (a) Absorption by the jacket-water;
  - (b) Conversion into work;
  - (c) Rejection in the exhaust;

- (2) The comparison of these results with the theoretic
     possibilities;

- (3) A clear understanding as to the causes of the results
     observed.

The observations which must be taken in order to com-
plete this programme are: —

(1) For determination of the heat supplied:

    (*a*) A gas-meter, protected by a gas-bag and properly calibrated;

    (*b*) A gas-calorimeter, preferably a Junkers;

    (*c*) An admission-counter, operated by the admission-valve.[1]

(2) For determination of the conversion of heat into work:

    (*a*) The indicator, equipped with two springs for use as noted below;

    (*b*) A Prony brake or absorption-dynamometer;

    (*c*) A revolution-counter;

    (*d*) An explosion-counter.

(3) For determination of the heat lost to the jacket-water:

    (*a*) The rate of flow, which should be kept constant during and for some time preceding the test;

    (*b*) The temperature at entrance to the jacket;

    (*c*) The temperature at exit from the jacket.

(4) General data:

    (*a*) The barometric pressure;

    (*b*) The temperature of the air entering the suction-pipe;

    (*c*) The temperature of the exhaust-gases (to be attempted only when the engine is running at maximum power, missing no possible explosions);

    (*d*) Chemical analysis of the exhaust-gases, including observation of the presence of uncombined carbon.

    (*e*) The dimensions of the engine, including the volume of the clearance.

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[1] Where the igniter is suspected of uncertainty of operation this instrument should be supplemented by an explosion-counter.

As to the taking of indicator-cards, two sets should be secured. One should be with the standard gas-engine indicator, with a spring heavy enough to keep the vibratory waves within proper limit. This will be used as already noted. The other set should be taken with a standard steam-engine indicator, with a spring so light that the compression of the charge brings the indicator-piston to, or quite near to, the top of its stroke. Such diagrams show the phenomena occurring during the exhaust, suction and compression strokes upon an exaggerated scale, with much greater accuracy than the former ones. Those, indeed, should be relied upon for the explosion and expansion curves only.

The diagrams from the light spring will also reveal the negative work involved in overcoming the fluid-friction of admission and exhaust. This should be carefully distinguished from the metallic friction of the engine, in making the report and drawing conclusions. It is usually high in gas-engines, apparently more so than is necessary.

If the spring used for the second set be too strong to let the compression bring the indicator-piston nearly against its upper stop before the explosion, the latter, when it occurs, will wreck the pencil-motion.

As to the explosion-counter, that is most easily rigged by using a second indicator which is always kept open to the engine-cylinder and the drum of which is given a slow continuous or ratchet motion by the engine just sufficient to cover a millimeter or so for each cycle. Its record will consist of a number of saw-teeth, high ones recording explosions and low ones signifying missed explosions. Such a record will check out any failure to ignite charges of gas which the admission-counter has recorded as entering the engine.

If a second indicator be not available, the reliability of the igniter may be tested in this way before and after the regular test.

The gas-engine test made in this way should result in a report recording : —

(1) The dimensions of the engine and the conditions of the test ;

(2) The observations themselves, reduced to averages and totals ;

(3) The calculated distribution of the heat to its four destinations, in percentages of the whole ;

(4) The four calculated efficiencies, viz. : —

    (a) The theoretic or ideal efficiency, calculated from the volume-ratio of compression ;

    (b) The thermodynamic efficiency, which is the same as the second item of paragraph (3), this page ;

    (c) The cylinder-efficiency, or (b) divided by (a) ;

    (d) The mechanical efficiency ;

(5) Average samples of cycle-diagrams, both from the indicator and in entropy-temperature coördinates ;

(6) The conclusions to be drawn from the foregoing.

(By the time this book has left the press it is probable that the committee of the Am. Soc. M. E. which was appointed to adopt standard methods of making and recording steam-engine and gas-engine tests will have made its final report. This report will be available to most students and should be consulted before undertaking a test.)

Degree of Compression

In the study of the attainment of maximum actual efficiency in the Otto gas-engine it would seem at first that the problem were a very simple one. The actual efficiency, though less than the theoretic, is very closely proportional to it. The latter is given by the expression

$$E = \frac{T_2 - T_1}{T_2} = 1 - \left(\frac{V_2}{V_1}\right)^{0.402}$$

wherein the subscripts 1 and 2 refer to the conditions at the beginning and end of the compression-stroke respectively. It is obviously easy, in the design of the engine, to make this expression take any value we choose, by variation in the proportion of clearance-volume.

In the development of the engine during the quarter-century which has elapsed since its first appearance, this policy of increasing the efficiency by decreasing the clearance has been steadily pursued. It has constituted the main line of progress up to about 1895. Since these thermodynamic principles were well known from the start, it is difficult to see at first why advantage of them was taken so slowly. The explanation is that the mechanical difficulties of construction multiply very rapidly as the degree of compression increases. The maximum pressure incurred at the moment of explosion rises very rapidly with a rise in initial compression. With it must rise the entire cost of construction of the engine; all parts must be strengthened and weighted and bearing-surfaces must be increased. Indeed, so close has constructive practice followed mechanical limitations in this matter that nothing has permitted the later advances into the higher degrees of compression but the steady rise in rotative and piston speeds which has also been a prime characteristic of progress, in gas-engines as in steam-engines. At the higher speeds the force needed to accelerate the reciprocating parts subtracts itself from the fluid-pressure on the piston and protects the bearing-surfaces from momentary over-pressure. It is to be noticed at the same time, however, that such inertia-forces fall upon the bearing-surfaces unmodified by fluid-forces during the suction and exhaust strokes; they may not be increased recklessly, therefore.

Another limit to the feasible degree of compression is found in the danger of pre-igniting the charge by the heat

of compression. This limit appears earlier in oil-engines than in gas-engines. It is a phenomenon which may be taken advantage of, however. All engines relying upon hot-tube ignition without the aid of timing-valves depend upon it, more or less unconsciously, for ignition. The Diesel engine relies upon it absolutely and completely; adopting a maximum pressure of compression of from 500 to 550 pounds per square inch, it insures at dead-centre an atmosphere within the cylinder all portions of which are above the temperature of ignition. Ignition is timed in this case by the preservation of the fuel separate from the air until the beginning of the working-stroke. It is then fed into the hot compressed air under control and ignites because of the high temperature of the air. The most valuable result yet derived from the Diesel engine is the demonstration that in such an atmosphere as this any sort of liquid fuel, whether light or heavy, crude or pure, may be injected without preliminary vaporization and the resultant combustion will be complete.[1] Referring again to the analysis of the theory of the Diesel cycle, which was presented on page 149, it is to be noted that its high efficiency is due primarily to its high degree of compression rather than to any peculiarities of form of cycle, method of ignition, "isothermal combustion," etc.

Complete Expansion. — The next line of effort at improvement of efficiency, taken in the chronological order of their appearance, was the avoidance of incomplete expansion and incomplete exhaust of the burned gases. These features, it will be remembered, are inseparable from an Otto-cycle engine in which all of the processes are enacted in a single cylinder. They are now accepted as such, but only after much time and money and some of the best skill in

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[1] "Mittheilungen über den Dieselschen, Wärmemotor," by Rudolf Diesel. *Zeitschrift d. Vereines Deutsche Ingenieure*, Vol. XXXXIII.

the land has been expended in proving the impracticability of their avoidance. Chief amongst the workers in this line was Mr. Atkinson, who made two full attempts at designing an engine which should completely expand and exhaust its burnt gases. In the first a horizontal cylinder open at both ends contained two trunk-pistons placed head to head. Each of these was connected to the engine-crank by a peculiar arrangement of links and walking-beams whereby each piston had a peculiar and independent motion of its own. Considering first the *relative* motion between the two pistons, it may be described as producing the following sequence of events : —

(1) The close juxtaposition of the two, which constituted the beginning of the suction-stroke ;

(2) Their separation by a certain distance, whereby the volume between them was filled with an explosive charge at atmospheric pressure ;

(3) Their approach by a lesser distance, whereby compression was performed, at the end of which process ignition took place ;

(4) Their separation by a distance greater than (2), whereby expansion took place much more completely than in the ordinary engine ;

(5) Their close approach, into position (1), whereby complete exhaust was effected.

This engine was known as Atkinson's "Differential" engine. Not proving a mechanical success, it was followed by his so-called "Cycle" engine, in which the same series of changes of volume was produced in a cylinder closed at one end and containing a single trunk-piston, by a peculiar radial gear between piston and crank. Both engines gave good thermodynamic results. Both were complete failures mechanically. The work was done so

well, however, that the failures are now regarded as inevitable in that line of effort.

Scavenging. — Although the completion of expansion in the Otto-type engine still remains an unsolved problem, the completion of exhaust has been accomplished in a simpler way. For a time a number of designers advocated the insertion between each two-revolution cycle of one complete idle revolution, the end in view being the pumping of a fresh charge of air through the engine, whereby the bulk of the burnt gases remaining in the clearance-volume should be swept out. But the gain was not found to be worth the mechanical and commercial disadvantages incurred, and the plan is now no longer followed. Later Mr. Atkinson evolved the beautifully simple plan of producing a partial vacuum in the cylinder during the exhaust-stroke by making the exhaust-pipe some sixty feet long or more, and quite straight and free. The inertia of this long column of gases, when once set into rapid motion by the first sharp puff of the exhaust, would draw into and through the cylinder quite a draft of pure air, washing out the bulk of the burnt gases in the clearance. The thermodynamic effect of this plan was excellent. His early experiments with it reported a saving of some 5%, a quite profitable one considering the ease with which it is attained.

Scavenging, in some form or other, is now considered an essential feature of all large explosive engines attempting refined efficiency. In the latest types of heavy engines for use with blast-furnace gases it is performed by a small separate blowing-cylinder.[1]

Regulation. — Of all the problems connected with the design of the Otto-type of gas-engine, however, none has

[1] See Humphrey's paper on "Recent Progress in Large Gas-engines," read before the British Association at Belfast, Sept. 11, 1902. Also abstracted in London *Engineering*, Sept. 19 *et seq.*

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a more urgent bearing upon the thermodynamics of the efficiency-problem than that of regulation. It will be remembered that all of the early designs operated upon what is called, most properly, the "hit-or-miss" plan of regulation. That is, so long as the engine were below normal speed there was no hindrance to, nor modification of, the development of full maximum power. When the speed rose above normal, all development of power was shut off completely by the omission of gas-supply on one or more of the suction-strokes. There was no gradation between these two extremes.

Many of the smaller engines, and a few of the larger ones, still adhere to this plan. When it is remembered that the single-cylinder gas-engine has three idle strokes to one working stroke, at the best, and that the dropping out of one working stroke leaves seven idle strokes before another impulse can be received ; when it is further remembered that this seven-to-one ratio applies to a reduction of power to only half the maximum, or about seven-tenths the proper rating, and that for further reductions the proportion of idle strokes to impulses may be increased to eleven, fifteen, or nineteen to one; when it is finally remembered that the power needed for compressing the next charge to be fired must be taken out of the fly-wheels at just the moment when they are the weakest and *after they have already fallen below normal speed ;* when all of these points are compared with modern demands and attainments in steam-engine regulation, the hit-or-miss plan of regulating gas-engines must appear as a crudity only comparable with steam-engine practice of a century ago.

Nor can any remedy possibly be hoped for in the governor. The above remarks would apply with complete force were the governor ideally perfect. It is true that in many small gas-engines the governor is abnormally and

inexcusably bad, and that the speed-variations are thereby much exaggerated over what is inevitable. In fact, no gas-engine operating upon this plan is fitted with a good governor. The governor is always one of two types, both of which are intrinsically unsuited to the work, viz.: (1) A very small and weak centrifugal governor, dimensioned so as to have great stability; (2) an inertia-governor, usually of the pendulum-type.

The latter is unsuited because its action depends upon acceleration; it can do nothing if not to keep the engine's acceleration, positive or negative, within certain limits. But the Otto-type of gas-engine necessarily involves a degree of acceleration within each cycle, positive during the expansion-stroke and negative during the compression-stroke, which far exceeds any degree of general acceleration in average speed which is permissible in good regulation.

The former type is unsuited because the range of speed-variation which is involved in stability and which constitutes a valuable factor for control in steam-engine governors is here entirely unavailing. The governor can do nothing but one extreme thing or the other, either to admit maximum power or cut off all of it, and a field of gradation between is powerless to accomplish gradation. Moreover, owing to these extremes in distribution of power, positive acceleration will be prompt and powerful; in addition, from one to four strokes must elapse before action on the part of the governor can become effective. The need is, therefore, for a governor which will signalize departure from normal speed with the utmost promptness and emphasis. Such is the perfectly isochronous pendulum governor, a type entirely unfit for steam-engine regulation. It is the only one fit for the hit-or-miss plan of controlling gas-engine speeds. It has never, so far as the writer's knowledge extends, been applied.

It is no object of the present text to properly discuss questions of regulation other than as they affect thermodynamic problems. But the latter cannot be correctly entered upon without previous acceptance of the premises that the hit-or-miss plan of governing is intolerable, according to all steam-engine standards of excellence, and must inevitably remain so. This being true, what other plans are available? Two only, viz.: —

(1) Reduction of amount of gas present at the moment of ignition.
(2) Reduction of the pressure prevailing at the moment of ignition, by the reduction of either the volume (by cut-off) or the pressure (by throttling) of the charge drawn in during suction.

Either (1) or (2) permits the gradual reduction of the vigor of each impulse, without diminishing their number, until only a small fraction of normal power is developed. Either plan inevitably involves some loss of efficiency from the maximum. In the case of the first plan this is due to the inevitably slower combustion in weak mixtures, and is so bad as to be intolerable. Reference to the theory of the Otto cycle will confirm the statement as to the second plan also. In the expression for its theoretic efficiency, $\frac{T_2 - T_1}{T_2}$, reduction of the degree of compression reduces the maximum efficiency attainable. With the second plan it is only a partial, but a valuable, compensation that decrease of the degree of compression (in a fixed clearance-volume) increases the ratio of expansion into that territory which was stated, on page 148, Part I, as belonging exclusively to the Lenoir cycle.

The objections to both plans can be modified by skilful design and particularly by a wise combination of the two.

Practically all of the larger and more important engines have accepted the premises stated above as to the hit-or-miss plan and rely upon graded impulses for regulation.

It is further to be remarked that variation of the power developed, with or without variation of the degree of compression prevailing at the moment of ignition, is obtainable by any one of three methods, viz.: —

(1) Gradation of *pressure* of charge during suction, or by *throttling*.

(2) Gradation of *volume* of atmospheric charge during suction, or by *variable cut-off*.

(3) Gradation of *final volume of compression*, or by *variable clearance-volume*.

The first two are very nearly alike; their difference is visible in Fig. 56. The second has the advantage over the first by a narrow margin of efficiency and the disadvantage of greater mechanical complexity. The third, so far as the writer is acquainted, has never been attempted.

To illustrate these points is published herewith a set of indicator-cards, with entropy-analyses thereof, from an 11″ × 12″ two-cylinder gas-engine, running at 290 revolutions per minute. The conditions under which the cards were taken were in every way normal and were graded from an overload of 17.4% above rating down to the friction of the engine alone. The fuel-consumption near rated load varies between 10 and 10.6 cubic feet of natural gas per brake horse-power, the gas having an average calorific power of 1000 B.t.u. per cubic foot. This represents a thermodynamic efficiency of from 24 to 25.5%. The variation of the fuel-rate with the load is remarkably constant. Calling the efficiency at rated load 100, the indicator-cards show it as rising to about 112 under the overload and falling to about 75 at half-load.

This is accomplished, so far as the cards reveal the method, by relying upon more or less throttling of the incoming charge under all conditions of load, even the maximum, and by thereby securing an expansion to a volume greater than the initial when the latter is measured at atmospheric pressure. The percentage-fraction of the stroke at which the charge is at atmospheric pressure is shown by the figures beneath the atmospheric line. It will be seen that only 71% of a cylinderful of fresh charge is taken in even under maximum load. At normal
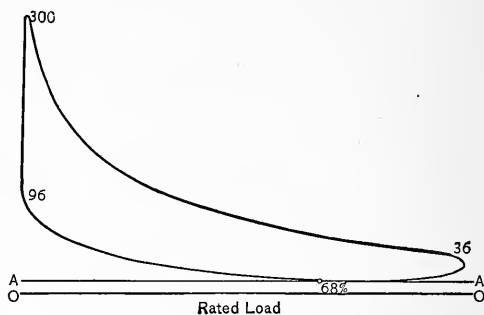
Fig. 50

load it is only 68%. At the same time, the clearance is made less than usual (only about 21.5%) so that the proper degree of compression is retained. The reduced pressure during suction, due to the throttling by the governor, and the work lost thereby, could be shown (without confusion of lines) only in Fig. 56, and there only approximately.

While the governor is reducing the power by throttling the charge, it is also reducing the proportion of gas present, making the inflammation slower, which is quite permissible in view of the excessive opportunity for expansion present during the lighter loads.

As these diagrams are typical representatives of the best

practice of the day with large engines, it is proper to use
them as a text for a discussion of the possibilities for further
progress, as revealed by thermodynamic considerations.

In the first place, the figures given above for the varia-
tion of efficiency in terms of variation of load are mislead-
ing, in that the efficiency at rated load was taken as the
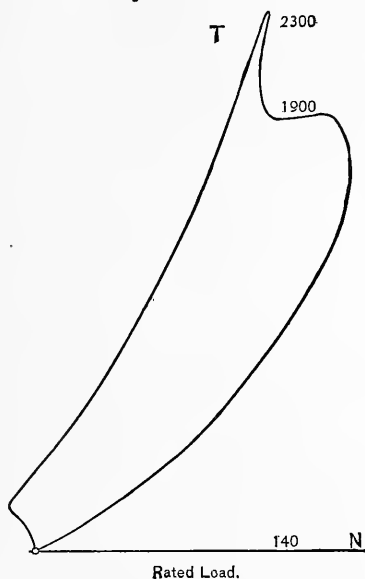
Rated Load.
FIG. 51

standard. It is one of the marked distinctions between
steam-engine and gas-engine practice that, whereas in
steam-engines (except in the almost obsolete throttle-
governed type) the maximum efficiency is found at or
slightly below rated load, decreasing as the load either
increases above or falls to a fraction of the rating, in gas-
engines the efficiency is always the greatest when the load
is the greatest. This fact led to the very unfortunate
practice, which is only now just dying out, of rating gas-

engines so high that they could seldom be driven appreciably above their rating, and often not fully up to it. Even the much more liberal practice shown in the accompanying diagrams reveals a maximum load only 18% above the rating; and, while the writer does not know that this is the extreme maximum capacity of the engine, it is plain, from the degree of compression incurred at this power, that it could not possibly rise much higher. This margin above rating has been obtained, too, by artificially adopting,
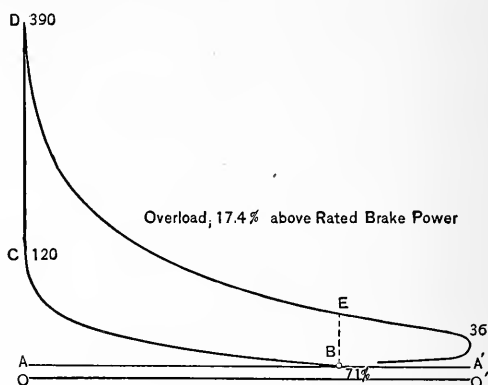
FIG. 52

as the standard efficiency of the engine, an efficiency which is only some 78% of what the engine plainly shows that it can do when given a chance. Yet, after all, the margin of power above the rating, 18%, which is purchased at the price of the loss of 22% of the efficiency, is very small when compared with standard steam-engine practice, and plainly reveals one of the chief reasons why the gas-engine fails to meet more universally the demands of the industrial world for a prime mover. It does not possess sufficient margin of power. (See curve W_B, Fig. 58.)

Does theory offer any promise of remedy for this situa-

tion? It does, very plainly. The remedy may not be easily attainable in constructive practice, but it is plainly there. In the writer's opinion, moreover, the reduction of it to practice does not involve any obstacles impossible of conquest.

Overload; 17.4 % above Rated Brake Power
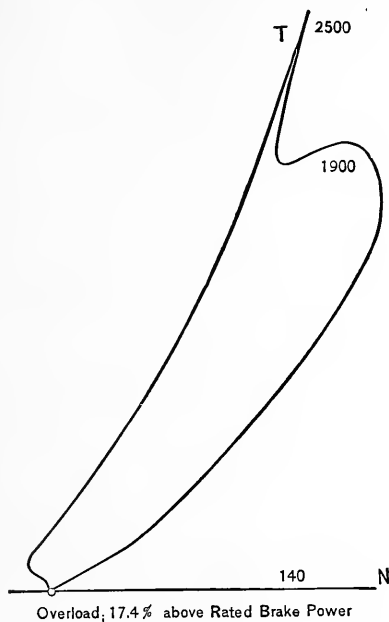
FIG. 53

The practice, as well as the theory, of the governing of Otto-cycle engines by throttling or cutting off the incoming charge shows that the maximum efficiency is obtained when the least throttling is done and the cycle approaches most nearly to the unmodified Beau-de-Rochas cycle. The departure from the latter is undertaken, very plainly, for the sake of margin of power and for better regulation, which are purely mechanical questions, and quite in the face of all considerations of efficiency.

The efficiency of the Beau-de-Rochas cycle is given by the expression $\dfrac{T_2 - T_1}{T_2}$, wherein the T's refer to temperatures before and after compression. Therefore, very plainly, the degree of compression should always be kept constant, at the maximum point permissible from mechanical considerations of bearing-surfaces and strength of parts or from danger of pre-ignition. At the same time, since any reduction of the proportion of gas in the fresh charge leads to a slower rate of combustion, which is fatal to efficiency, that should be kept constant too. Since any reduction of the pressure of the fresh
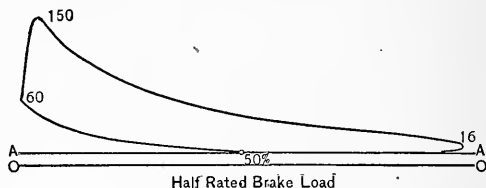
FIG. 54

charge below atmospheric leads to loss of power in recompression, not to mention the much greater loss due to the diminished pressure at the instant of ignition which has been discussed above, that, too, should be avoided; yet it is the least objectionable in the list.

If all of these rules are to be observed, while the power developed and the gas taken in are reduced to suit a diminished resistance, only the following policy is left open to the designer : —

(1) To reduce the volume of the atmospheric charge of standard mixture taken in ;

(2) To reduce the clearance-volume in like proportion, in order to maintain a constant degree of compression.

The first cannot be accomplished, in the standard Otto engine, except by recourse to an automatic cut-off. As the throttling of the charge is a method only slightly inferior to this thermodynamically, and superior to it mechanically, this first point will be disposed of by leaving these alternatives open to the designer without further discussion.

2200

140 N

Half Rated Brake Load
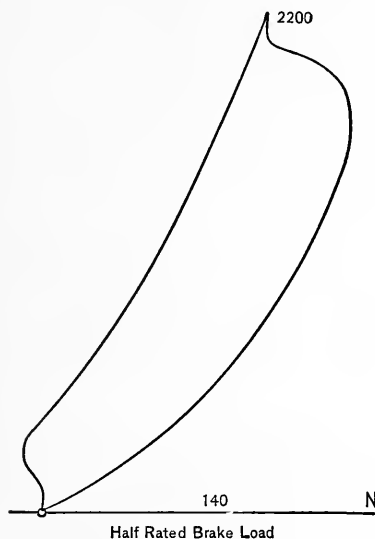
FIG. 55

(It should be said, however, that the friction in getting the gases into and out of a gas-engine cylinder usually constitutes as much as two-thirds of its entire frictional loss, and this is much exaggerated where throttling is substituted for automatic cut-off. The entire question of engine-friction is one of the most unsatisfactory ones connected with gas-engine design. In the present case the mechanical efficiency is between 75 and 80%, which is certainly much below what engineering skill ought to be able to com-

mand. The reasons for the present faulty situation are
two: (1) Combustion within the cylinder, demanding small
poppet-valves for admission and exhaust; (2) the double
transmission of the energy required for compression, first
from the piston through all the oblique connections and
bearings to the fly-wheel and afterward back again.
Neither of these may be remedied without radical depar-
ture from existing arrangement of parts.)

As to the second rule essential to efficiency, the varia-
tion of clearance-volume, as this is not a book upon con-
structive mechanics, the question as to how variation of the
clearance-volume is to be accomplished will not be opened.

No Brake Load: Engine-Friction Only.
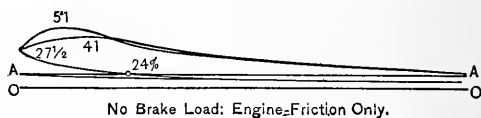
FIG. 56

It is quite proper, however, to inquire as to what would be
the thermodynamic results were such a construction opera-
tively accomplished.

For this purpose let Fig. 52 be taken as representing the
best of modern practice in the handling of gas-engine fluids
in an engine-cylinder. Let the new engine be arranged
to develop, under maximum load, the facsimile of Fig. 52
chopped off at BE, or the indicator-diagram $BCDE$. This
would call for a clearance of about 30%. But this 30%
must be supposed to be the maximum limit of a clearance
which is variable by the governor from 30% down to the
minimum practicable.

If, now, for all lesser loads than maximum the initial
volume of the charge AB be diminished in somewhat equal
proportion, and the clearance-volume be diminished in exact

proportion with *AB*, the indicator-cards developed would be the exact likeness of *BCDE*, Fig. 52, upon a diminished longitudinal scale, and would develop a like efficiency; *except* that now the terminal pressure *EB* has opportunity for further expansion during the remainder of the stroke, to the right of the new positions of *E*. This added expansion is not obtained, as is the case in all existing engines, at the expense of diminished efficiency of the portion *BCDE*, due to diminished compression. It is all pure gain, and serves

No Brake Load. Engine Friction Only
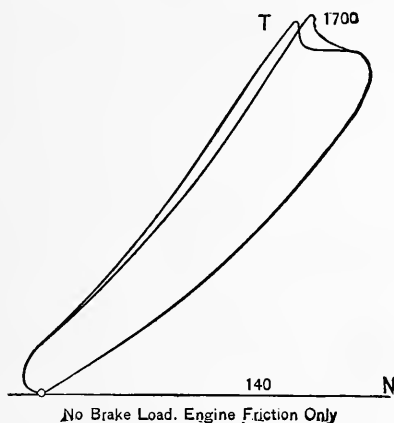
FIG. 57

to make the efficiency an increasing instead of a decreasing function of the load as the latter falls from maximum.

The movement of *B* toward *A* under the influence of the governor as the load diminishes would result in the variation of efficiency with load according to the curves of Fig. 58. In Fig. 58 the actual diagram *BCDE* of Fig. 52 is taken in each case as a standard for comparison of 100. The curve W_B shows the approximate relation of the brake-efficiency of the existing gas-engine under discussion to its brake-load. The curve N_B shows the same relation for

the proposed new engine with variable clearance-volume
controlled by the governor.   The curve N_I shows the rela-
tion of indicated efficiency to indicated power for the new
engine.

In considering these curves, it must be remembered that
the curve N_B is a theoretic one only in the supposition of
altered dimensions of clearance-space.   All the inevitable
departures of actual results from theoretic possibilities are
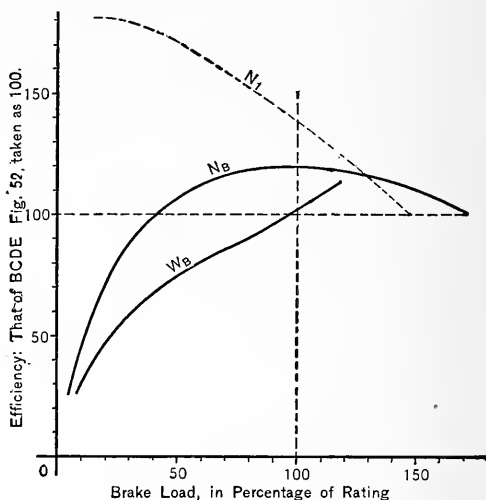
FIG. 58

included in the premises.   It is also to be noted that the
curve W_B represents, not poor engineering practice which
needs rebuke, but what is possibly the best, and surely
near to the best, of modern practice.   The curve N_B shows
in comparison the undeveloped possibilities of the Otto
engine, so far as the writer yet perceives them.

In comparing the curves W_B and N_B, the point of prob-
ably greater importance than the generally higher average
efficiency of the latter is the wide range of its capacity.

The rating as chosen shows a surplus margin of capacity of 70%. It is probable that the average builder would prefer to rate such an engine at about 120 (on the arbitrary scale adopted), reducing its margin of surplus of maximum above rated power to some 41%, but at the same time reducing the lower limit of fractional load which might be imposed without perceptible detriment to the efficiency to 50%. The limit to reduction of power without passing below what is now first-class gas-engine efficiency at rated load would be thereby reduced to 33%. In other words, an engine consisting of a pair of $13'' \times 14''$ single-acting cylinders running at 275 r.p.m., if equipped with governor-controlled variable clearance, would rate at 60 horse-power and would develop that power at a fuel-rate of $8\frac{1}{4}$ to 9 cubic feet of natural gas per brake horse-power. It would develop a maximum brake horse-power of 85 without losing more than 17% of its best efficiency, and could develop as little as 30 horse-power without falling below an efficiency now attained only at the maximum capacity of the actual engine adopted as a basis for comparison. It could develop as little as 20 horse-power without falling below the fuel-rate which it demanded at its maximum capacity, and which is the same as that of the best existing engines under rated load.

The curve N_B is of the same general form as that which has always been revealed by the standard steam-engine. It expresses characteristics which are essential to the broad success of any prime-mover. Their incorporation into the explosive gas-engine would constitute a long stride forward toward that mechanical parallelism with the steam-engine which alone can mark the completion of its growth.

Before leaving the set of diagrams which have formed the text of the foregoing discussion, attention should be

given to the entropy-diagrams, Figs. 51, 53, 55, and 57. There lies in them much food for instructive thought. The only point to which particular notice is to be called at present is the persistent concavity (toward the right) of the expansion-lines, in spite of the greatest variety of form, from sharp peak to flatly rounded top, on the part of the pressure-volume curve. This, and the approach of the later stages of expansion to isothermal conditions, hold true of practically every card the writer has ever analyzed. They leave no room for doubt as to the entire completion of combustion almost immediately after ignition and the sole activity of cylinder-wall influence in the deformation of the expansion-line from a true adiabatic. They also prove beyond question that the indicator-card alone, untranslated into the entropy-temperature diagram, throws about as brilliant a light upon the real nature of the processes transpiring within the cylinder as would a German work upon thermodynamics to one who knew no German.

The isothermal character of the expansion in its later stages would go to show that the gain in efficiency due to prolonging it by means of variable clearance might easily be greater than that calculated above.

APPENDIX

APPENDIX

TABLE OF SPECIFIC AND LATENT HEATS OF VARIOUS SUBSTANCES

SPECIFIC HEATS

SOLIDS

Aluminum .	. 0.21	Iron, at 32°	. 0.110	Silver .	.	. 0.057
Brass .	. 0.092	Iron, at 600°	. 0.130	Clay .	.	. 0.055
Copper, at 32°	. 0.090	Tin .	. 0.055	Stone .	.	. 0.20
Copper, at 600° .	0.099	Zinc .	. 0.095	Glass .	.	. 0.18
Mercury .	. 0.033	Lead ..	. 0.030	Ice .	.	. 0.50

LIQUIDS

Anhydrous ammonia	. 1.23	Alcohol . . .	. . 0.6
Bisulphide of carbon	. 0.24	Oils . . .	. 0.4 to 0.5
Ether . .	. 0.53		

GASES

	Under Const. Press.	Under Const. Vol.	Ratio	Density
Atmospheric air .	0.237	0.169	1.4024	1.000
Ammonia (anhydrous) .	0.52	0.41	1.27	0.59
Bisulphide of carbon .	0.16	——	——	1.266
Carbonic acid .	0.217	0.167	1.30	1.520
Carbonic oxide .	0.242	0.173	1.40	0.967
Ether . . .	0.47	0.45	1.04	2.557
Hydrogen, at 0° .	3.40	2.41	1.41	0.069
Hydrogen, at 100° .	3.41	2.42	1.41	——
Methane . .	0.593	0.450	1.316	0.555
Nitrogen . .	0.244	0.173	1.41	0.971
Oxygen . . .	0.217	0.155	1.40	1.106
Superheated steam .	0.6	0.46	1.3	0.622
Alcohol . . .	0.453	0.398	1.14	1.589

LATENT HEATS

Of the Melting of Solids

	Temperature of the Melting Point	B.t.u.
Cast-iron, white	60	——
Cast-iron, gray	42	——
Lead	10	617°
Mercury	5	−40°
Silver	38	1830°
Tin	25	414°
Zinc	50	780°
Ice	142	32°
Sea-ice	98	16°

Of the Vaporization of Liquids

Ammonia (anhydrous)	554–578
Bisulphide of carbon	150–180
Carbonic acid	110
Ether	162

For the above values the author is chiefly indebted to the authorities cited in Landolt & Börnstein. He has also drawn upon the table in Pupin's "Thermodynamics."

THE VARIABILITY OF THE SPECIFIC HEATS

In the preceding work it was invariably assumed, except in the case of water, that the specific heats of all substances were constant for all temperatures. In the thermodynamics of superheated steam and of the gas- and oil-engines the truth or falsity of this assumption becomes important. It would not, however, affect the graphical methods or the general conclusions reached in the discussions presented, and for this reason its brief treatment has been reserved to this point, as a refinement which might be added by those so inclined.

Several scientific investigators, including Regnault, have maintained the constancy of the specific heats under varying pressure and temperature. Nevertheless, the weight of evidence is overwhelmingly to the opposite. But the variations appear to be peculiar to each gas investigated, and no general laws have been established. It generally appears that the specific heats increase with both pressure and temperature, at least for all gases of interest to the engineer. The maximum is reached at a pressure of from 350 to 400 atmospheres; with temperatures it seems to rise indefinitely. A few experimental results are given, to indicate the state of the science.

The Permanent Gases. — As these become of interest chiefly in connection with gas-engine problems, the reader is referred to the able work done by Professor Burstall for the Gas-engine Research Committee of the Brit. Inst. M. E. Professor Burstall quotes the results given by Mallard and Le Chatelier for S_v as : —

Oxygen, . . $0.1488 + 0.0000763\ t$;

Nitrogen, . . $0.170\ \ + 0.0000872\ t$;

Carbonic acid, . $0.1477 + 0.000176\ t$;

Superheated steam, $0.321\ \ + 0.000219\ t$;

wherein t is temperature Fahrenheit.

Professor Burstall adds the conclusions which he reached as to the variability of the specific heat of gas-engine mixtures in terms of the richness of mixture. The writer has taken the liberty of presenting them, for convenience, in the form of Fig. 59. It gives the values (on the vertical scale) of A and B in the formula —

$$S_p = 0.2400 + \frac{A}{10,000} + \frac{B}{1,000,000},$$

in terms of the proportion of air to gas in the mixture by volume. (See also Batelli, in *Rapports Présentés au Congrès International de Physique*, 1900, and to the authorities referred to therein.)

Superheated Steam. — Aside from the value of Mallard and Le Chatelier given above only one set of results is given for super-
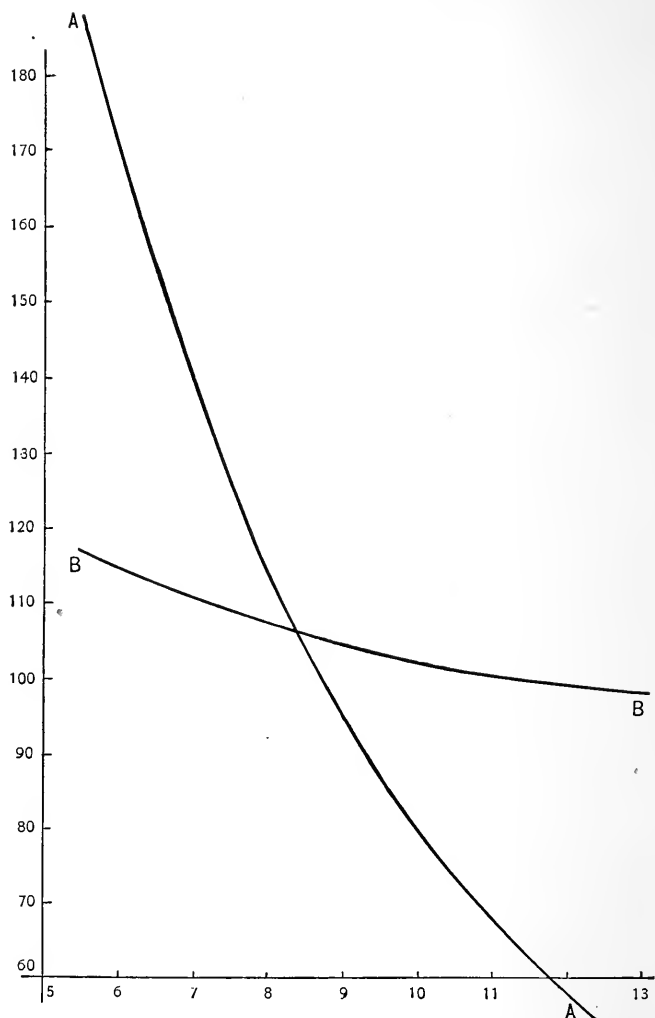
FIG. 59

PROPORTION OF AIR TO GAS, BY VOLUME.

heated steam, that reported by Grindley (*Phil. Trans.*, Vol. 194). These experiments are very carefully made ; they are quite recent ; they should be significant. The investigator reports, as his general conclusions, that the specific heat under constant pressure is independent of the pressure, but varies with the temperature. His conclusions, and also his tabulation of his calculations from Hirn's experiments, are appended.

VALUES OF THE MEAN SPECIFIC HEAT, UNDER CONSTANT PRESSURE, OF SUPERHEATED STEAM AT ATMOSPHERIC TEMPERATURE

Hirn

Temperatures between which the Specific Heat is taken	Mean Specific Heat
260.0 to 263.1	0.3048
263.1 " 271.4	0.6742
271.4 " 279.9	0.5362
279.9 " 287.1	0.5428
287.1 " 291.4	0.7874
291.4 " 296.6	0.5839
296.6 " 301.2	0.5977
301.2 " 306.5	0.9258
306.5 " 312.0	0.8028
312.0 " 314.1	0.9577
314.1 " 316.0	0.9309

Grindley

Temperatures between which the Specific Heat is taken	Mean Specific Heat
230.7 to 246.5	0.4317
246.5 " 260.8	0.4778
260.8 " 269.7	0.5152
269.7 " 295.0	0.5646
295.0 " 311.5	0.6482

Because the above results do not seem to be altogether consistent, the writer has taken the liberty of recalculating the values which seem to him to be pointed to by the data which Grindley records as the result of his investigation, and without reducing them to atmospheric pressure. These values are displayed in Fig. 60, which shows that portion of the entropy-temperature, or, to give it its proper name, the thermal, diagram covering the field investigated. SS is the saturation-curve. $PPPP$ are constant-pressure lines, their level being noted where they cross SS. $QQQQ$ are the constant-heat curves upon which Grindley conducted his inquiry. The temperature-scale is in degrees Fahrenheit.

Had Grindley chosen a fixed set of lower pressures to which to wire-draw the steam, the field would have been checkered by these P and Q lines, between each pair of intersections of which would have been known the specific heat with great accuracy, averaged over a comparatively slight distance, viz., that between the lines. As it is, the results given are located in no regular distribution over the field. They are most of them, too, not only averages over quite extended temperature-ranges, but over ranges which lap one over the other. Nevertheless, enough is shown definitely to prove conclusively : —

(1) That the specific heat is not independent of pressure, but varies directly with it.

(2) That it varies directly with the temperature.

(3) That it varies inversely with the entropy. Lines of constant specific heat cannot be located definitely ; but it is clear that they would have a generally northeasterly trend.

Whether law (3) above, taken with the hypothesis that entropy is a close function of molecular mass, has any connection with the relation between specific heat and molecular weight is a question for the physicists to answer.

In view of the important rôle which superheated steam is beginning to play in the steam-engineering industry, the need for further investigation is obvious.
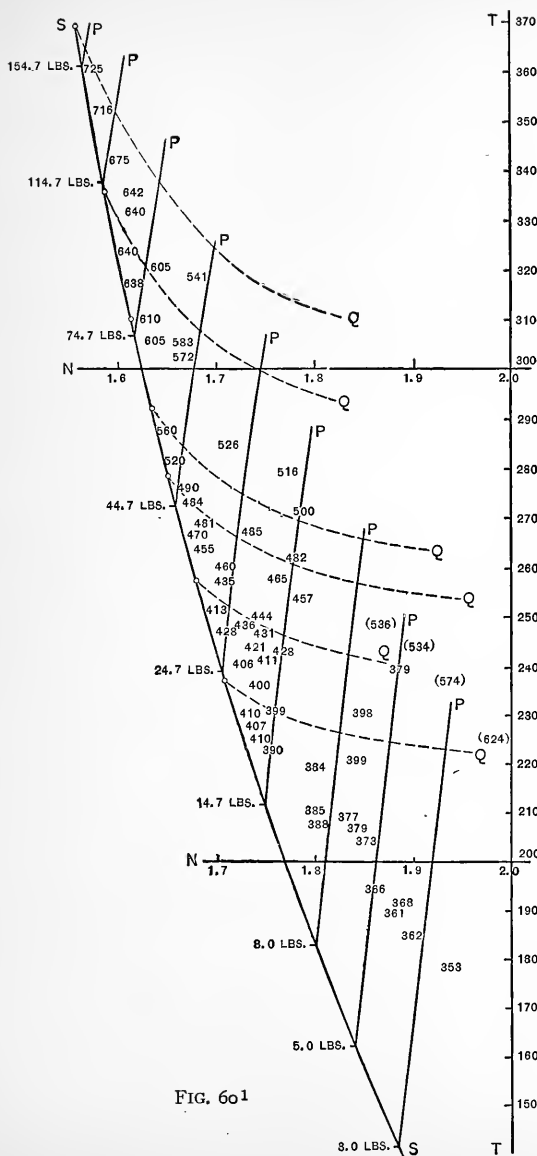
FIG. 60[1]

[1] The numbers placed in parentheses are those reported by C. Bach from the laboratory of the Royal Technical High School at Stuttgart (*Zeit. d. Ver. Deutscher Ingenieure*, May, 1902, page 729), from experiments to which the determination of the specific heat was only incidental. They are reported with the others as a matter of comparative interest. The numbers displayed in Fig. 60 are, of course, to be divided by 1000 before use.

A TABLE OF NATURAL LOGARITHMS

Note. — The student is reminded that in using *natural* logarithms the logarithm of a number ten or one hundred times as great as any number contained in the table cannot be gotten, as in common logarithms, by adding one or two integers, which are the common logarithms of 10 and 100, to the tabulated logarithm. Instead, to the tabulated logarithm must be added one or two or more times the *natural* logarithm of 10, which is 2.302585.

	0	1	2	3	4	5	6	7	8	9
1.00	0.00000	0.00100	0.00200	0.00300	0.00399	0.00499	0.00598	0.00698	0.00797	0.00896
1.01	0.00995	0.01094	0.01193	0.01292	0.01390	0.01489	0.01587	0.01686	0.01784	0.01882
1.02	0.01980	0.02078	0.02176	0.02274	0.02372	0.02469	0.02567	0.02664	0.02762	0.02859
1.03	0.02956	0.03053	0.03150	0.03247	0.03344	0.03440	0.03537	0.03633	0.03730	0.03826
1.04	0.03922	0.04018	0.04114	0.04210	0.04306	0.04402	0.04497	0.04593	0.04688	0.04784
1.05	0.04879	0.04974	0.05069	0.05164	0.05259	0.05354	0.05449	0.05543	0.05638	0.05733
1.06	0.05827	0.05921	0.06015	0.06110	0.06204	0.06298	0.06391	0.06485	0.06579	0.06672
1.07	0.06766	0.06859	0.06953	0.07046	0.07139	0.07232	0.07325	0.07418	0.07511	0.07603
1.08	0.07696	0.07789	0.07881	0.07974	0.08066	0.08158	0.08250	0.08342	0.08434	0.08526
1.09	0.08618	0.08709	0.08801	0.08893	0.08984	0.09075	0.09167	0.09258	0.09349	0.09440
1.10	0.09531	0.09622	0.09713	0.09803	0.09894	0.09985	0.10075	0.10165	0.10256	0.10346
1.1	0.09531	0.1044	0.1133	0.1222	0.1310	0.1398	0.1484	0.1570	0.1655	0.1739
1.2	0.1823	0.1906	0.1988	0.2070	0.2151	0.2231	0.2311	0.2390	0.2469	0.2546
1.3	0.2624	0.2700	0.2776	0.2852	0.2927	0.3001	0.3075	0.3148	0.3221	0.3293
1.4	0.3365	0.3436	0.3507	0.3577	0.3646	0.3716	0.3784	0.3853	0.3920	0.3988
1.5	0.4055	0.4121	0.4187	0.4253	0.4318	0.4382	0.4447	0.4511	0.4574	0.4637
1.6	0.4700	0.4762	0.4824	0.4886	0.4947	0.5008	0.5068	0.5128	0.5188	0.5247
1.7	0.5306	0.5365	0.5423	0.5481	0.5539	0.5596	0.5653	0.5710	0.5766	0.5822
1.8	0.5878	0.5933	0.5988	0.6043	0.6098	0.6152	0.6206	0.6259	0.6313	0.6366
1.9	0.6418	0.6471	0.6523	0.6575	0.6627	0.6678	0.6729	0.6780	0.6831	0.6881
2.0	0.6931	0.6981	0.7031	0.7080	0.7129	0.7178	0.7227	0.7275	0.7324	0.7372
2.1	0.7419	0.7467	0.7514	0.7561	0.7608	0.7655	0.7701	0.7747	0.7793	0.7839
2.2	0.7884	0.7930	0.7975	0.8020	0.8065	0.8109	0.8154	0.8198	0.8242	0.8286
2.3	0.8329	0.8372	0.8416	0.8459	0.8502	0.8544	0.8587	0.8629	0.8671	0.8713
2.4	0.8755	0.8796	0.8838	0.8879	0.8920	0.8961	0.9002	0.9042	0.9083	0.9123
2.5	0.9163	0.9203	0.9243	0.9282	0.9322	0.9361	0.9400	0.9439	0.9478	0.9517
2.6	0.9555	0.9594	0.9632	0.9670	0.9708	0.9746	0.9783	0.9821	0.9858	0.9895
2.7	0.9933	0.9969	1.0006	1.0043	1.0080	1.0116	1.0152	1.0188	1.0225	1.0260
2.8	1.0296	1.0332	1.0367	1.0403	1.0438	1.0473	1.0508	1.0543	1.0578	1.0613
2.9	1.0647	1.0682	1.0716	1.0750	1.0784	1.0818	1.0852	1.0886	1.0919	1.0953
3.0	1.0986	1.1019	1.1053	1.1086	1.1119	1.1151	1.1184	1.1217	1.1249	1.1282
3.1	1.1314	1.1346	1.1378	1.1410	1.1442	1.1474	1.1506	1.1537	1.1569	1.1600
3.2	1.1632	1.1663	1.1694	1.1725	1.1756	1.1787	1.1817	1.1848	1.1878	1.1909
3.3	1.1939	1.1969	1.2000	1.2030	1.2060	1.2090	1.2119	1.2149	1.2179	1.2208
3.4	1.2238	1.2267	1.2296	1.2326	1.2355	1.2384	1.2413	1.2442	1.2470	1.2499
3.5	1.2528	1.2556	1.2585	1.2613	1.2641	1.2669	1.2698	1.2726	1.2754	1.2782
3.6	1.2809	1.2837	1.2865	1.2892	1.2920	1.2947	1.2975	1.3002	1.3029	1.3056

	0	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
3.7	1.3083	1.3110	1.3137	1.3164	1.3191	1.3218	1.3244	1.3271	1.3297	1.3324
3.8	1.3350	1.3376	1.3403	1.3429	1.3455	1.3481	1.3507	1.3533	1.3558	1.3584
3.9	1.3610	1.3635	1.3661	1.3686	1.3712	1.3737	1.3762	1.3788	1.3813	1.3838
4.0	1.3863	1.3888	1.3913	1.3938	1.3962	1.3987	1.4012	1.4036	1.4061	1.4085
4.1	1.4110	1.4134	1.4159	1.4183	1.4207	1.4231	1.4255	1.4279	1.4303	1.4327
4.2	1.4351	1.4375	1.4398	1.4422	1.4446	1.4469	1.4493	1.4516	1.4540	1.4563
4.3	1.4586	1.4609	1.4633	1.4656	1.4679	1.4702	1.4725	1.4748	1.4770	1.4793
4.4	1.4816	1.4839	1.4861	1.4884	1.4907	1.4929	1.4951	1.4974	1.4996	1.5019
4.5	1.5041	1.5063	1.5085	1.5107	1.5129	1.5151	1.5173	1.5195	1.5217	1.5239
4.6	1.5261	1.5282	1.5304	1.5326	1.5347	1.5369	1.5390	1.5412	1.5433	1.5454
4.7	1.5476	1.5497	1.5518	1.5539	1.5560	1.5581	1.5602	1.5623	1.5644	1.5665
4.8	1.5686	1.5707	1.5728	1.5748	1.5769	1.5790	1.5810	1.5831	1.5851	1.5872
4.9	1.5892	1.5913	1.5933	1.5953	1.5974	1.5994	1.6014	1.6034	1.6054	1.6074
5.0	1.6094	1.6114	1.6134	1.6154	1.6174	1.6194	1.6214	1.6233	1.6253	1.6273
5.1	1.6292	1.6312	1.6332	1.6351	1.6371	1.6390	1.6409	1.6429	1.6448	1.6467
5.2	1.6487	1.6506	1.6525	1.6544	1.6563	1.6582	1.6601	1.6620	1.6639	1.6658
5.3	1.6677	1.6696	1.6715	1.6734	1.6752	1.6771	1.6790	1.6808	1.6827	1.6845
5.4	1.6864	1.6882	1.6901	1.6919	1.6938	1.6956	1.6974	1.6993	1.7011	1.7029
5.5	1.7047	1.7066	1.7084	1.7102	1.7120	1.7138	1.7156	1.7174	1.7192	1.7210
5.6	1.7228	1.7246	1.7263	1.7281	1.7299	1.7317	1.7334	1.7352	1.7370	1.7387
5.7	1.7405	1.7422	1.7440	1.7457	1.7475	1.7492	1.7509	1.7527	1.7544	1.7561
5.8	1.7579	1.7596	1.7613	1.7630	1.7647	1.7664	1.7681	1.7699	1.7716	1.7733
5.9	1.7750	1.7766	1.7783	1.7800	1.7817	1.7834	1.7851	1.7867	1.7884	1.7901
6.0	1.7918	1.7934	1.7951	1.7967	1.7984	1.8001	1.8017	1.8034	1.8050	1.8066
6.1	1.8083	1.8099	1.8116	1.8132	1.8148	1.8165	1.8181	1.8197	1.8213	1.8229
6.2	1.8245	1.8262	1.8278	1.8294	1.8310	1.8326	1.8342	1.8358	1.8374	1.8390
6.3	1.8405	1.8421	1.8437	1.8453	1.8469	1.8485	1.8500	1.8516	1.8532	1.8547
6.4	1.8563	1.8579	1.8594	1.8610	1.8625	1.8641	1.8656	1.8672	1.8687	1.8703
6.5	1.8718	1.8733	1.8749	1.8764	1.8779	1.8795	1.8810	1.8825	1.8840	1.8856
6.6	1.8871	1.8886	1.8901	1.8916	1.8931	1.8946	1.8961	1.8976	1.8991	1.9006
6.7	1.9021	1.9036	1.9051	1.9066	1.9081	1.9095	1.9110	1.9125	1.9140	1.9155
6.8	1.9169	1.9184	1.9199	1.9213	1.9228	1.9242	1.9257	1.9272	1.9286	1.9301
6.9	1.9315	1.9330	1.9344	1.9359	1.9373	1.9387	1.9402	1.9416	1.9430	1.9445
7.0	1.9459	1.9473	1.9488	1.9502	1.9516	1.9530	1.9544	1.9559	1.9573	1.9587
7.1	1.9601	1.9615	1.9629	1.9643	1.9657	1.9671	1.9685	1.9699	1.9713	1.9727
7.2	1.9741	1.9755	1.9769	1.9782	1.9796	1.9810	1.9824	1.9838	1.9851	1.9865
7.3	1.9879	1.9892	1.9906	1.9920	1.9933	1.9947	1.9961	1.9974	1.9988	2.0001
7.4	2.0015	2.0028	2.0042	2.0055	2.0069	2.0082	2.0096	2.0109	2.0122	2.0136
7.5	2.0149	2.0162	2.0176	2.0189	2.0202	2.0215	2.0229	2.0242	2.0255	2.0268
7.6	2.0281	2.0295	2.0308	2.0321	2.0334	2.0347	2.0360	2.0373	2.0386	2.0399
7.7	2.0412	2.0425	2.0438	2.0451	2.0464	2.0477	2.0490	2.0503	2.0516	2.0528
7.8	2.0541	2.0554	2.0567	2.0580	2.0592	2.0605	2.0618	2.0631	2.0643	2.0656
7.9	2.0668	2.0681	2.0694	2.0707	2.0719	2.0732	2.0744	2.0757	2.0769	2.0782
8.0	2.0794	2.0807	2.0819	2.0832	2.0844	2.0857	2.0869	2.0881	2.0894	2.0906
8.1	2.0919	2.0931	2.0943	2.0956	2.0968	2.0980	2.0992	2.1005	2.1017	2.1029
8.2	2.1041	2.1054	2.1066	2.1078	2.1090	2.1102	2.1114	2.1126	2.1138	2.1150
8.3	2.1163	2.1175	2.1187	2.1199	2.1211	2.1223	2.1235	2.1247	2.1258	2.1270

	0	1	2	3	4	5	6	7	8	9
8.4	2.1282	2.1294	2.1306	2.1318	2.1330	2.1342	2.1353	2.1365	2.1377	2.1389
8.5	2.1401	2.1412	2.1424	2.1436	2.1448	2.1459	2.1471	2.1483	2.1494	2.1506
8.6	2.1518	2.1529	2.1541	2.1552	2.1564	2.1576	2.1587	2.1599	2.1610	2.1622
8.7	2.1633	2.1645	2.1656	2.1668	2.1679	2.1691	2.1702	2.1713	2.1725	2.1736
8.8	2.1748	2.1759	2.1770	2.1782	2.1793	2.1804	2.1815	2.1827	2.1838	2.1849
8.9	2.1861	2.1872	2.1883	2.1894	2.1905	2.1917	2.1928	2.1939	2.1950	2.1961
9.0	2.1972	2.1983	2.1994	2.2006	2.2017	2.2028	2.2039	2.2050	2.2061	2.2072
9.1	2.2083	2.2094	2.2105	2.2116	2.2127	2.2138	2.2148	2.2159	2.2170	2.2181
9.2	2.2192	2.2203	2.2214	2.2225	2.2235	2.2246	2.2257	2.2268	2.2279	2.2289
9.3	2.2300	2.2311	2.2222	2.2332	2.2343	2.2354	2.2364	2.2375	2.2386	2.2396
9.4	2.2407	2.2418	2.2428	2.2439	2.2450	2.2460	2.2471	2.2481	2.2492	2.2502
9.5	2.2513	2.2523	2.2534	2.2544	2.2555	2.2565	2.2576	2.2586	2.2597	2.2607
9.6	2.2618	2.2628	2.2638	2.2649	2.2659	2.2670	2.2680	2.2690	2.2701	2.2711
9.7	2.2721	2.2732	2.2742	2.2752	2.2762	2.2773	2.2783	2.2793	2.2803	2.2814
9.8	2.2824	2.2834	2.2844	2.2854	2.2865	2.2875	2.2885	2.2895	2.2905	2.2915
9.9	2.2925	2.2935	2.2946	2.2956	2.2966	2.2976	2.2986	2.2996	2.3006	2.3016
10.0	2.3026	2.3036	2.3046	2.3056	2.3066	2.3076	2.3086	2.3096	2.3105	2.3115

THE STEAM-TABLE

A TABLE OF THE THERMAL AND PHYSICAL
PROPERTIES OF SATURATED STEAM-
VAPOR AND OF THE SPECIFIC
HEAT OF WATER

COMPILED FROM VARIOUS SOURCES BY

SIDNEY A. REEVE

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POLYTECHNIC INSTITUTE

New York
THE MACMILLAN COMPANY
LONDON: MACMILLAN & CO., Ltd.
1903

THE STEAM–TABLE

NOTES EXPLANATORY OF THE STEAM-TABLE

A BRIEF investigation of the present knowledge of the thermal properties of water and steam leaves much to be desired. The various authorities are far from agreement. In consulting them, either all except one must be discarded as worthless, or else a highly unscientific compromise or average must be effected in order to recognize them all. The following table represents a combination of both courses. Nothing more can be claimed for it than a conscientious effort after probability of truth.

The British thermal unit in which it is expressed is the specific heat of water at 59° Fahrenheit or 15° Centigrade. The mechanical equivalent of heat is taken as 778. The weight of one cubic inch of mercury is taken as 0.4912 pound.

In making use of the steam-table it is first to be noted that the high temperatures and pressures are at the top and the lower ones at the bottom. The index figures from which the table is to be entered are in pressure and temperature both, and are printed in heavy type in the middle of the page. Either index may be used, according to convenience. From a given index the desired value will be found by passing horizontally to right or left. Immediately above the value sought is printed, in small type, the difference needed to carry the value to that for the next

integral index above the one chosen *and of the same sort*. For instance, if it be elected to make use of a pressure-index, and 21 pounds per square inch be the integral pressure involved, opposite 21 pounds will be found the values for that pressure. Above them are the differences required to carry those values to the correct ones for 22 pounds per square inch, irrespective of the fact that the values for 22 pounds lie several lines above. By the aid of this difference any fractional pressures between 21 and 22 pounds may easily be interpolated.

The arrangement of the indexes in pressure and temperature side by side will be found to reduce the need for and the work of interpolation to a minimum.

All pressures near or below atmospheric are expressed in inches of mercury-column, thus avoiding the necessity for translation of observations into pounds per square inch; at the same time the gaps which involve the work and the error of interpolation are reduced by half.

The column of logarithms at the right will be found convenient in handling temperature-ratios, both for entropy calculations and for ordinary purposes. While these are themselves logarithms of ratios, it is plain that the difference between any two is itself the logarithm of the ratio of the corresponding temperatures. This column, in fact, reduces all temperature-calculations to the basis of a scale in which the absolute temperature of melting ice is unity, which is the only natural temperature-scale.

	QUANTITIES OF HEAT-ENERGY					DIMENSIONS							Natural Logarithms of the Ratio of Absolute Temperature to that of Melting Ice
Specific Heat of Water	Sensible Heat of the Liquid above 32° F.	Disgregation Energy	Heat-Equivalent of External Work	Latent Heat of Vaporization	Total Heat, from Water at 32°	Of the Body		Of the Heat-Energy					
						Volume Cu. Ft. per Pound	Pressure Lbs. per Sq. In. abs.	Temperatures		Entropies			
								Fahr.	Abs.	Of the Water	Of Vaporization	Of Saturated Steam	
S	l	D	X	L	H	V	P	t	T	N_w	N_v	N_s	$\mathrm{Log}_e \dfrac{T}{T_0}$
1.0562	420.6	711.9	85.18	797.1	1217.7	1.167	400.9	**445**	905.8	0.6184	0.8800	1.4984	0.60871
1.0561	420.4	712.1	85.18	797.3	1217.6	1.170	**400**	444.8	905.6	0.6181	0.8804	1.4985	0.60849
1.0560	420.1	712.3	85.18	797.5	1217.5	1.173	**399**	444.5	905.3	0.6179	0.8808	1.4987	0.60816
1.0560	419.9	712.4	85.19	797.6	1217.5	1.176	**398**	444.3	905.1	0.6175	0.8813	1.4988	0.60794
1.0559	419.6	712.6	85.19	797.8	1217.4	1.179	**397**	444.0	904.8	0.6172	0.8817	1.4990	0.60761
1.0559	419.6	712.6	85.19	797.8	1217.4	1.179	396.8	**444**	904.8	0.6172	0.8818	1.4990	0.60761
1.0559	419.4	712.8	85.19	798.0	1217.3	1.182	**396**	443.8	904.6	0.6170	0.8821	1.4991	0.60739
1.0558	419.1	712.9	85.19	798.2	1217.2	1.185	**395**	443.5	904.3	0.6167	0.8826	1.4993	0.60706
1.0558	418.8	713.1	85.20	798.3	1217.1	1.188	**394**	443.3	904.1	0.6164	0.8830	1.4994	0.60684
1.0557	418.6	713.3	85.20	798.5	1217.0	1.191	**393**	443.0	903.8	0.6161	0.8834	1.4996	0.60650
1.0557	418.5	713.3	85.20	798.5	1217.0	1.191	392.8	**443**	903.8	0.6160	0.8835	1.4996	0.60650
1.0557	418.3	713.4	85.20	798.7	1216.9	1.194	**392**	442.8	903.6	0.6158	0.8839	1.4997	0.60628
1.0556	418.0	713.6	85.20	798.8	1216.9	1.197	**391**	442.6	903.4	0.6155	0.8843	1.4999	0.60606
1.0556	417.8	713.8	85.21	799.0	1216.8	1.200	**390**	442.3	903.1	0.6152	0.8848	1.5000	0.60573
1.0555	417.5	714.0	85.21	799.2	1216.7	1.203	**389**	442.1	902.9	0.6149	0.8852	1.5002	0.60551
1.0554	417.5	714.0	85.21	799.2	1216.7	1.203	388.8	**442**	902.8	0.6149	0.8853	1.5002	0.60540
1.0554	417.2	714.1	85.21	799.3	1216.6	1.206	**388**	441.8	902.6	0.6147	0.8856	1.5003	0.60517
1.0553	417.0	714.3	85.21	799.5	1216.5	1.209	**387**	441.5	902.3	0.6144	0.8861	1.5004	0.60484
1.0553	416.7	714.5	85.22	799.7	1216.4	1.212	**386**	441.3	902.1	0.6140	0.8865	1.5005	0.60462

[111] 0.66650	[0] 1.4996	[47] 0.8835	[12] 0.6160	903.8	**443**	[+0] 392.8	1.191	[+0] 1217.0	798.5	85.20	713.3	418.5	1.0557
[22] 0.66628	[1] 1.4997	[5] 0.8839	[3] 0.6158	[2] 903.6	[2] 442.8	**392**	[3] 1.194	[1] 1216.9	[2] 798.7	[0] 85.20	[1] 713.4	[3] 418.3	[0] 1.0557
[22] 0.60606	[2] 1.4999	[4] 0.8843	[3] 0.6155	[2] 903.4	[2] 442.6	**391**	[3] 1.197	[0] 1216.9	[1] 798.8	[0] 85.20	[2] 713.6	[3] 418.0	[1] 1.0556
[33] 0.60573	[1] 1.5000	[5] 0.8848	[3] 0.6152	[3] 903.1	[3] 442.3	**390**	[3] 1.200	[1] 1216.8	[2] 799.0	[1] 85.21	[2] 713.8	[2] 417.8	[0] 1.0556
[22] 0.60551	[2] 1.5002	[4] 0.8852	[3] 0.6149	[2] 902.9	[2] 442.1	**389**	[3] 1.203	[1] 1216.7	[2] 799.2	[0] 85.21	[2] 714.0	[3] 417.5	[1] 1.0555
[110] 0.60540	[6] 1.5002	[18] 0.8853	[11] 0.6149	[1.0] 902.8	**442**	[4.0] 388.8	[12] 1.203	[3] 1216.7	[7] 799.2	[0] 85.21	[7] 714.0	[1.0] 417.5	[3] 1.0554
[34] 0.60517	[1] 1.5003	[4] 0.8856	[2] 0.6147	[3] 902.6	[3] 441.8	**388**	[3] 1.206	[1] 1216.6	[1] 799.3	[1] 85.21	[2] 714.1	[3] 417.2	[1] 1.0554
[33] 0.60484	[1] 1.5004	[5] 0.8861	[3] 0.6144	[3] 902.3	[3] 441.5	**387**	[3] 1.209	[1] 1216.5	[2] 799.5	[0] 85.21	[2] 714.3	[2] 417.0	[1] 1.0554
[22] 0.60462	[1] 1.5005	[4] 0.8865	[4] 0.6140	[2] 902.1	[2] 441.3	**386**	[3] 1.212	1216.4	[3] 799.7	[1] 85.22	[3] 714.5	[3] 416.7	[0] 1.0553
													1.0553

33 0.60429	2 1.5007	5 0.8870	3 0.6137	38 901.8	3 441.0	**385**	3 1.215	1 1216.3	2 799.9	0 85.22	2 714.7	3 416.4	1 1.0552
111 0.60429	5 1.5007	17 0.8870	12 0.6137	1.0 901.8	2 **441**	3.9 384.9	12 1.215	4 1216.3	7 799.9	1 85.22	7 714.7	111 416.4	2 1.0552
22 0.60407	1 1.5008	4 0.8874	3 0.6134	2 901.6	2 440.8	**384**	3 1.218	2 1216.2	1 800.1	0 85.22	2 714.8	2 416.2	0 1.0552
34 0.60373	2 1.5010	5 0.8879	3 0.6131	3 901.3	3 440.5	**383**	3 1.221	0 1216.2	2 800.2	1 85.22	2 715.0	2 415.9	1 1.0551
22 0.60351	2 1.5012	4 0.8883	3 0.6128	1.1 901.1	3 440.3	**382**	4 1.225	1 1216.1	2 800.4	1 85.23	2 715.2	7 415.7	1 1.0550
111 0.60318	6 1.5013	18 0.8888	12 0.6125	1.0 900.8	**440**	3.9 381.0	13 1.228	3 1216.0	7 800.6	0 85.23	4 715.4	1.0 415.4	0 1.0550
33 0.60318	1 1.5013	0.8888	3 0.6125	3 900.8	3 440.0	**381**	3 1.228	1 1216.0	2 800.6	1 85.23	2 715.5	3 415.4	0 1.0550
33 0.60285	2 1.5015	5 0.8893	3 0.6122	3 900.5	3 439.7	**380**	3 1.231	1 1215.9	2 800.8	0 85.23	7 715.7	3 415.1	1 1.0549
23 0.60262	1 1.5016	4 0.8897	2 0.6120	2 900.3	3 439.5	**379**	3 1.234	1 1215.8	2 801.0	1 85.24	2 715.9	3 414.8	0 1.0548
34 0.60228	2 1.5018	5 0.8902	3 0.6117	3 900.0	3 439.2	**378**	3 1.237	3 1215.8	2 801.2	1 85.24	7 716.1	5 414.5	3 1.0548
111 0.60207	6 1.5019	18 0.8906	11 0.6114	1.0 899.8	**439**	3.9 377.1	12 1.240	3 1215.7	7 801.3	0 85.24	1 716.1	1.1 414.3	2 1.0547
21 0.60207	1 1.5019	0.8906	4 0.6113	2 899.8	2 439.0	**377**	3 1.240	1 1215.7	2 801.4	1 85.24	3 716.3	2 414.2	0 1.0547
34 0.60173	2 1.5021	5 0.8911	3 0.6110	3 899.5	3 438.7	**376**	4 1.244	1 1215.6	1 801.6	0 85.24	2 716.4	2 414.0	1 1.0547
22 0.60151	1 1.5022	4 0.8915	3 0.6107	3 899.3	2 438.5	**375**	3 1.247	1 1215.5	1 801.7	1 85.24	6 716.6	3 413.7	1 1.0546
33 0.60118	2 1.5024	5 0.8920	3 0.6104	3 899.0	3 438.2	**374**	3 1.250	4 1215.4	2 801.9	0 85.24	8 716.8	3 413.4	4 1.0545
111 0.60096	6 1.5025	17 0.8923	12 0.6102	1.0 898.8	**438**	3.9 373.2	13 1.253	3 1215.3	7 802.0	1 85.24	2 716.8	1.1 413.2	0 1.0545
34 0.60084	1 1.5025	4 0.8924	3 0.6101	3 898.7	3 437.9	**373**	4 1.254	1 1215.3	1 802.0	0 85.24	1 717.0	3 413.2	1 1.0545
22 0.60062	2 1.5027	5 0.8929	3 0.6098	3 898.5	3 437.7	**372**	3 1.257	2 1215.2	2 802.2	1 85.25	1 717.1	3 412.9	1 1.0544
33 0.60029	1 1.5028	5 0.8934	3 0.6095	3 898.2	3 437.4	**371**	4 1.261	1 1215.1	4 802.4			2 412.6	4 1.0543

0.60007	1.5029	0.8938	0.6091	898.0	437.2	**370**	1.264	1215.0	802.5	85.25	717.3	412.4	1.0543
0.59984	1.5030	0.8941	0.6089	897.8	**437**	369.4	1.266	1214.9	802.6	85.25	717.4	412.2	1.0542
0.59973	1.5031	0.8943	0.6088	897.7	436.9	**369**	1.267	1214.9	802.7	85.25	717.5	412.1	1.0542
0.59940	1.5033	0.8948	0.6085	897.4	436.6	**368**	1.271	1214.8	802.9	85.25	717.7	411.8	1.0542
0.59917	1.5035	0.8952	0.6082	897.2	436.4	**367**	1.274	1214.7	803.1	85.26	717.9	411.5	1.0541
0.59884	1.5036	0.8957	0.6079	896.9	436.1	**366**	1.278	1214.6	803.3	85.26	718.1	411.2	1.0540
0.59873	1.5037	0.8959	0.6077	896.8	**436**	365.6	1.279	1214.6	803.4	85.26	718.1	411.1	1.0540
0.59862	1.5038	0.8962	0.6075	896.7	435.9	**365**	1.281	1214.5	803.5	85.26	718.2	410.9	1.0540
0.59828	1.5039	0.8967	0.6072	896.4	435.6	**364**	1.284	1214.4	803.7	85.26	718.4	410.7	1.0539
0.59795	1.5041	0.8971	0.6069	896.1	435.3	**363**	1.288	1214.3	803.9	85.27	718.6	410.4	1.0538
0.59772	1.5042	0.8976	0.6066	895.9	435.1	**362**	1.292	1214.2	804.1	85.27	718.8	410.2	1.0538
0.59762	1.5043	0.8977	0.6066	895.8	**435**	361.8	1.292	1214.2	804.1	85.27	718.8	410.1	1.0538
0.59739	1.5044	0.8981	0.6063	895.6	434.8	**361**	1.295	1214.1	804.3	85.27	719.0	409.9	1.0537
0.59705	1.5046	0.8986	0.6060	895.3	434.5	**360**	1.299	1214.1	804.5	85.27	719.2	409.6	1.0536
0.59672	1.5047	0.8991	0.6057	895.0	434.2	**359**	1.303	1214.0	804.7	85.28	719.4	409.3	1.0536
0.59650	1.5049	0.8995	0.6054	894.8	**434**	358.1	1.306	1213.9	804.8	85.28	719.5	409.0	1.0535
0.59650	1.5049	0.8996	0.6053	894.8	434.0	**358**	1.306	1213.9	804.8	85.28	719.6	409.0	1.0535
0.59616	1.5050	0.9000	0.6050	894.5	433.7	**357**	1.310	1213.8	805.0	85.28	719.8	408.7	1.0535
0.59582	1.5052	0.9005	0.6047	894.2	433.4	**356**	1.313	1213.7	805.2	85.28	720.0	408.4	1.0534

0.59560 (22)	1.5054 (2)	0.9010 (5)	0.6044 (3)	894.0 (2)	433.2 (2)	**355**	1.317 (4)	1213.6 (1)	805.4 (2)	85.28 (0)	720.2 (2)	408.2 (2)	1.0533 (1)
0.59538 (112)	1.5055 (6)	0.9013 (18)	0.6042 (12)	893.8 (1.0)	**433**	3.7 354.4	1.319 (13)	1213.5 (4)	805.5 (8)	85.28 (0)	720.3 (8)	408.0 (1.0)	1.0533 (2)
0.59527 (33)	1.5055 (1)	0.9015 (5)	0.6041 (3)	893.7 (3)	432.9 (3)	**354**	1.320 (3)	1213.5 (1)	805.6 (2)	85.28 (1)	720.4 (2)	407.9 (3)	1.0533 (0)
0.59493 (34)	1.5057 (2)	0.9020 (0)	0.6037 (4)	893.4 (3)	432.6 (2)	**353**	1.324 (4)	1213.4 (3)	805.8 (2)	85.29 (0)	720.6 (2)	407.6 (3)	1.0532 (0)
0.59471 (22)	1.5059 (2)	0.9025 (5)	0.6034 (3)	893.2 (3)	432.4 (2)	**352**	1.328 (4)	1213.3 (1)	806.0 (2)	85.29 (0)	720.8 (2)	407.3 (3)	1.0531 (0)
0.59437 (34)	1.5061 (2)	0.9030 (18)	0.6031 (3)	892.9 (2)	432.1	**351**	1.332 (4)	1213.2 (3)	806.2 (2)	85.29 (1)	721.0 (2)	407.0 (3)	1.0531 (0)
0.59426 (12)	1.5062 (1)	0.9031 (5)	0.6030 (12)	892.8 (1.0)	**432**	3.7 350.7	1.333 (14)	1213.2 (1)	806.3 (7)	85.29 (0)	721.0 (7)	406.9 (1.1)	1.0531 (0)
0.59403 (34)	1.5063 (2)	0.9035 (5)	0.6028 (3)	892.6 (3)	431.8 (3)	**350**	1.336 (4)	1213.1 (1)	806.4 (2)	85.29 (1)	721.1 (1)	406.7 (3)	1.0530 (1)
0.59370 (33)	1.5065 (2)	0.9040 (5)	0.6025 (3)	892.3 (3)	431.5 (3)	**349**	1.339 (3)	1213.1 (0)	806.6 (4)	85.29 (0)	721.3 (2)	406.4 (3)	1.0530 (1)
0.59336 (34)	1.5066 (1)	0.9045 (18)	0.6021 (11)	892.0 (3)	431.2 (2)	**348**	1.343 (4)	1213.0 (3)	806.8 (2)	85.30 (1)	721.5 (2)	406.1 (3)	1.0529 (1)
0.59314 (112)	1.5068 (2)	0.9049 (5)	0.6019 (3)	891.8 (1.0)	**431**	3.6 347.1	1.347 (14)	1212.9 (1)	807.0 (7)	85.30 (0)	721.7 (2)	405.9 (1.0)	1.0529 (1)
0.59314 (22)	1.5068 (2)	0.9050 (5)	0.6018 (3)	891.8 (2)	431.0 (2)	**347**	1.347 (4)	1212.8 (1)	807.2 (2)	85.30 (0)	721.7 (2)	405.8 (3)	1.0528 (1)
0.59280 (34)	1.5070 (1)	0.9055 (5)	0.6015 (3)	891.5 (3)	430.7 (3)	**346**	1.351 (4)	1212.7 (1)	807.4 (2)	85.30 (1)	721.9 (3)	405.6 (3)	1.0527 (1)
0.059246 (34)	1.5071 (2)	0.9060 (5)	0.6012 (3)	891.2 (3)	430.4 (3)	**345**	1.355 (4)	1212.6 (1)	807.7 (3)	85.31 (0)	722.1 (2)	405.3 (3)	1.0527 (1)
0.59224 (22)	1.5073 (6)	0.9065 (19)	0.6009 (12)	891.0 (2)	430.2 (2)	**344**	1.359 (14)	1212.6 (1)	807.8 (7)	85.31 (0)	722.4 (8)	405.0 (3)	1.0526 (1)
0.59202 (112)	1.5074 (2)	0.9068 (5)	0.6007 (3)	890.8 (1.0)	**430**	3.7 343.4	1.361 (4)	1212.6 (1)	807.9 (2)	85.31 (1)	722.5 (2)	404.8 (1.1)	1.0526 (2)
0.59190 (34)	1.5075 (2)	0.9070 (5)	0.6005 (4)	890.7 (3)	429.9 (3)	**343**	1.363 (4)	1212.6 (1)	808.1 (2)	85.31 (0)	722.6 (2)	404.7 (3)	1.0525 (0)
0.59157 (33)	1.5077 (2)	0.9075 (5)	0.6002 (3)	890.4 (3)	429.6 (3)	**342**	1.367 (4)	1212.5 (1)	808.3 (3)	85.31 (1)	722.8 (2)	404.4 (3)	1.0525 (1)
0.59123 (34)	1.5079	0.9080	0.5999	890.1	429.3	**341**	1.370 (3)	1212.4		85.31	723.0	404.1	1.0524

0.59089	1.5080	0.9085	0.5995	889.8	429.0	**340**	1.374	1212.3	808.5	85.31	723.2	403.8	1.0523
0.59089	1.5081	0.9086	0.5995	889.8	**429**	339.8	1.375	1212.2	808.5	85.31	723.2	403.8	1.0523
0.59067	1.5082	0.9090	0.5992	889.6	428.8	**339**	1.379	1212.1	808.7	85.31	723.4	403.5	1.0523
0.59033	1.5084	0.9095	0.5989	889.3	428.5	**338**	1.383	1212.1	808.9	85.32	723.6	403.2	1.0522
0.58999	1.5086	0.9101	0.5985	889.0	428.2	**337**	1.387	1212.0	809.1	85.32	723.8	402.9	1.0522
0.58977	1.5087	0.9104	0.5983	888.8	**428**	336.3	1.390	1211.9	809.2	85.32	723.9	402.7	1.0521
0.58965	1.5088	0.9106	0.5982	888.7	427.9	**336**	1.391	1211.9	809.3	85.32	723.9	402.6	1.0521
0.58932	1.5090	0.9111	0.5979	888.4	427.6	**335**	1.395	1211.8	809.5	85.32	724.1	402.3	1.0521
0.58909	1.5092	0.9116	0.5975	888.2	427.4	**334**	1.399	1211.7	809.7	85.32	724.3	402.0	1.0521
0.58875	1.5094	0.9122	0.5972	887.9	427.1	**333**	1.403	1211.6	809.9	85.32	724.5	401.7	1.0520
0.58864	1.5094	0.9123	0.5971	887.8	**427**	332.8	1.404	1211.6	809.9	85.32	724.6	401.7	1.0519
0.58842	1.5095	0.9127	0.5969	887.6	426.8	**332**	1.407	1211.5	810.1	85.32	724.7	401.4	1.0519
0.58808	1.5097	0.9132	0.5965	887.3	426.5	**331**	1.411	1211.4	810.3	85.33	724.9	401.1	1.0519
0.58774	1.5099	0.9137	0.5962	887.0	426.2	**330**	1.416	1211.3	810.5	85.33	725.1	400.8	1.0518
0.58751	1.5100	0.9141	0.5959	886.8	**426**	329.3	1.419	1211.2	810.6	85.33	725.3	400.6	1.0518
0.58740	1.5101	0.9143	0.5958	886.7	425.9	**329**	1.420	1211.2	810.7	85.33	725.3	400.5	1.0517
0.58706	1.5103	0.9148	0.5955	886.4	425.6	**328**	1.424	1211.1	810.9	85.33	725.6	400.2	1.0517
0.58684	1.5105	0.9153	0.5952	886.2	425.4	**327**	1.429	1211.0	811.1	85.33	725.8	399.9	1.0515
0.58650	1.5107	0.9159	0.5948	885.9	425.1	**326**	1.433	1210.9	811.3	85.33	726.0	399.6	1.0514

0.58639	1.5107	0.9160	0.5947	885.8	**425**	325.8	1.434	1210.9	811.4	85.33	726.1	399.6	1.0514
0.58616	1.5109	0.9164	0.5945	885.6	424.8	**325**	1.437	1210.9	811.5	85.33	726.2	399.3	1.0513
0.58582	1.5111	0.9170	0.5941	885.3	424.5	**324**	1.442	1210.8	811.7	85.34	726.4	399.0	1.0512
0.58548	1.5113	0.9175	0.5938	885.0	424.2	**323**	1.446	1210.7	812.0	85.34	726.6	398.7	1.0512
0.58526	1.5114	0.9179	0.5936	884.8	**424**	322.3	1.449	1210.6	812.1	85.34	726.8	398.5	1.0511
0.58514	1.5115	0.9180	0.5934	884.7	423.9	**322**	1.451	1210.6	812.2	85.34	726.8	398.4	1.0511
0.58480	1.5117	0.9186	0.5931	884.4	423.6	**321**	1.455	1210.5	812.4	85.34	727.0	398.1	1.0511
0.58447	1.5119	0.9191	0.5927	884.1	423.3	**320**	1.460	1210.4	812.6	85.34	727.2	397.8	1.0510
0.58413	1.5121	0.9197	0.5924	883.8	423.0	**319**	1.465	1210.3	812.8	85.35	727.4	397.5	1.0510
0.58413	1.5121	0.9197	0.5924	883.8	**423**	318.9	1.465	1210.3	812.8	85.35	727.5	397.5	1.0510
0.58379	1.5123	0.9202	0.5920	883.5	422.7	**318**	1.469	1210.2	813.0	85.35	727.7	397.2	1.0509
0.58345	1.5125	0.9208	0.5917	883.2	422.4	**317**	1.474	1210.1	813.2	85.35	727.9	396.9	1.0509
0.58311	1.5127	0.9214	0.5913	882.9	422.1	**316**	1.478	1210.0	813.5	85.36	728.1	396.6	1.0508
0.58299	1.5128	0.9216	0.5912	882.8	**422**	315.5	1.480	1210.0	813.6	85.36	728.2	396.4	1.0508
0.58277	1.5129	0.9219	0.5910	882.6	421.8	**315**	1.483	1209.9	813.7	85.36	728.4	396.2	1.0507
0.58243	1.5131	0.9225	0.5906	882.3	321.5	**314**	1.487	1209.8	814.0	85.36	728.6	395.9	1.0506
0.58209	1.5133	0.9230	0.5903	882.0	421.2	**313**	1.492	1209.8	814.2	85.36	728.8	395.6	1.0506
0.58186	1.5135	0.9235	0.5900	881.8	**421**	312.2	1.496	1209.7	814.4	85.36	729.0	395.4	1.0505
0.58175	1.5135	0.9236	0.5899	881.7	420.9	**312**	1.497	1209.7	814.4	85.37	729.1	395.3	1.0504

34 0.58141	2 1.5137	6 0.9242	4 0.5895	3 881.4	3 420.6	**311**	5 1.502	1.6 1209.6	3 814.7	0 85.37	2 729.3	3 395.0	1 1.0503
34 0.58107	2 1.5139	5 0.9247	3 0.5892	3 881.1	3 420.3	**310**	5 1.507	1 1209.5	2 814.9	0 85.37	2 729.5	4 394.7	1 1.0502
34 0.58073	3 1.5142	6 0.9253	3 0.5889	3 880.8	3 420.0	**309**	4 1.511	1 1209.4	3 815.1	0 85.37	2 729.7	4 394.3	0 1.0502
113 0.58073	2 1.5142	19 0.9254	12 0.5888	1.0 880.8	**420**	3.3 308.9	16 1.512	1 1209.4	2 815.1	0 85.37	2 729.7	1.1 394.3	3 1.0501
34 0.58039	2 1.5144	2 0.9259	4 0.5885	3 880.5	3 419.7	**308**	5 1.516	1 1209.3	3 815.3	1 85.38	2 729.9	3 394.0	1 1.0501
35 0.58004	2 1.5146	6 0.9265	4 0.5881	3 880.2	3 419.4	**307**	5 1.521	1 1209.2	2 815.5	0 85.38	2 730.1	3 393.7	1 1.0500
34 0.57970	2 1.5148	0 0.9270	12 0.5878	3 879.9	3 419.1	**306**	5 1.526	1 1209.1	3 815.7	0 85.37	2 730.3	3 393.4	1 1.0500
114 0.57959	7 1.5149	19 0.9273	4 0.5876	1.0 879.8	**419**	3.3 305.6	16 1.528	1 1209.1	3 815.8	0 85.37	2 730.4	1.0 393.3	1 1.0499
34 0.57936	2 1.5150	6 0.9276	4 0.5874	3 879.6	3 418.8	**305**	5 1.531	1 1209.0	2 815.9	0 85.37	2 730.6	3 393.1	0 1.0499
34 0.57902	2 1.5152	6 0.9282	4 0.5870	3 879.3	3 418.5	**304**	5 1.536	0 1209.0	3 816.2	0 85.37	2 730.8	3 392.8	1 1.0499
34 0.57868	2 1.5155	19 0.9288	12 0.5867	3 879.0	3 418.2	**303**	5 1.541	3 1208.9	3 816.4	1 85.37	8 731.0	4 392.4	1 1.0498
114 0.57845	7 1.5156	6 0.9292	4 0.5864	1.0 878.8	**418**	3.3 302.3	17 1.545	1.8 1208.8	8 816.6	1 85.36	2 731.2	1.1 392.2	1 1.0497
34 0.57834	2 1.5157	6 0.9294	3 0.5863	3 878.7	3 417.9	**302**	5 1.546	1 1208.8	3 816.7	1 85.36	2 731.3	3 392.1	3 1.0496
34 0.57800	2 1.5159	6 0.9300	4 0.5859	3 878.4	3 417.6	**301**	5 1.551	1 1208.7	3 816.9	0 85.36	2 731.5	3 391.8	1 1.0496
34 0.57766	2 1.5161	5 0.9305	4 0.5856	3 878.1	3 417.3	**300**	5 1.556	1 1208.6	2 817.1	0 85.36	2 731.8	3 391.5	0 1.0495
35 0.57731	2 1.5163	2 0.9311	4 0.5852	3 877.8	3 417.0	**299**	5 1.561	1 1208.5	3 817.4	1 85.36	2 732.0	3 391.2	1 1.0495
114 0.57731	7 1.5163	19 0.9311	12 0.5852	1.0 877.8	**417**	3.3 299.0	16 1.561	1 1208.5	3 817.4	1 85.36	2 732.0	1.0 391.2	1 1.0494
34 0.57697	3 1.5166	6 0.9317	3 0.5849	3 877.5	3 416.7	**298**	6 1.567	1 1208.4	2 817.6	0 85.36	2 732.3	4 390.8	2 1.0494
34 0.57663	2 1.5168	6 0.9323	4 0.5845	3 877.2	2 416.4	**297**	5 1.572	1 1208.3	2 817.8	1 85.35	2 732.5	3 390.5	1 1.0493
													1 1.0492

0.57629 ^{34}	1.5170 ^{2}	0.9329 ^{6}	0.5841 ^{4}	876.9 ^{3}	416.1 ^{3}	**296**	1.577 ^{5}	1208.2 ^{1}	818.0 ^{2}	85.35 ^{0}	732.7 ^{2}	390.2 ^{3}	1.0491 ^{1}
0.57617 ^{114}	1.5170 ^{7}	0.9330 ^{19}	0.5840 ^{12}	876.8 $^{1.0}$	**416**	295.8 $^{3.2}$	1.578 ^{17}	1208.2 ^{3}	818.1 ^{7}	85.35 ^{1}	732.8 ^{8}	390.1 $^{1.1}$	1.0491 ^{3}
0.57595 ^{34}	1.5172 ^{2}	0.9335 ^{2}	0.5837 ^{4}	876.6 ^{3}	415.8 ^{8}	**295**	1.582 ^{5}	1208.1 ^{1}	818.3 ^{1}	85.35 ^{1}	733.0 ^{2}	389.8 ^{4}	1.0491 ^{1}
0.57549 ^{46}	1.5174 ^{2}	0.9341 ^{6}	0.5834 ^{3}	876.2 ^{4}	415.4 ^{4}	**294**	1.588 ^{6}	1208.0 ^{1}	818.5 ^{2}	85.34 ^{1}	733.2 ^{2}	389.5 ^{3}	1.0491 ^{4}
0.57515 ^{34}	1.5177 ^{2}	0.9347 ^{6}	0.5830 ^{4}	875.9 ^{3}	415.1 ^{3}	**293**	1.593 ^{5}	1207.9 $^{1.0}$	818.7 ^{2}	85.34 ^{1}	733.4 ^{2}	389.2 ^{3}	1.0490 ^{3}
0.57503 ^{114}	1.5178 ^{2}	0.9349 ^{6}	0.5828 ^{12}	875.8 $^{1.0}$	**415**	292.6 $^{3.2}$	1.595 ^{5}	1207.9 ^{1}	818.8 ^{2}	85.34 ^{1}	733.5 ^{2}	389.1 $^{1.0}$	1.0490 ^{3}
0.57480 ^{35}	1.5179 ^{2}	0.9353 ^{6}	0.5826 ^{4}	875.6 ^{3}	414.8 ^{8}	**292**	1.598 ^{5}	1207.8 ^{3}	818.9 ^{3}	85.33 ^{1}	733.6 ^{3}	388.9 ^{2}	1.0489 ^{2}
0.57446 ^{34}	1.5181 ^{3}	0.9359 ^{6}	0.5822 ^{4}	875.3 ^{3}	414.5 ^{3}	**291**	1.603 ^{5}	1207.7 ^{1}	819.2 ^{2}	85.33 ^{1}	733.9 ^{3}	388.5 ^{4}	1.0489 $^{1.0}$
0.57412 ^{114}	1.5184 ^{7}	0.9365 ^{20}	0.5818 ^{12}	875.0 $^{1.0}$	414.2 ^{3}	**290**	1.609 ^{6}	1207.6 ^{1}	819.4 ^{2}	85.32 ^{0}	734.1 ^{4}	388.2 ^{5}	1.0488 ^{2}
0.57389 ^{114}	1.5185 ^{2}	0.9369 ^{6}	0.5816 ^{4}	874.8 ^{3}	**414**	289.4 $^{3.2}$	1.612 ^{17}	1207.6 ^{3}	819.6 ^{8}	85.32 ^{2}	734.3 ^{8}	388.0 $^{1.1}$	1.0487 ^{1}
0.57378 ^{34}	1.5186 ^{2}	0.9371 ^{6}	0.5815 ^{3}	874.7 ^{3}	413.9 ^{3}	**289**	1.614 ^{5}	1207.6 $^{1.0}$	819.7 ^{3}	85.32 ^{0}	734.4 ^{3}	387.9 ^{3}	1.0486 ^{1}
0.57343 ^{35}	1.5188 ^{2}	0.9377 ^{6}	0.5811 ^{4}	874.4 ^{3}	413.6 ^{3}	**288**	1.619 ^{5}	1207.5 ^{1}	819.9 ^{2}	85.31 ^{1}	734.6 ^{3}	387.6 ^{3}	1.0486 ^{3}
0.57309 ^{34}	1.5190 ^{7}	0.9383 ^{19}	0.5807 ^{12}	874.1 ^{3}	413.3 ^{3}	**287**	1.625 ^{17}	1207.4 ^{1}	820.1 ^{2}	85.31 ^{0}	734.8 ^{4}	387.2 ^{4}	1.0486 ^{0}
0.57275 ^{114}	1.5192 ^{2}	0.9388 ^{6}	0.5804 ^{4}	873.8 $^{1.0}$	**413**	286.2 $^{3.2}$	1.629 ^{3}	1207.3 ^{3}	820.3 ^{7}	85.30 ^{2}	735.0 ^{7}	387.0 $^{1.0}$	1.0485 ^{1}
0.57263 ^{46}	1.5193 ^{3}	0.9389 ^{6}	0.5804 ^{4}	873.7 ^{4}	412.9 ^{4}	**286**	1.630 ^{5}	1207.3 ^{1}	820.4 ^{2}	85.30 ^{0}	735.1 ^{3}	386.9 ^{3}	1.0484 ^{1}
0.57229 ^{34}	1.5195 ^{2}	0.9395 ^{6}	0.5800 ^{4}	873.4 ^{3}	412.6 ^{3}	**285**	1.635 ^{6}	1207.2 ^{1}	820.6 ^{2}	85.29 ^{1}	735.3 ^{3}	386.6 ^{4}	1.0483 ^{1}
0.57195 ^{34}	1.5197 ^{2}	0.9401 ^{6}	0.5796 ^{4}	873.1 ^{3}	412.3 ^{3}	**284**	1.641 ^{5}	1207.1 ^{1}	820.8 ^{2}	85.29 ^{0}	735.6 ^{3}	386.2 ^{3}	1.0482 ^{0}
0.57160 ^{35}	1.5200 ^{8}	0.9407 ^{19}	0.5792 ^{12}	872.8 $^{1.0}$	412.0 ^{3}	**283**	1.646 ^{17}	1207.0 ^{1}	821.1 ^{8}	85.28 ^{2}	735.8 ^{8}	385.9 ^{3}	1.0482 ^{1}
0.57160 ^{115}	1.5200	0.9407	0.5792	872.8	**412**	283.0 $^{3.2}$	1.646	1207.0 ^{3}	821.1	85.28	735.8	385.9 $^{1.1}$	1.0481 ^{2}

0.57126	1.5202	0.9414	0.5788	872.5	411.7	**282**	1.652	1206.9	821.3	85.27	736.1	385.6	1.0480
0.57091	1.5204	0.9420	0.5784	872.2	411.4	**281**	1.658	1206.8	821.5	85.27	736.3	385.2	1.0479
0.57046	1.5207	0.9426	0.5781	871.8	411.0	**280**	1.663	1206.7	821.6	85.26	736.5	384.9	1.0478
0.57046	1.5207	0.9427	0.5780	871.8	**411**	**279.9**	1.664	1206.7	821.8	85.26	736.5	384.9	1.0478
0.57011	1.5209	0.9432	0.5777	871.5	410.7	**279**	1.669	1206.6	822.0	85.25	736.7	384.6	1.0477
0.56977	1.5211	0.9438	0.5773	871.2	410.4	**278**	1.675	1206.5	822.3	85.25	737.0	384.2	1.0477
0.56942	1.5214	0.9445	0.5769	870.9	410.1	**277**	1.681	1206.4	822.5	85.24	737.2	383.9	1.0476
0.56931	1.5214	0.9446	0.5768	870.8	**410**	**276.8**	1.682	1206.4	822.6	85.24	737.3	383.8	1.0476
0.56908	1.5216	0.9451	0.5765	870.6	409.8	**276**	1.686	1206.3	822.8	85.24	737.5	383.5	1.0475
0.56862	1.5218	0.9457	0.5761	870.2	409.4	**275**	1.692	1206.2	823.0	85.23	737.7	383.2	1.0474
0.56827	1.5221	0.9463	0.5757	869.9	409.1	**274**	1.698	1206.1	823.2	85.23	738.0	382.9	1.0473
0.56816	1.5222	0.9465	0.5756	869.8	**409**	**273.7**	1.700	1206.1	823.3	85.23	738.1	382.8	1.0473
0.56793	1.5224	0.9470	0.5754	869.6	408.8	**273**	1.704	1206.0	823.5	85.23	738.2	382.5	1.0472
0.56747	1.5226	0.9476	0.5750	869.2	408.4	**272**	1.710	1205.9	823.7	85.22	738.5	382.2	1.0472
0.56712	1.5228	0.9483	0.5746	868.9	408.1	**271**	1.717	1205.8	824.0	85.21	738.8	381.8	1.0471
0.56701	1.5229	0.9485	0.5744	868.8	**408**	**270.6**	1.719	1205.8	824.1	85.21	738.9	381.7	1.0470
0.56678	1.5231	0.9489	0.5742	868.6	407.8	**270**	1.723	1205.7	824.2	85.21	739.0	381.5	1.0470
0.56643	1.5234	0.9496	0.5738	868.3	407.5	**269**	1.729	1205.6	824.5	85.20	739.3	381.1	1.0469
0.56597	1.5236	0.9502	0.5734	867.9	407.1	**268**	1.735	1205.5	824.7	85.20	739.5	380.8	1.0468

115 0.56586	8 1.5237	20 0.9505	12 0.5732	1.0 867.8	**407** 3	3.0 267.6	18 1.737	3 1205.5	824.8 3	1 85.20	7.6 739.6	1.0 380.7	2 1.0468
34 0.56563	2 1.5238	7 0.9509	5 0.5729	3.6 867.6	406.8	267	6 1.741	1 1205.4	825.0 2	0 85.20	3.8 739.8	4 380.4	1 1.0467
47 0.56516	1 1.5241	6 0.9515	4 0.5725	4.2 867.2	406.4 4	**266**	7 1.748	3 1205.3	825.2 3	1 85.19	3 740.1	3 380.1	1 1.0466
34 0.56482	2 1.5243	7 0.9522	1 0.5721	3 866.9	406.1 3	**265** 2.9	6 1.754	1 1205.2	825.5 8	2 85.18	2 740.3	4 379.7	0 1.0466
16 0.56470	7 1.5244	19 0.9524	12 0.5720	1.0 866.8	**406** 3	264.7	19 1.756	3 1205.2	825.6 2	1 85.18	8 740.4	1.1 379.6	3 1.0465
35 0.56447	2 1.5246	4 0.9528	4 0.5717	3 866.6	405.8 4	**264**	7 1.761	1 1205.1	825.7 3	0 85.17	3 740.6	3 379.4	1 1.0465
46 0.56401	2 1.5248	7 0.9535	4 0.5713	4 866.2	405.4 3	**263**	6 1.767	3 1205.0	826.0 2	1 85.17	2 740.8	4 379.0	1 1.0464
35 0.56366	3 1.5251	42 0.9542	4 0.5709	3 865.9	405.1 4	**262** 2.9	7 1.774	1 1204.9	826.2 7	0 85.17	3 741.1	3 378.7	1 1.0463
35 0.56355	7 1.5251	19 0.9543	5 0.5708	3 865.8	**405** 4	261.8	7 1.775	3 1204.9	826.3 3	1 85.16	7 741.1	1.0 378.6	1 1.0463
46 0.56320	2 1.5253	7 0.9549	4 0.5704	5 865.5	404.7 3	**261**	5 1.780	1 1204.8	826.5 3	0 85.16	1 741.3	4 378.3	1 1.0462
34 0.56286	2 1.5255	6 0.9555	4 0.5700	3 865.2	404.4 4	**260**	7 1.787	1 1204.7	826.7 3	1 85.15	6 741.6	3 377.9	1 1.0461
47 0.56239	3 1.5258	2 0.9562	4 0.5696	4.8 864.8	404.0 4	**259** 2.9	6 1.793	3 1204.6	827.0 7	2 85.15	2 741.8	6 377.6	0 1.0461
16 0.56239	1 1.5259	20 0.9563	12 0.5696	3 864.8	**404** 3	258.9	19 1.794	3 1204.6	827.0 3	1 85.15	8 741.9	1.1 377.5	2 1.0461
34 0.56205	3 1.5261	7 0.9569	4 0.5692	3.7 864.5	403.7 3	**258**	7 1.800	1 1204.5	827.3 2	0 85.14	3 742.1	4 377.2	1 1.0460
47 0.56158	3 1.5264	6 0.9576	4 0.5688	4.1 864.1	403.3 3	**257** 2.8	8 1.808	1 1204.4	827.5 8	1 85.14	3 742.4	3 376.8	1 1.0459
15 0.56124	1 1.5266	20 0.9583	12 0.5684	1.0 863.8	**403** 3	256.1	7 1.814	3 1204.3	827.8 3	1 85.14	3 742.7	1.0 376.5	1 1.0458
34 0.56124	2 1.5266	7 0.9583	4 0.5684	3 863.8	403.0 4	**256**	7 1.815	1 1204.3	827.8 3	0 85.14	3 742.7	4 376.4	1 1.0458
47 0.56077	3 1.5269	7 0.9590	5 0.5679	4 863.4	402.6 4	**255**	5 1.822	1 1204.2	828.1 2	1 85.13	2 742.9	3 376.1	1 1.0457
46 0.56031	2 1.5272	7 0.9597	4 0.5675	4 863.0	402.2	**254**	7 1.829	1 1204.1	828.3	0 85.13	3 743.2	4 375.7	0 1.0457

¹¹⁶0.56008	⁸1.5274	¹⁹0.9602	¹²0.5672	¹·⁰862.8	**402**	²·⁸253.3	²⁰1.834	³1204.0	⁷828.5	²85.12	⁷743.4	¹·¹375.4	²1.0456
³⁵0.55996	³1.5275	⁷0.9604	⁵0.5670	³862.7	³401.9	**253**	⁷1.836	¹1204.0	³828.6	¹85.12	³743.5	⁴375.3	¹1.0456
⁴⁶0.55950	³1.5278	⁸0.9612	⁴0.5666	⁴862.3	⁴401.5	**252**	⁶1.843	¹1203.9	³828.3	⁰85.12	³743.8	³375.0	¹1.0455
³⁵0.55915	³1.5281	⁷0.9619	⁴0.5662	³862.0	³401.2	**251** ²·⁸250.5	⁸1.851	³1203.8	³829.2	¹85.11	³744.1	⁴374.6	⁴1.0454
¹¹⁶0.55892	⁸1.5282	²⁰0.9622	¹³0.5659	¹·⁰861.8	**401**	**250**	⁷1.854	³1203.7	³829.3	¹85.11	³744.2	¹·⁰374.4	²1.0454
⁴⁶0.55869	³1.5283	⁷0.9626	⁵0.5657	⁴861.6	⁴400.8	**249**	⁸1.858	²1203.6	²829.4	⁰85.11	²744.3	⁴374.2	⁰1.0454
⁴⁷0.55822	³1.5286	⁷0.9633	⁵0.5652	⁴861.2	⁴400.4	**248** ²·⁷247.8	⁷1.865	¹1203.5	³829.7	¹85.11	³744.6	⁴373.8	¹1.0454
³⁵0.55787	³1.5289	⁸0.9641	⁴0.5648	³860.9	³400.1	**247**	⁷1.872	¹1203.4	³830.0	¹85.10	³744.9	³373.4	¹1.0453
¹¹⁶0.55776	³1.5289	²⁰0.9642	¹²0.5647	¹·⁰860.8	**400**	**246** ²·⁸245.0	²⁰1.874	¹1203.4	⁷830.0	¹85.10	³744.9	⁴373.4	¹1.0452
⁴⁶0.55741	³1.5292	⁷0.9648	⁴0.5644	⁴860.5	⁴399.7	**245**	⁸1.880	¹1203.3	²830.2	¹85.10	²745.1	³373.1	²1.0452
⁴⁷0.55694	²1.5294	⁷0.9655	⁷0.5639	⁴860.1	³399.3	**244**	⁸1.888	¹1203.2	³830.5	¹85.09	³745.4	⁴372.7	¹1.0451
¹¹⁷0.55659	⁸1.5297	²⁰0.9662	¹²0.5635	³859.8	**399**	**243** ²·⁷242.3	²¹1.895	¹1203.1	³830.8	¹85.09	³745.7	¹·¹372.3	⁰1.0451
³⁵0.55659	³1.5297	⁷0.9662	⁴0.5635	³859.8	³399.0	**242**	⁷1.895	¹1203.1	³830.8	¹85.08	³745.7	⁴372.3	¹1.0451
⁴⁶0.55613	³1.5300	⁸0.9670	⁴0.5631	⁴859.4	⁴398.6	**241**	⁸1.903	¹1203.0	³831.1	¹85.08	³746.0	⁴371.9	⁰1.0450
⁴⁷0.55566	³1.5303	⁷0.9677	⁵0.5626	⁴859.0	⁴398.2	**240**	⁸1.911	¹1202.9	⁷831.3	¹85.07	³746.3	⁴371.5	¹1.0450
¹¹⁶0.55543	⁸1.5305	²⁰0.9682	¹²0.5623	¹·⁰858.8	**398**		²¹1.916	³1202.8	³831.6	¹85.06	³746.5	¹·⁰371.3	⁰1.0449
³⁵0.55531	³1.5305	⁷0.9684	⁴0.5622	³858.7	³397.9		⁷1.918	¹1202.8	³831.8	³85.05	²746.5	³371.2	²1.0448
⁴⁶0.55485	³1.5308	⁸0.9692	⁵0.5617	⁴858.3	⁴397.5		⁸1.926	¹1202.7	³832.1	¹85.05	³746.8	⁴370.8	²1.0448
⁴⁷0.55438	³1.5311	⁷0.9699	⁴0.5613	⁴857.9	⁴397.1		⁸1.934	²1202.5		²85.04 ⁸⁵·⁰²	³747.1	⁴370.4	¹1.0446

116 0.55427	8 1.5313	20 0.9702	12 0.5611	1.0 857.8	**397**	2.7 239.6	21 1.937	3 1202.5	7 832.2	3 85.02	7.2 747.2	1.1 370.2	2 1.0446
35 0.55403	3 1.5314	7 0.9706	4 0.5609	3 857.6	3 396.8	**239**	8 1.942	1 1204.2	3 832.4	1 85.01	3 747.4	4 370.0	0 1.0446
46 0.55357	3 1.5317	8 0.9714	5 0.5604	4 857.2	4 396.4	**238**	7 1.949	4 1202.3	3 832.7	1 85.00	3 747.7	3 369.6	1 1.0445
47 0.55310	3 1.5320	7 0.9721	5 0.5599	6 856.8	4 396.0	**237**	7 1.957	2 1202.2	7 833.0	4 84.99	7 748.0	6 369.2	1 1.0444
117 0.55310	7 1.5320	20 0.9722	12 0.5599	1.0 856.8	**396**	2.7 236.9	21 1.958	3 1202.2	8 833.0	4 84.98	8 748.0	1.0 369.2	2 1.0444
35 0.55275	3 1.5323	8 0.9728	4 0.5595	3 856.5	3 395.7	**236**	8 1.965	1 1202.1	2 833.2	2 84.97	3 748.3	4 368.8	1 1.0443
47 0.55228	3 1.5326	8 0.9736	4 0.5591	5 856.1	4 395.3	**235**	8 1.973	2 1202.0	3 833.5	1 84.96	2 748.5	4 368.4	1 1.0443
117 0.55193	8 1.5328	20 0.9742	13 0.5586	1.0 855.8	**395**	2.7 234.2	22 1.980	3 1201.9	7 833.7	6 84.95	5 748.8	1.1 368.1	2 1.0442
47 0.55181	3 1.5329	7 0.9743	5 0.5586	4 855.7	4 394.9	**234**	9 1.982	1 1201.9	8 833.8	3 84.95	8 748.8	4 368.0	1 1.0442
35 0.55146	2 1.5331	5 0.9751	5 0.5581	7 855.4	4 394.6	**233**	8 1.990	3 1201.8	2 834.0	4 84.94	3 749.1	3 367.7	1 1.0441
46 0.55100	3 1.5334	7 0.9758	4 0.5577	4 855.0	4 394.2	**232**	8 1.998	1 1201.7	3 834.3	1 84.93	3 749.4	3 367.3	1 1.0440
117 0.55076	8 1.5336	20 0.9762	12 0.5574	1.0 854.8	**394**	2.7 231.5	22 2.002	2 1201.6	5 834.5	3 84.92	8 749.6	1.0 367.1	2 1.0440
47 0.55053	3 1.5337	7 0.9765	5 0.5572	6 854.6	4 393.8	**231**	8 2.006	5 1201.5	6 834.6	2 84.91	6 749.7	4 366.9	1 1.0440
47 0.55006	3 1.5340	8 0.9773	5 0.5568	4 854.2	4 393.4	**230**	8 2.014	1 1201.4	3 834.9	1 84.90	3 750.0	5 366.5	1 1.0439
35 0.54971	3 1.5343	7 0.9780	5 0.5563	3 853.9	3 393.1	**229**	8 2.022	3 1201.3	2 835.1	3 84.89	2 750.2	4 366.1	8 1.0438
20 0.54959	8 1.5344	20 0.9782	12 0.5562	1.0 853.8	**393**	2.7 228.8	22 2.024	3 1201.3	7 835.2	3 84.89	7 750.3	1.1 366.0	8 1.0438
47 0.54924	3 1.5346	8 0.9788	5 0.5558	4 853.5	4 392.7	**228**	9 2.031	2 1201.2	3 835.4	1 84.88	3 750.5	4 365.7	1 1.0437
47 0.54877	3 1.5349	7 0.9795	4 0.5554	4 853.1	4 392.3	**227**	8 2.039	1 1201.1	7 835.7	1 84.87	4 750.9	4 365.3	0 1.0437
117 0.54842	2 1.5352	20 0.9802	5 0.5550	1.0 852.8	**392**	2.7 226.1	22 2.046	3 1201.0	0 836.0	3 84.86	8 751.1	1.0 365.0	1 1.0436

0.54842	1.5352	0.9803	0.5549	852.8	392.0	**226**	2.047	1201.0	836.0	84.86	751.2	365.0	1.0436
0.54795	1.5355	0.9811	0.5545	852.4	391.6	**225**	2.056	1200.9	836.3	84.85	751.5	364.6	1.0436
0.54748	1.5358	0.9818	0.5540	852.0	391.2	**224** 223.5	2.064	1200.8	836.6	84.84	751.8	364.2	1.0436
0.54725	1.5360	0.9822	0.5538	851.8	**391**	**223**	2.069	1200.7	836.7	84.83	751.9	364.0	1.0436
0.54701	1.5361	0.9826	0.5535	851.6	390.8	**222**	2.073	1200.6	836.8	84.82	752.0	363.8	1.0436
0.54654	1.5364	0.9834	0.5530	851.2	390.4	**221** 220.9	2.082	1200.5	837.1	84.81	752.3	363.4	1.0436
0.54619	1.5364	0.9842	0.5526	850.9	390.1	**220**	2.091	1200.4	837.4	84.79	752.6	363.0	1.0436
0.54607	1.5368	0.9843	0.5525	850.8	**390**	**219** 218.3	2.092	1200.4	837.4	84.79	752.6	362.9	1.0436
0.54572	1.5368	0.9849	0.5521	850.5	389.7	**218**	2.100	1200.3	837.7	84.78	752.9	362.6	1.0436
0.54525	1.5371	0.9857	0.5517	850.1	389.3	**217**	2.109	1200.2	838.0	84.76	753.2	362.2	1.0436
0.54490	1.5374	0.9863	0.5513	849.8	**389**	**216** 215.7	2.115	1200.1	838.2	84.75	753.4	361.9	1.0436
0.54478	1.5376	0.9865	0.5512	849.7	388.9	**215**	2.118	1200.1	838.3	84.75	753.5	361.8	1.0436
0.54431	1.5377	0.9873	0.5507	849.3	388.5	**214** 213.2	2.127	1199.9	838.5	84.73	753.8	361.3	1.0436
0.54384	1.5380	0.9881	0.5502	848.9	388.1	**213**	2.136	1199.8	838.8	84.71	754.1	360.9	1.0436
0.54372	1.5383	0.9883	0.5501	848.8	**388**		2.139	1199.8	838.9	84.70	754.2	360.8	1.0436
0.54337	1.5384	0.9889	0.5497	848.5	387.7		2.146	1199.7	839.1	84.69	754.5	360.5	1.0436
0.54289	1.5386	0.9897	0.5492	848.1	387.3		2.155	1199.6	839.4	84.67	754.8	360.1	1.0436
0.54254	1.5389	0.9904	0.5488	847.8	**387**		2.163	1199.5	839.7	84.65	755.0	359.8	1.0436
0.54242	1.5392	0.9905	0.5487	847.7	386.9		2.165	1199.5	839.7	84.65	755.1	359.7	1.0436

47 0.54195	4 1.5396	8 0.9913	4 0.5483	4 847.3	4 386.5	**212**	9 2.174	2 1199.3	3 840.0	2 84.63	3 755.4	4 359.3	1.0436
47 0.54148	3 1.5399	8 0.9921	5 0.5478	3 846.9	4 386.1	**211**	10 2.184	1 1199.2	3 840.3	2 84.61	3 755.7	4 358.9	1.0436
118 0.54136	8 1.5400	20 0.9924	12 0.5476	1.0 846.8	**386**	2.5 210.7	24 2.187	4 1199.1	7 840.4	5 84.60	8 755.8	1.1 358.7	1.0436
47 0.54101	4 1.5403	9 0.9930	5 0.5473	4 846.5	4 385.7	**210**	10 2.194	1 1199.0	3 840.6	2 84.59	5 756.0	5 358.4	1.0436
48 0.54053	5 1.5406	8 0.9938	5 0.5468	4 846.1	4 385.3	**209**	10 2.204	9 1198.9	3 840.9	5 84.57	3 756.3	4 358.0	1.0436
118 0.54018	8 1.5408	20 0.9944	12 0.5464	1.0 845.8	**385**	2.5 208.2	25 2.212	3 1198.8	7 841.1	5 84.55	8 756.6	1.0 357.7	1.0436
47 0.54006	3 1.5409	8 0.9946	5 0.5463	4 845.7	4 384.9	**208**	10 2.214	1 1198.8	3 841.2	2 84.55	3 756.7	4 357.6	1.0436
47 0.53959	3 1.5412	9 0.9955	5 0.5458	4 845.3	4 384.5	**207**	11 2.225	1 1198.7	3 841.5	2 84.52	3 756.7	4 357.2	1.0436
48 0.53911	4 1.5416	8 0.9963	5 0.5453	4 844.9	4 384.1	**206**	10 2.235	2 1198.5	8 841.8	5 84.50	3 757.0	5 356.7	1.0436
118 0.53900	8 1.5416	21 0.9965	12 0.5452	1.0 844.8	**384**	2.4 205.8	25 2.237	1 1198.5	8 841.9	5 84.50	3 757.3	1.0 356.7	1.0436
47 0.53864	1 1.5419	8 0.9972	5 0.5447	4 844.5	4 383.7	**205**	10 2.245	1 1198.4	1 842.1	2 84.48	4 757.4	4 356.3	1.0436
59 0.53805	4 1.5423	8 0.9980	5 0.5442	4 844.0	5 383.2	**204**	11 2.256	3 1198.3	3 842.4	2 84.46	7 757.7	4 355.9	1.0436
119 0.53781	9 1.5425	20 0.9985	13 0.5439	1.0 843.8	**383**	2.4 203.4	25 2.262	2 1198.2	7 842.6	5 84.45	3 758.0	1.1 355.6	1.0436
48 0.53757	6 1.5426	8 0.9989	5 0.5437	4 843.6	4 382.8	**203**	11 2.266	1 1198.1	3 842.7	2 84.44	8 758.2	5 355.4	1.0436
47 0.53710	4 1.5430	9 0.9998	5 0.5432	4 843.2	4 382.4	**202**	11 2.277	1 1198.0	3 843.0	2 84.42	3 758.3	4 355.0	1.0436
119 0.53662	8 1.5433	21 1.0006	12 0.5427	1.0 842.8	**382**	2.4 201.0	25 2.287	3 1197.9	7 843.3	5 84.40	6 758.6	1.0 354.6	1.0436
48 0.53662	3 1.5433	9 1.0007	5 0.5427	4 842.8	4 382.0	**201**	11 2.288	1 1197.9	3 843.3	2 84.40	7 758.9	4 354.5	1.0436
47 0.53615	4 1.5437	8 1.0016	6 0.5421	4 842.4	4 381.6	**200**	11 2.299	1 1197.8	6 843.6	2 84.38	3 758.9	4 354.1	1.0436
59 0.53556	3 1.5440	8 1.0024	5 0.5416	5 841.9	5 381.1	**199**	2.310	1 1197.7	4 844.0	84.36	759.5	3 353.7	1.0436

^{118}0.53544	^{8}1.5441	^{21}1.0027	^{13}0.5414	$^{1.0}$841.8	**381**^{4}	$^{2.3}$198.7	^{26}2.313	^{3}1197.6	^{8}844.1	^{5}84.35	^{8}759.7	$^{1.1}$353.5	1.0436
^{48}0.53508	^{4}1.5444	^{9}1.0033	^{5}0.5411	^{4}841.5	380.7	**198**	^{10}2.320	^{1}1197.5	^{3}844.3	^{2}84.34	^{4}759.9	^{5}353.2	1.0436
^{47}0.53461	^{3}1.5447	^{9}1.0042	^{6}0.5405	^{4}841.1	380.3	**197**	^{11}2.331	^{1}1197.4	^{3}844.6	^{3}84.31	^{4}760.3	^{4}352.8	1.0436
^{119}0.53425	^{8}1.5449	^{20}1.0047	^{12}0.5402	$^{1.0}$840.8	**380**	$^{2.4}$196.3	^{25}2.338	^{3}1197.3	^{7}844.8	^{4}84.30	^{8}760.5	$^{1.0}$352.5	1.0436
^{60}0.53401	^{5}1.5450	^{8}1.0050	^{5}0.5400	^{4}840.6	379.8	**196**	^{12}2.342	^{2}1197.3	^{3}844.9	^{2}84.29	^{3}760.6	^{5}352.3	1.0436
^{47}0.53354	^{4}1.5454	^{9}1.0059	^{5}0.5395	^{4}840.2	379.4	**195**	^{12}2.354	^{1}1197.1	^{4}845.3	^{1}84.28	^{4}761.0	^{4}351.9	1.0436
^{119}0.53306	^{9}1.5458	^{21}1.0068	^{12}0.5390	$^{1.0}$839.8	**379**	$^{2.3}$194.0	^{28}2.366	^{3}1197.0	^{8}845.6	^{4}84.26	^{8}761.3	$^{1.1}$351.4	1.0436
^{48}0.53306	^{8}1.5458	^{8}1.0068	^{5}0.5390	^{4}839.8	379.0	**194**	^{12}2.366	^{1}1197.0	^{3}845.6	^{2}84.26	^{3}761.3	^{5}351.4	1.0436
^{48}0.53258	^{3}1.5461	^{9}1.0077	^{6}0.5384	^{4}839.4	378.6	**193**	^{12}2.378	^{1}1196.9	^{9}845.9	^{2}84.24	^{3}761.6	^{4}351.0	1.0436
^{59}0.53199	^{4}1.5465	^{21}1.0086	^{5}0.5379	^{5}838.9	378.1	**192**	^{27}2.389	^{2}1196.7	^{2}846.2	^{1}84.23	^{4}762.0	^{5}350.5	1.0436
^{119}0.53187	^{5}1.5466	^{8}1.0089	^{13}0.5377	$^{1.0}$838.8	**378**	$^{2.3}$191.7	^{12}2.393	^{3}1196.7	^{7}846.3	^{4}84.22	^{8}762.1	$^{1.0}$350.4	1.0436
^{48}0.53151	^{4}1.5469	^{9}1.0095	^{5}0.5374	^{4}838.5	377.7	**191**	^{12}2.401	^{1}1196.6	^{3}846.5	^{2}84.21	^{3}762.3	^{4}350.1	1.0436
^{48}0.53103	^{3}1.5472	^{9}1.0104	^{6}0.5368	^{4}838.1	377.3	**190**	^{12}2.413	^{2}1196.4	^{8}846.8	^{2}84.19	^{7}762.6	^{5}349.6	1.0436
^{120}0.53067	^{8}1.5474	^{21}1.0110	^{12}0.5365	$^{1.0}$837.8	**377**	$^{2.3}$189.4	^{27}2.420	^{4}1196.3	^{7}847.0	^{1}84.18	^{8}762.8	$^{1.1}$349.3	1.0436
^{59}0.53044	^{3}1.5475	^{9}1.0113	^{5}0.5362	^{5}837.6	376.8	**189**	^{12}2.425	^{1}1196.3	^{3}847.1	^{2}84.17	^{3}762.9	^{4}349.2	1.0436
^{48}0.52996	^{4}1.5479	^{22}1.0122	^{5}0.5357	^{4}837.2	376.4	**188**	^{12}2.437	^{2}1196.1	^{3}847.4	^{1}84.16	^{5}763.3	^{5}348.7	1.0436
^{48}0.52948	^{3}1.5483	^{20}1.0130	^{28}0.5352	$^{1.0}$836.8	**376**	$^{2.3}$187.1	^{28}2.448	^{3}1196.0	^{7}847.7	^{4}84.14	^{8}763.6	$^{1.0}$348.3	1.0436
^{60}0.52936	^{4}1.5483	^{9}1.0131	^{5}0.5352	^{5}836.7	375.9	**187**	^{12}2.449	^{1}1196.0	^{3}847.7	^{2}84.14	^{3}763.6	^{5}348.2	1.0436
^{48}0.52888	^{4}1.5487	^{10}1.0141	^{6}0.5346	^{4}836.3	375.5	**186**	^{13}2.462	^{1}1195.9	^{4}848.1	^{2}84.12	^{4}764.0	^{4}347.8	1.0436

0.52840^{48}	1.5490	1.0150^{9}	0.5341^{5}	835.9	375.1	**185**	2.474^{12}	1195.8	848.4	84.10	764.3	347.3	1.0436
0.52828^{120}	1.5491	1.0151^{21}	0.5340^{12}	835.8	**375**	184.8	2.476^{28}	1195.7	848.5	84.10	764.4	347.3	1.0436
0.52781^{59}	1.5494	1.0159^{9}	0.5335^{6}	835.4	374.6	**184**	2.487^{13}	1195.6	848.7	84.09	764.7	346.9	1.0436
0.52733^{119}	1.5498	1.0168^{9}	0.5329^{13}	835.0	374.2	**183**	2.500^{13}	1195.5	849.1	84.07	765.0	346.4	1.0436
0.52709^{60}	1.5499	1.0172^{21}	0.5327^{6}	834.8	**374**	182.6	2.505^{29}	1195.4	849.2	84.06	765.1	346.2	1.0436
0.52673^{48}	1.5502	1.0178^{10}	0.5323^{5}	834.5	373.7	**182**	2.513^{13}	1195.3	849.4	84.05	765.3	345.9	1.0436
0.52625^{120}	1.5506	1.0188^{10}	0.5318^{5}	834.1	373.3	**181**	2.526^{13}	1195.2	849.7	84.03	765.7	345.4	1.0436
0.52589^{60}	1.5508	1.0193^{21}	0.5315^{12}	833.8	**373**	180.4	2.534^{29}	1195.1	849.9	84.02	765.9	345.2	1.0436
0.52565^{60}	1.5510	1.0197^{9}	0.5312^{6}	833.6	372.8	**180**	2.540^{14}	1195.0	850.1	84.01	766.1	345.0	1.0436
0.52505^{120}	1.5514	1.0207^{10}	0.5306^{13}	833.1	372.3	**179**	2.554^{29}	1194.9	850.4	83.99	766.4	344.5	1.0436
0.52469^{60}	1.5516	1.0214^{21}	0.5302^{6}	832.8	**372**	178.3	2.563^{14}	1194.8	850.7	83.98	766.7	344.1	1.0436
0.52445^{48}	1.5517	1.0217^{10}	0.5300^{6}	832.6	371.8	**178**	2.568^{14}	1194.8	850.8	83.97	766.8	344.0	1.0436
0.52397^{120}	1.5521	1.0227^{10}	0.5294^{12}	832.2	371.4	**177**	2.582^{30}	1194.6	851.1	83.95	767.2	343.5	1.0436
0.52349^{60}	1.5525	1.0235^{21}	0.5290^{5}	831.8	**371**	176.2	2.593^{14}	1194.5	851.4	83.94	767.5	343.1	1.0436
0.52337^{60}	1.5526	1.0237^{10}	0.5289^{6}	831.7	370.9	**176**	2.596^{14}	1194.5	851.5	83.93	767.6	343.0	1.0436
0.52277^{121}	1.5530	1.0247^{21}	0.5283^{13}	831.2	370.4	**175**	2.610^{30}	1194.3	851.9	83.91	768.0	342.5	1.0436
0.52228^{49}	1.5533	1.0256^{10}	0.5277^{6}	830.8	**370**	174.1	2.623^{14}	1194.2	852.2	83.89	768.3	342.0	1.0436
	1.5534	1.0257^{10}	0.5277^{6}	830.8	370.0	**174**	2.624^{15}	1194.2	852.2	83.89	768.3	342.0	1.0436
0.52168^{60}	1.5538	1.0267	0.5271	830.3	369.5	**173**	2.639	1194.0	852.6	83.86	768.7	341.5	1.0436

0.52108^{60}	1.5542^{4}	1.0277^{10}	0.5265^{6}	829.8^{5}	369.0^{5}	**172**	2.653^{14}	1193.9^{1}	852.9^{3}	83.84^{2}	769.1^{4}	341.0^{5}	1.0436
0.52108^{120}	1.5542^{9}	1.0278^{22}	0.5265^{12}	829.8^{10}	**369**	$172.0^{2.1}$	2.654^{31}	1193.9^{3}	852.9^{7}	83.84^{5}	769.1^{8}	341.0^{10}	1.0436
0.52108^{60}	1.5546^{4}	1.0287^{10}	0.5259^{6}	829.3^{5}	368.5^{5}	**171**	2.668^{15}	1193.8^{1}	853.2^{3}	83.81^{3}	769.4^{3}	340.5^{5}	1.0436
0.52048^{60}	1.5550^{6}	1.0297^{10}	0.5253^{6}	828.9^{5}	368.1^{5}	**170**	2.682^{14}	1193.6^{2}	853.6^{4}	83.79^{2}	769.8^{4}	340.0^{5}	1.0436
0.51999^{49}	1.5551^{9}	1.0299^{21}	0.5252^{13}	828.8^{10}	**368**	$169.8^{2.2}$	2.685^{32}	1193.6^{3}	853.6^{7}	83.78^{3}	769.9^{8}	340.0^{10}	1.0436
0.51987^{121}	1.5554^{4}	1.0307^{10}	0.5247^{6}	828.4^{5}	367.6^{5}	**169**	2.697^{15}	1193.5^{1}	853.9^{3}	83.76^{3}	770.2^{4}	339.5^{5}	1.0436
0.51939^{60}	1.5558^{4}	1.0317^{10}	0.5241^{6}	827.9^{5}	367.1^{5}	**168**	2.712^{15}	1193.3^{1}	854.3^{4}	83.76^{3}	770.6^{4}	339.0^{5}	1.0436
0.51879^{60}	1.5560^{9}	1.0320^{21}	0.5239^{13}	827.8^{10}	**367**	$167.7^{2.1}$	2.716^{31}	1193.3^{3}	854.4^{3}	83.73^{3}	770.7^{8}	338.9^{11}	1.0436
0.51867^{120}	1.5563^{5}	1.0328^{11}	0.5235^{6}	827.5^{4}	366.7^{4}	**167**	2.727^{15}	1193.1^{2}	854.6^{4}	83.72^{3}	770.9^{3}	338.5^{5}	1.0436
0.51830^{49}	1.5567^{4}	1.0338^{22}	0.5229^{12}	827.0^{5}	366.2^{5}	**166**	2.742^{15}	1193.1^{1}	855.0^{4}	83.70^{3}	771.3^{4}	338.1^{4}	1.0436
0.51770^{60}	1.5569^{9}	1.0342^{10}	0.5227^{6}	826.8^{10}	**366**	$165.6^{2.1}$	2.748^{32}	1193.0^{2}	855.1^{7}	83.67^{6}	771.5^{8}	337.9^{10}	1.0436
0.51746^{121}	1.5571^{5}	1.0348^{10}	0.5223^{6}	826.5^{5}	365.7^{5}	**165**	2.757^{15}	1192.9^{2}	855.3^{3}	83.66^{3}	771.7^{4}	337.6^{5}	1.0436
0.51710^{60}	1.5575^{4}	1.0358^{10}	0.5217^{13}	826.1^{5}	365.3^{5}	**164**	2.772^{15}	1192.8^{1}	855.6^{3}	83.64^{2}	772.1^{4}	337.1^{5}	1.0436
0.51661^{49}	1.5578^{4}	1.0363^{21}	0.5214^{6}	825.8^{10}	**365**	$163.5^{2.1}$	2.780^{32}	1192.7^{3}	855.8^{4}	83.62^{3}	772.3^{8}	336.8^{11}	1.0436
0.51625^{121}	1.5579^{5}	1.0368^{10}	0.5211^{6}	825.6^{5}	364.8^{5}	**163**	2.787^{15}	1192.6^{2}	856.0^{4}	83.60^{3}	772.4^{3}	336.6^{5}	1.0436
0.51601^{60}	1.5584^{9}	1.0379^{11}	0.5205^{13}	825.1^{5}	364.3^{5}	**162**	2.803^{16}	1192.5^{1}	856.4^{4}	83.59^{3}	772.8^{4}	336.1^{5}	1.0436
0.51540^{61}	1.5586^{5}	1.0385^{22}	0.5201^{6}	824.8^{10}	**364**	$161.4^{2.1}$	2.813^{33}	1192.4^{3}	856.6^{3}	83.56^{3}	773.1^{8}	335.8^{10}	1.0436
0.51504^{121}	1.5588^{4}	1.0389^{10}	0.5199^{6}	824.6^{5}	363.8^{5}	**161**	2.819^{16}	1192.3^{2}	856.7^{4}	83.54^{3}	773.2^{4}	335.6^{5}	1.0436
0.51479^{61}	1.5592^{4}	1.0399^{10}	0.5193	824.1	363.3	**160**	2.835^{16}	1192.2^{1}	857.1	83.53^{3}	773.6	335.1	1.0436
0.51419^{60}										83.50			

C1	C2	C3	C4	C5	C6	C7	C8	C9	C10	C11	C12	C13	C14
¹²²0.51382	⁹1.5595	²¹1.0406	¹²0.5189	¹·⁰823.8	**363**	²·¹159.3	³³2.846	³1192.1	⁷857.3	⁶83.48	⁸773.9	¹·¹334.7	1.0436
⁶¹0.51358	⁵1.5597	¹¹1.0410	⁶0.5187	823.6	⁵362.8	**159**	¹⁶2.851	¹1192.1	³857.4	³83.47	⁴774.0	⁵334.6	1.0436
⁶¹0.51297	⁴1.5601	¹¹1.0421	¹³0.5181	⁵823.1	⁵362.3	**158**	¹⁷2.868	²1191.9	⁴857.8	³83.44	⁴774.4	⁵334.1	1.0436
¹²¹0.51261	⁹1.5604	²²1.0428	⁷0.5176	¹·⁰822.8	**362**	²·⁰157.3	³⁴2.880	³1191.8	⁸857.8	⁶83.42	⁸774.7	¹·⁰333.7	1.0436
⁶⁰0.51237	⁹1.5606	¹¹1.0432	⁶0.5174	822.6	⁵361.8	**157**	¹⁷2.885	¹1191.8	²858.1	³83.41	⁷774.8	⁵333.6	1.0436
⁶¹0.51176	⁴1.5610	¹¹1.0443	¹³0.5168	822.1	⁵361.3	**156**	¹⁷2.902	²1191.6	³858.2	³83.38	⁴775.2	⁵333.5	1.0436
¹²²0.51139	⁹1.5613	²²1.0450	⁷0.5163	¹·⁰821.8	**361**	²·⁰155.3	³⁴2.914	³1191.5	⁷858.5	⁶83.36	⁸775.5	¹·¹333.0	1.0436
⁶¹0.51115	⁵1.5615	¹¹1.0454	⁶0.5161	⁵821.6	⁵360.8	**155**	¹⁸2.920	¹1191.4	⁴858.8	³83.35	⁴775.6	⁵332.6	1.0436
⁶¹0.51054	⁴1.5619	¹¹1.0465	¹²0.5155	821.1	⁵360.3	**154**	³⁵2.938	²1191.3	⁴859.3	³83.32	⁸776.0	⁵332.5	1.0436
¹²²0.51017	⁹1.5622	²²1.0472	⁷0.5151	¹·⁰820.8	**360**	¹·⁹153.4	¹⁸2.949	³1191.2	³859.6	⁶83.30	³776.3	¹·⁰332.0	1.0436
⁶¹0.50993	⁵1.5624	¹²1.0477	¹³0.5148	820.6	⁵359.8	**153**	¹⁸2.956	¹1191.1	⁴859.7	³83.29	⁸776.4	⁵331.6	1.0436
⁶¹0.50932	⁹1.5629	¹¹1.0488	⁷0.5141	820.1	⁵359.3	**152**	³⁵2.974	²1191.0	⁷860.1	³83.26	⁴776.9	⁵331.4	1.0436
¹²²0.50895	⁹1.5631	²¹1.0493	¹³0.5138	¹·⁰819.8	**359**	¹·⁹151.5	¹⁹2.984	³1190.9	⁴860.3	⁶83.24	⁸777.1	¹·⁰330.9	1.0436
⁷³0.50859	⁵1.5634	¹¹1.0499	⁷0.5135	⁶819.6	⁵358.7	**151**	¹⁹2.993	¹1190.8	⁴860.5	³83.23	³777.3	⁵330.6	1.0436
⁶¹0.50798	⁷1.5638	²²1.0511	¹³0.5128	⁵819.0	⁵358.2	**150**	³⁶3.012	²1190.7	⁷860.9	⁶83.19	⁸777.7	⁵330.3	1.0436
¹²²0.50773	⁹1.5640	¹²1.0515	⁷0.5125	818.8	**358**	¹·⁹149.6	²⁰3.020	³1190.6	³861.0	³83.18	⁷777.9	¹·¹329.7	1.0436
⁶¹0.50737	⁵1.5643	¹²1.0523	¹²0.5121	818.5	⁵357.7	**149**	²⁰3.032	¹1190.5	⁴861.3	³83.16	⁸778.1	⁵329.5	1.0436
⁷³0.50664	¹⁰1.5649	²²1.0535	¹³0.5114	¹·⁰817.9	⁶357.1	**148**	¹⁹3.051	³1190.4	⁷861.7	⁶83.13	⁸778.6	²329.2	1.0436
¹²²0.50651	1.5650	1.0537	0.5112	817.8	**357**	¹·⁸147.8	³⁶3.056	³1190.3	⁸861.8	³83.12	778.7	¹·⁰328.6	1.0436

0.50602 (62)	1.5654 (5)	1.0547 (12)	0.5107 (7)	817.4 (5)	356.6 (5)	**147**	3.071 (20)	1190.2 (2)	862.1 (4)	83.09 (4)	779.0 (4)	328.0 (6)	1.0436
0.50529 (73)	1.5659 (5)	1.0559 (12)	0.5100 (7)	816.8 (6)	356.0 (6)	**146**	3.091 (20)	1190.1 (1)	862.5 (4)	83.06 (6)	779.5 (5)	327.5 (5)	1.0436
0.50529 (122)	1.5659 (9)	1.0559 (22)	0.5100 (12)	816.8 (1.0)	**356**	1.8 140.0	3.091 (36)	1190.0 (1)	862.5 (7)	83.06 (6)	779.5 (5)	327.4 (1.1)	1.0436
0.50468 (61)	1.5664 (5)	1.0571 (12)	0.5093 (13)	816.3 (10)	355.5 (5)	**145**	3.092 (21)	1189.9 (2)	862.9 (8)	83.03 (3)	779.9 (4)	326.9 (6)	1.0436
0.50406 (123)	1.5668 (5)	1.0581 (22)	0.5087 (7)	815.8 (1.0)	**355**	1.8 144.2	3.112 (38)	1189.7 (3)	863.3 (8)	83.00 (6)	780.3 (5)	326.4 (1.0)	1.0436
0.50394 (74)	1.5669 (5)	1.0583 (12)	0.5086 (7)	815.7 (6)	354.9 (6)	**144**	3.130 (21)	1189.7 (2)	863.3 (4)	82.99 (4)	780.4 (4)	326.3 (6)	1.0436
0.50321 (73)	1.5674 (10)	1.0596 (13)	0.5079 (13)	815.1 (1.0)	354.3 (6)	**143**	3.133 (22)	1189.5 (3)	863.7 (7)	82.96 (6)	780.8 (8)	325.7 (6)	1.0436
0.50284 (122)	1.5678 (6)	1.0608 (22)	0.5071 (8)	814.8 (1.0)	**354**	1.8 142.4	3.155 (38)	1189.4 (3)	864.0 (7)	82.94 (3)	781.0 (8)	325.3 (1.0)	1.0436
0.50259 (62)	1.5680 (5)	1.0621 (12)	0.5064 (7)	814.6 (6)	353.8 (5)	**142**	3.168 (21)	1189.3 (2)	864.2 (5)	82.93 (4)	781.3 (4)	325.1 (6)	1.0436
0.50186 (73)	1.5685 (5)	1.0626 (13)	0.5061 (13)	814.0 (1.0)	353.2 (6)	**141**	3.176 (21)	1189.2 (1)	864.6 (6)	82.89 (6)	781.7 (8)	324.5 (1.0)	1.0436
0.50161 (123)	1.5687 (5)	1.0633 (23)	0.5057 (7)	813.8 (1.0)	**353**	1.0 140.6	3.197 (38)	1189.1 (1)	864.8 (8)	82.88 (3)	781.9 (4)	324.3 (6)	1.0436
0.50124 (62)	1.5690 (5)	1.0646 (12)	0.5050 (13)	813.5 (5)	352.7 (6)	**140**	3.206 (22)	1189.0 (2)	865.0 (4)	82.86 (6)	782.2 (5)	324.0 (6)	1.0436
0.50050 (74)	1.5695 (5)	1.0648 (13)	0.5048 (8)	812.9 (6)	352.1 (6)	**139**	3.219 (22)	1188.8 (2)	865.4 (7)	82.83 (3)	782.6 (8)	323.4 (1.0)	1.0436
0.50038 (123)	1.5696 (5)	1.0658 (22)	0.5042 (12)	812.8 (1.0)	**352**	1.8 138.8	3.241 (39)	1188.8 (1)	865.5 (7)	82.82 (4)	782.7 (5)	323.3 (6)	1.0436
0.49976 (74)	1.5700 (10)	1.0670 (12)	0.5036 (7)	812.3 (6)	351.5 (5)	**138**	3.245 (22)	1188.7 (3)	865.9 (8)	82.79 (5)	783.1 (8)	322.8 (1.1)	1.0436
0.49915 (123)	1.5706 (5)	1.0671 (22)	0.5036 (13)	811.8 (1.0)	**351**	1.8 137.0	3.263 (39)	1188.5 (3)	866.3 (4)	82.76 (4)	783.5 (4)	322.2 (6)	1.0436
0.49915 (61)	1.5706 (5)	1.0683 (13)	0.5036 (8)	811.8 (1.0)	351.0 (6)	**137**	3.284 (22)	1188.5 (2)	866.3 (4)	82.76 (3)	783.5 (5)	322.2 (6)	1.0436
0.49841 (74)	1.5711 (9)	1.0683 (12)	0.5028 (8)	811.2 (1.0)	350.4 (6)	**136**	3.307 (40)	1188.3 (3)	866.7 (7)	82.72 (4)	784.0 (8)	321.6 (1.0)	1.0436
0.49792 (123)	1.5715	1.0693 (23)	0.5023 (13)	810.8	**350**	1.8 135.2	3.324	1188.2	867.0	82.69	784.3	321.2	1.0436

1.0436	321.0 (6)	784.4 (4)	82.68 (3)	867.1 (4)	1188.2 (1)	3.329 (22)	135	349.9 (5)	810.7 (5)	0.5021 (7)	1.0696 (13)	1.5716 (5)	0.49779 (62)
1.0436	320.5 (5)	784.9 (5)	82.65 (6)	867.6 (5)	1188.0 (2)	3.352 (23)	134 (1.8)	349.3 (6)	810.1 (6)	0.5014 (13)	1.0708 (12)	1.5722 (6)	0.49705 (74)
1.0436	320.1 (1.1)	785.1 (1)	82.63 (3)	867.8 (8)	1187.9 (3)	3.365 (41)	133.4	349	809.8 (1.0)	0.5010 (7)	1.0715 (22)	1.5725 (10)	0.49668 (124)
1.0436	319.9 (6)	785.3 (4)	82.61 (4)	868.0 (4)	1187.8 (1)	3.375 (23)	133	348.7 (6)	809.5 (6)	0.5007 (8)	1.0721 (13)	1.5728 (6)	0.49631 (74)
1.0436	319.3 (9)	785.8 (5)	82.57 (4)	868.4 (4)	1187.7 (1)	3.399 (24)	132 (1.7)	348.2 (6)	809.0 (6)	0.4999 (13)	1.0734 (13)	1.5733 (9)	0.49569 (62)
1.0436	319.1 (1.0)	785.9 (8)	82.56 (5)	868.5 (7)	1187.6 (2)	3.407 (42)	131.7	348	808.8 (6)	0.4997 (7)	1.0738 (23)	1.5734 (5)	0.49545 (123)
1.0436	318.7 (6)	786.3 (5)	82.53 (4)	868.8 (8)	1187.5 (3)	3.423 (24)	131	347.6 (6)	808.4 (6)	0.4992 (7)	1.0747 (12)	1.5738 (6)	0.49495 (74)
1.0436	318.1 (6)	786.7 (3)	82.49 (4)	869.3 (5)	1187.3 (2)	3.447 (24)	130 (1.8)	347.1 (5)	807.9 (5)	0.4985 (13)	1.0759 (6)	1.5744 (5)	0.49433 (62)
1.0436	318.0 (1.1)	786.8 (7)	82.49 (7)	869.3 (8)	1187.3 (3)	3.449 (42)	129.9	347	807.8 (6)	0.4984 (8)	1.0760 (22)	1.5744 (10)	0.49421 (124)
1.0436	317.5 (6)	787.2 (8)	82.45 (4)	869.7 (4)	1187.1 (3)	3.471 (24)	129 (1.7)	346.5 (6)	807.3 (6)	0.4977 (13)	1.0772 (13)	1.5749 (5)	0.49359 (74)
1.0436	317.0 (1.0)	787.6 (5)	82.42 (8)	870.0 (7)	1187.0 (2)	3.491 (42)	128.2	346	806.8 (1.0)	0.4971 (7)	1.0783 (23)	1.5754 (10)	0.49297 (23)
1.0436	316.9 (6)	787.7 (6)	82.41 (4)	870.1 (4)	1187.0 (3)	3.495 (24)	128	345.9 (6)	806.7 (6)	0.4970 (8)	1.0785 (13)	1.5755 (5)	0.49285 (74)
1.0436	316.3 (9)	788.1 (4)	82.37 (4)	870.5 (5)	1186.8 (2)	3.520 (25)	127 (1.7)	345.3 (6)	806.1 (6)	0.4962 (13)	1.0798 (13)	1.5760 (9)	0.49210 (75)
1.0436	316.0 (1.0)	788.4 (8)	82.35 (8)	870.8 (8)	1186.7 (3)	3.534 (43)	126.5	345	805.8 (1.0)	0.4958 (7)	1.0805 (22)	1.5763 (5)	0.49173 (124)
1.0436	315.7 (6)	788.6 (5)	82.33 (4)	871.0 (5)	1186.6 (2)	3.546 (26)	126	344.7 (6)	805.5 (6)	0.4955 (8)	1.0812 (14)	1.5766 (6)	0.49136 (74)
1.0436	315.1 (1.1)	789.1 (8)	82.29 (5)	871.4 (4)	1186.4 (3)	3.572 (44)	125 (1.7)	344.1 (6)	804.9 (6)	0.4947 (13)	1.0825 (13)	1.5772 (5)	0.49061 (75)
1.0436	314.9 (9)	789.2 (5)	82.28 (7)	871.5 (7)	1186.4 (4)	3.578 (26)	124.8	344	804.8 (1.0)	0.4945 (7)	1.0828 (23)	1.5773 (10)	0.49049 (124)
1.0436	314.4 (6)	789.6 (8)	82.25 (7)	871.9 (5)	1186.3 (3)	3.599 (27)	124 (1.7)	343.5 (6)	804.3 (6)	0.4939 (13)	1.0839 (14)	1.5778 (5)	0.48987 (74)
1.0436	313.9 (1.0)	790.0 (8)	82.21 (6)	872.3 (8)	1186.1 (1)	3.623 (45)	123.1	343	803.8 (1.0)	0.4932 (7)	1.0851 (23)	1.5783 (10)	0.48925 (124)

1.0436	313.8 [6]	790.1 [5]	82.21 [4]	872.3 [4]	1186.1 [2]	3.626 [27]	**123**	342.9 [6]	803.7 [6]	0.4931 [8]	1.0853 [14]	1.5784 [6]	0.48912 [75]
1.0436	313.1 [7]	790.6 [6]	82.16 [5]	872.8 [5]	1185.9 [2]	3.654 [28]	**122** [1.6]	342.3	803.1 [1.0]	0.4923 [8]	1.0867 [14]	1.5790 [10]	0.48837 [75]
1.0436	312.8 [1.1]	790.8 [8]	82.14 [7]	873.0 [7]	1185.8 [2]	3.668 [45]	121.5	**342**	802.8 [6]	0.4919 [13]	1.0874 [23]	1.5793	0.48800 [45/125]
1.0436	312.5 [6]	791.1 [5]	82.12 [4]	873.2 [4]	1185.7	3.682 [28]	**121**	341.7 [6]	802.5 [1.9]	0.4915 [8]	1.0881 [14]	1.5796 [6]	0.48763 [74]
1.0436	311.8 [7]	791.6 [6]	82.08 [5]	873.7 [5]	1185.5 [2]	3.711 [29]	**120** [1.6]	341.1	801.9 [1.0]	0.4907 [13]	1.0895 [14]	1.5802 [6]	0.48688 [75]
1.0436	311.8 [1.0]	791.6 [8]	82.07 [7]	873.7 [7]	1185.5 [3]	3.714 [46]	119.9	**341**	801.8 [1.2]	0.4906 [13]	1.0896 [22]	1.5802 [9]	0.48675 [125]
1.0436	311.2 [6]	792.1 [5]	82.03 [5]	874.2 [5]	1185.3 [2]	3.740 [29]	**119** [1.6]	340.4 [7]	801.2 [1.0]	0.4899 [13]	1.0910 [15]	1.5808 [6]	0.48601 [87]
1.0436	310.7 [1.1]	792.5 [9]	82.00 [7]	874.5 [8]	1185.2 [3]	3.760 [46]	118.3	**340** [6]	800.8 [6]	0.4893 [9]	1.0919 [23]	1.5812 [10]	0.48551 [124]
1.0436	310.5 [6]	792.7 [6]	81.99 [4]	874.6 [5]	1185.1 [2]	3.770 [30]	**118**	339.8 [7]	800.6 [6]	0.4890 [8]	1.0924 [14]	1.5814 [7]	0.48526 [75]
1.0436	309.8 [1.0]	793.2 [8]	81.94 [7]	875.1 [7]	1184.9 [2]	3.801 [31]	**117** [1.5]	339.1	799.9 [1.0]	0.4882 [13]	1.0939 [23]	1.5821 [10]	0.48438 [88]
1.0436	309.7 [6]	793.3 [5]	81.93 [5]	875.2 [5]	1184.9 [3]	3.807 [47]	116.8	**339** [6]	799.8 [6]	0.4880 [9]	1.0942 [15]	1.5822 [6]	0.48426 [125]
1.0436	309.1 [1.1]	793.7 [8]	81.89 [7]	875.6 [7]	1184.7 [2]	3.832 [31]	**116**	338.5	799.3 [1.0]	0.4873 [23]	1.0954 [23]	1.5827 [10]	0.48363 [75]
1.0436	308.6 [7]	794.1 [5]	81.86 [5]	875.9 [5]	1184.6	3.855 [48]	115.3	**338** [7]	798.8 [7]	0.4867 [9]	1.0965 [15]	1.5832 [6]	0.48301 [125]
1.0436	308.4 [7]	794.2 [6]	81.85 [7]	876.1 [7]	1184.5 [2]	3.864 [32]	**115**	337.8	798.6 [7]	0.4864 [8]	1.0969 [15]	1.5833 [7]	0.48276 [87]
1.0436	307.7 [1.0]	794.8 [6]	81.80 [5]	876.6 [5]	1184.3 [3]	3.897 [33]	**114** [1.5]	337.1	797.9 [1.0]	0.4856 [13]	1.0984 [23]	1.5840 [10]	0.48188 [88]
1.0436	307.6 [6]	794.9 [8]	81.79 [7]	876.7 [8]	1184.3	3.904 [49]	113.8	**337** [6]	797.8 [6]	0.4854 [9]	1.0988 [16]	1.5842 [7]	0.48175 [126]
1.0436	307.1 [1.0]	795.3 [5]	81.75 [5]	877.1 [5]	1184.1 [3]	3.930 [33]	**113** [1.5]	336.5	797.3 [1.0]	0.4847 [13]	1.1000 [23]	1.5847 [10]	0.48113 [75]
1.0436	306.6 [1.0]	795.7 [8]	81.72 [7]	877.4 [7]	1184.0 [2]	3.954 [50]	112.3	**336**	796.8 [6]	0.4841 [15]	1.1011 [23]	1.5852 [6]	0.48050 [125]
1.0436	306.4 [7]	795.8	81.71	877.5	1183.9	3.964 [34]	**112**	335.8	796.6	0.4838	1.1015	1.5853	0.48025 [88]

0.47937	1.5860	1.1031	0.4829	795.9	335.1	111	3.997	1183.7	878.0	81.66	796.4	305.7	1.0436
0.47924	1.5862	1.1034	0.4828	795.8	335	110.8	4.004	1183.7	878.1	81.65	796.5	305.5	1.0436
0.47861	1.5867	1.1040	0.4821	795.3	334.5	110	4.031	1183.5	878.5	81.61	796.9	305.0	1.0436
0.47799	1.5872	1.1057	0.4815	794.8	334	109.3	4.055	1183.4	878.9	81.58	797.3	304.5	1.0436
0.47773	1.5874	1.1062	0.4812	794.6	333.8	109	4.066	1183.3	879.0	81.57	797.5	304.3	1.0436
0.47685	1.5880	1.1077	0.4803	793.9	333.1	108	4.101	1183.0	879.5	81.52	798.0	303.6	1.0436
0.47673	1.5882	1.1080	0.4802	793.8	333	107.8	4.107	1183.0	879.6	81.51	798.1	303.4	1.0436
0.47597	1.5887	1.1093	0.4794	793.2	332.4	107	4.136	1183.0	880.0	81.47	798.5	302.8	1.0436
0.47547	1.5892	1.1103	0.4788	792.8	332	106.4	4.159	1182.8	880.3	81.44	798.9	302.4	1.0436
0.47521	1.5894	1.1109	0.4785	792.6	331.8	106	4.172	1182.7	880.5	81.42	799.1	302.1	1.0436
0.47433	1.5901	1.1125	0.4776	791.9	331.1	105	4.209	1182.6	881.0	81.37	799.6	301.4	1.0436
0.47420	1.5902	1.1127	0.4775	791.8	331	104.9	4.213	1182.4	881.1	81.37	799.7	301.3	1.0436
0.47345	1.5908	1.1141	0.4767	791.2	330.4	104	4.247	1182.4	881.5	81.33	800.2	300.7	1.0436
0.47294	1.5912	1.1150	0.4762	790.8	330	103.4	4.268	1182.2	881.8	81.30	800.5	300.3	1.0436
0.47256	1.5915	1.1158	0.4758	790.5	329.7	103	4.285	1182.1	882.1	81.28	800.8	300.0	1.0436
0.47167	1.5922	1.1174	0.4749	789.8	329.0	102	4.323	1182.0	882.6	81.23	801.3	299.3	1.0436
0.47167	1.5922	1.1174	0.4749	789.8	329	102.0	4.323	1181.8	882.6	81.23	801.3	299.3	1.0436
0.47079	1.5930	1.1190	0.4740	789.1	328.3	101	4.362	1181.8	883.1	81.18	801.9	298.5	1.0436
0.47041	1.5933	1.1197	0.4736	788.8	328	100.6	4.379	1181.6	883.3	81.16	802.1	298.2	1.0436

0.46990 ⁸⁹	1.5937 ⁷	1.1207 ¹⁷	0.4730 ¹⁰	788.4 ¹·⁰	327.6 ⁷	**100** ¹·⁴³	4.402 ⁴⁰	1181.4 ²	883.6 ⁷	81.13 ⁷	802.4 ⁵	297.8 ¹·⁰	1.0436
0.46914 ¹²⁷	1.5943 ¹⁰	1.1221 ²⁴	0.4722 ¹⁴	787.8 ¹·⁰	**327**	99.17	4.436 ⁵⁷	1181.2 ³	884.0 ⁵	81.09 ⁵	802.9 ⁸	297.2 ¹·⁰	1.0436
0.46901 ⁸⁹	1.5944 ⁷	1.1224 ¹⁷	0.4720 ¹⁰	787.7 ⁷	326.9 ⁸	**99**	4.443 ⁴¹	1181.2 ²	884.1 ⁶	81.08 ⁵	803.0 ⁶	297.0 ⁸	1.0436
0.46800 ¹⁰¹	1.5952 ¹¹	1.1241 ¹⁷	0.4720 ⁹	786.9 ¹·⁰	326.1 ⁵	**98** ¹·³⁸	4.485 ⁴²	1180.9 ³	884.7 ⁵	81.03 ⁵	803.6 ⁶	296.3 ¹·¹	1.0436
0.46787 ¹²⁷	1.5954 ¹¹	1.1244 ²³	0.4711 ¹³	786.8 ¹·⁰	**326**	97.79	4.494 ⁵⁸	1180.9 ³	884.8 ⁵	81.02 ⁷	803.7 ⁸	296.1 ¹·¹	1.0436
0.46711 ⁸⁹	1.5959 ¹⁰	1.1258 ¹⁷	0.4709 ¹³	786.2 ⁷	325.4 ⁸	**97**	4.528 ⁵⁸	1180.7 ³	885.2 ⁷	80.98 ⁵	804.2 ⁸	295.5 ¹·⁰	1.0436
0.46660 ¹²⁷	1.5964 ⁸	1.1268 ²⁴	0.4701 ¹³	785.8 ¹·⁰	**325**	**96** ¹·³⁶	4.552 ⁴⁴	1180.6 ²	885.5 ⁵	80.95 ⁷	804.5 ⁵	295.1 ¹·⁰	1.0436
0.46622 ⁸⁹	1.5967 ¹¹	1.1276 ¹⁸	0.4696 ¹⁰	785.5 ⁷	324.7 ⁷	96.43	4.572 ⁶⁰	1180.5 ³	885.7 ⁶	80.93 ⁵	804.8 ⁸	294.7 ¹·¹	1.0436
0.46532 ¹²⁸	1.5975 ⁹	1.1292 ²⁴	0.4691 ¹³	784.8 ⁸	**324**	**95**	4.612 ⁴⁵	1180.3 ³	886.2 ⁶	80.88 ⁶	805.3 ⁸	294.0 ¹·¹	1.0436
0.46520 ¹⁰²	1.5976 ⁸	1.1294 ¹⁸	0.4683 ⁹	784.7 ⁷	323.9 ⁷	95.10 ¹·³³	4.617 ⁴⁶	1180.3 ²	886.3 ⁸	80.88 ⁸	805.4 ⁷	294.0 ⁷	1.0436
0.46430 ⁹⁰	1.5984 ⁸	1.1312 ²⁴	0.4682 ¹⁰	784.0 ¹·⁰	323.2 ⁸	**94** ¹·³²	4.663 ⁶¹	1180.1 ³	886.9 ⁸	80.82 ⁶	806.1 ⁹	293.2 ¹·⁰	1.0436
0.46405 ¹²⁷	1.5985 ¹¹	1.1331 ¹⁹	0.4672 ¹⁴	783.8 ⁸	**323**	93.78	4.673 ⁴⁷	1180.0 ³	887.0 ⁵	80.81 ⁷	806.2 ⁶	293.0 ¹·⁰	1.0436
0.46328 ¹⁰²	1.5992 ¹¹	1.1340 ²⁴	0.4669 ¹¹	783.2 ⁸	322.4 ⁸	**93**	4.710 ⁶²	1179.8 ³	887.4 ⁶	80.77 ⁷	806.7 ⁸	292.4 ¹·¹	1.0436
0.46277 ¹²⁸	1.5996 ¹¹	1.1349 ¹⁸	0.4661 ¹³	782.8 ⁸	**322**	**92** ¹·²⁹	4.735 ⁴⁹	1179.7 ³	887.6 ⁸	80.74 ⁷	807.0 ⁶	291.9 ⁹	1.0436
0.46226 ¹⁰²	1.6000 ⁹	1.1364 ²⁴	0.4656 ¹³	782.4 ¹·⁰	321.6 ⁸	92.49	4.759 ⁶³	1179.6 ³	888.0 ⁸	80.71 ⁵	807.3 ³	291.5 ¹·⁰	1.0432 ⁴
0.46149 ¹²⁸	1.6007 ⁸	1.1368 ¹⁹	0.4651 ¹³	781.8 ⁸	**321**	**91** ¹·²⁷	4.798 ⁴⁹	1179.4 ³	888.5 ⁶	80.67 ⁶	807.8 ⁸	290.9 ⁹	1.0427 ⁹
0.46124 ¹⁰²	1.6009 ¹⁰	1.1388 ²⁰	0.4643 ¹⁰	781.6 ⁸	320.8 ⁸	91.22	4.709 ⁵⁰	1179.3 ³	888.6 ⁷	80.66 ⁷	807.9 ⁶	290.7 ¹·⁰	1.0424 ¹²
0.46021 ¹⁰³	1.6017 ⁸	1.1388 ²⁴	0.4641 ¹¹	780.8 ⁸	**320**	**90** ¹·²⁵	4.760 ⁶³	1179.1 ³	889.2 ⁶	80.60 ⁶	808.6 ⁶	289.9 ¹·⁰	1.0412 ¹⁵
0.46021 ¹²⁸	1.6017 ¹⁰	1.1388 ²⁴	0.4630 ¹⁴	780.8	320.0	89.97	4.861	1179.1	889.2	80.60	808.6	289.9	**1.0412**

Col 1	Col 2	Col 3	Col 4	Col 5	Col 6	Col 7	Col 8	Col 9	Col 10	Col 11	Col 12	Col 13	Col 14
102 0.45919	9 1.6026	19 1.1407	11 0.4619	8 780.0	8 319.2	89 (1.24)	52 4.812	2 1178.9	6 889.8	5 80.55	6 809.2	8 289.1	13 1.0399
128 0.45893	11 1.6028	24 1.1412	13 0.4616	1.0 779.8	8 319	88.73	64 4.925	3 1178.8	8 890.0	7 80.53	8 809.4	1.1 288.8	16 1.0396
103 0.45816	11 1.6034	19 1.1426	11 0.4608	1.0 779.2	8 318.4	88 (1.23)	52 4.964	3 1178.6	6 890.4	6 80.49	9 809.9	9 288.2	13 1.0386
128 0.45765	11 1.6039	24 1.1436	13 0.4603	8 778.8	8 318	87.50	66 4.991	3 1178.5	7 890.7	6 80.46	8 810.2	1.0 287.8	17 1.0379
102 0.45714	9 1.6043	20 1.1446	10 0.4598	8 778.4	8 317.6	87 (1.24)	53 5.017	4 1178.4	6 891.0	6 80.43	6 810.6	1.0 287.4	14 1.0372
129 0.45636	11 1.6050	25 1.1461	14 0.4589	1.0 777.8	8 317	86.26	67 5.058	3 1178.2	8 891.5	7 80.39	9 811.1	1.0 286.8	18 1.0361
103 0.45611	9 1.6052	19 1.1466	10 0.4586	1.0 777.6	8 316.8	86 (1.23)	55 5.072	3 1178.1	6 891.6	6 80.38	7 811.3	1.1 286.5	15 1.0357
128 0.45508	11 1.6061	24 1.1485	13 0.4576	8 776.8	8 316	85.03	68 5.126	3 1177.9	7 892.2	7 80.32	8 811.9	1.0 285.7	20 1.0341
103 0.45508	9 1.6061	20 1.1486	10 0.4576	1.0 776.8	8 316.0	85	20 5.128	2 1177.9	6 892.2	6 80.32	9 811.9	9 285.7	1.0341
103 0.45392	11 1.6071	24 1.1506	11 0.4565	9 775.9	9 315.1	84 (1.21)	57 5.185	2 1177.7	6 892.9	6 80.26	7 812.6	1.0 284.8	19 1.0322
129 0.45379	11 1.6072	20 1.1509	13 0.4563	1.0 775.8	8 315	83.82	69 5.195	2 1177.7	8 893.0	7 80.25	8 812.7	1.0 284.7	22 1.0319
103 0.45289	10 1.6080	20 1.1526	14 0.4554	8 775.1	8 314.3	83 (1.21)	58 5.243	3 1177.5	6 893.5	6 80.20	7 813.2	1.0 284.0	19 1.0303
129 0.45250	9 1.6083	25 1.1534	11 0.4549	8 774.8	8 314	82.61	70 5.265	3 1177.4	7 893.7	6 80.18	8 813.5	9 283.7	23 1.0296
104 0.45185	11 1.6089	20 1.1546	11 0.4543	1.0 774.3	8 313.5	82 (1.20)	59 5.302	3 1177.2	6 894.1	6 80.14	7 813.9	1.1 283.1	19 1.0284
129 0.45121	11 1.6094	21 1.1558	9 0.4536	9 773.8	8 313	81.41	72 5.337	3 1177.1	7 894.4	7 80.11	8 814.3	8 282.6	23 1.0273
116 0.45069	9 1.6098	25 1.1567	21 0.4531	1.0 773.4	9 312.6	81 (1.19)	60 5.362	2 1177.0	8 894.7	6 80.08	6 814.6	1.0 282.3	19 1.0265
129 0.44992	12 1.6106	20 1.1583	13 0.4523	1.0 772.8	8 312	80.22	73 5.410	3 1176.8	6 895.2	7 80.04	8 815.1	9 281.6	23 1.0250
103 0.44966	10 1.6108	20 1.1587	11 0.4520	8 772.6	8 311.8	80 (1.19)	62 5.424	3 1176.7	7 895.3	5 80.03	7 815.3	1.0 281.4	20 1.0245
130 0.44862	1.6117	24 1.1607	0.4510	1.0 771.8	311	79.03	74 5.484	1176.5	895.9	79.97	8 815.9	1.0 280.6	24 1.0226

A	B	C	D	E	F	G	H	I	J	K	L	M	N
104 0.44862	10 1.6118	21 1.1608	10 0.4510	8 771.8	8 311.0	**79**	62 5.486	2 1176.5	6 895.9	6 79.97	6 815.9	8 280.6	20 1.0225
117 0.44745	9 1.6127	21 1.1629	12 0.4498	9 770.9	9 310.1	**78**	64 5.550	3 1176.2	6 896.5	6 79.91	7 816.6	9 279.7	22 1.0203
130 0.44732	11 1.6128	25 1.1632	14 0.4496	10 770.8	**310**	1.16 77.87	66 5.559	3 1176.2	6 896.6	7 79.90	8 816.7	10 279.6	25 1.0201
117 0.44628	9 1.6136	21 1.1650	12 0.4486	9 770.0	9 309.2	**77**	66 5.616	9 1175.9	7 897.1	7 79.85	7 817.3	9 278.8	43 1.0160
129 0.44603	11 1.6139	24 1.1656	13 0.4483	10 769.8	**309**	1.15 76.72	76 5.635	4 1175.8	7 897.3	7 79.83	8 817.5	11 278.5	54 1.0147
103 0.44525	11 1.6147	22 1.1672	11 0.4475	8 769.2	8 308.4	**76**	68 5.684	3 1175.6	7 897.7	7 79.79	7 817.9	9 277.9	32 1.0128
130 0.44473	9 1.6151	25 1.1681	13 0.4470	10 768.8	**308**	1.13 75.59	77 5.712	6 1175.5	7 898.0	7 79.76	9 818.2	10 277.5	32 1.0115
117 0.44408	10 1.6157	22 1.1694	12 0.4463	9 768.3	9 307.5	**75**	71 5.755	5 1175.3	7 898.4	7 79.72	8 818.6	9 277.0	21 1.0107
131 0.44342	11 1.6162	24 1.1705	13 0.4457	10 767.8	**307**	1.11 74.48	79 5.791	3 1175.2	7 898.7	7 79.69	8 819.0	10 276.5	15 1.0100
131 0.44277	10 1.6167	25 1.1716	12 0.4451	10 767.3	10 306.5	**74**	72 5.827	2 1175.1	7 899.0	7 79.66	7 819.3	10 276.1	9 1.0098
130 0.44212	11 1.6173	25 1.1730	14 0.4443	10 766.8	**306**	1.09 73.39	80 5.871	3 1174.9	7 899.4	7 79.62	8 819.8	10 275.5	5 1.0095
117 0.44160	10 1.6177	23 1.1739	12 0.4439	9 766.4	9 305.6	**73**	74 5.901	9 1174.8	7 899.7	7 79.59	8 820.1	9 275.1	5 1.0093
130 0.44082	12 1.6185	24 1.1754	13 0.4430	10 765.8	**305**	1.07 72.32	81 5.952	8 1174.6	6 900.1	7 79.55	8 820.6	10 274.5	5 1.0090
118 0.44042	11 1.6188	23 1.1762	13 0.4426	9 765.5	9 304.7	**72**	77 5.978	3 1174.5	6 900.3	7 79.53	9 820.8	9 274.2	5 1.0088
131 0.43951	11 1.6196	25 1.1779	13 0.4417	10 764.8	**304** 1.0	1.05 71.27	83 6.035	3 1174.3	7 900.8	7 79.48	7 821.3	10 273.5	5 1.0085
130 0.43912	11 1.6199	24 1.1786	13 0.4413	10 764.5	10 303.7	**71**	80 6.058	2 1174.2	7 901.0	7 79.46	8 821.5	10 273.2	4 1.0084
131 0.43820	11 1.6207	25 1.1804	13 0.4404	10 763.8	**303**	1.03 70.24	84 6.119	3 1174.0	7 901.5	7 79.41	8 822.1	10 272.5	5 1.0080
118 0.43794	11 1.6210	24 1.1810	12 0.4401	9 763.6	9 302.8	**70**	82 6.140	3 1173.9	7 901.7	7 79.39	8 822.3	10 272.2	5 1.0079
131 0.43689	12 1.6219	24 1.1828	13 0.4391	10 762.8	**302**	1.02 69.22	86 6.205	7 1173.7	7 902.2	7 79.34	8 822.9	10 271.5	4 1.0076

0.43663 ¹³¹	1.6221 ¹¹	1.1834 ²⁴	0.4388 ¹³	762.6 ¹·⁰	301.8 ¹·⁰	**69** ¹·⁰⁰	6,224 ⁸⁴	1173.6 ³	902.4 ⁷	79.32 ⁸	823.0 ⁷	271.3 ¹·⁰	1.0076 ³
0.43558 ¹³¹	1.6230 ¹¹	1.1853 ²⁵	0.4377 ¹⁴	761.8 ¹·⁰	**301** ¹·⁰⁰	68.22	6,292 ⁸⁷	1173.4 ³	902.9 ⁷	79.26 ⁷	823.6 ⁶	270.5 ¹·⁰	1.0076
0.43532 ¹³²	1.6233 ¹²	1.1859 ²⁵	0.4374 ¹³	761.6 ¹·⁰	300.8 ¹·⁰	**68** ¹·⁰⁰	6,311 ⁸⁷	1173.3 ³	903.1 ⁷	79.25 ⁷	823.8 ⁸	270.2 ¹·⁰	1.0076
0.43427 ¹³¹	1.6242 ¹²	1.1878 ²⁴	0.4361 ¹³	760.8 ¹·⁰	**300** ¹·⁰	67.22	6,380 ⁸⁸	1173.1 ⁴	903.6 ⁷	79.19 ⁸	824.4 ⁶	269.5 ¹·¹	1.0076
0.43400 ¹³²	1.6244 ¹¹	1.1883 ²⁴	0.4361 ¹³	760.6 ¹·⁰	299.8 ¹·⁰	**67** ⁹⁹	6,400 ⁸⁹	1172.9 ³	903.8 ⁷	79.17 ⁷	824.6 ⁸	269.2 ¹·¹	1.0076
0.43295 ¹³²	1.6253 ¹²	1.1902 ²⁵	0.4351 ¹³	760.6 ¹·⁰	**299** ¹·⁰	66.23	6,470 ⁹⁰	1172.7 ⁴	904.3 ⁷	79.12 ⁸	825.2 ⁶	268.4 ¹·⁰	1.0076
0.43269 ¹³¹	1.6256 ¹²	1.1908 ²⁵	0.4348 ¹³	759.8 ¹·⁰	298.8 ¹·⁰	**66** ⁹⁹	6,491 ⁹¹	1172.6 ³	904.5 ⁷	79.09 ⁸	825.4 ⁸	268.2 ¹·⁰	1.0076
0.43163 ¹³²	1.6265 ¹²	1.1927 ²⁵	0.4338 ¹³	759.6 ¹·⁰	**298** ¹·¹	65.24	6,562 ⁹²	1172.4 ³	905.0 ⁷	79.04 ⁷	826.0 ⁸	267.4 ¹·⁰	1.0076
0.43124 ¹⁴⁵	1.6268 ¹¹	1.1933 ²⁵	0.4335 ¹⁴	758.8 ¹·⁰	297.7 ¹·¹	**65** ⁹⁸	6,585 ⁹⁴	1172.3 ³	905.2 ⁷	79.02 ⁷	826.2 ⁷	267.2 ¹·⁰	1.0076
0.43031 ¹³²	1.6276 ¹¹	1.1952 ²⁶	0.4324 ¹⁵	758.5 ¹·⁰	**297** ¹·⁰	64.26	6,655 ⁹³	1172.1 ³	905.7 ⁷	78.97 ⁸	826.7 ⁷	266.4 ¹·⁰	1.0076
0.42992 ¹³²	1.6279 ¹²	1.1959 ²⁵	0.4320 ¹³	757.8 ¹·⁰	296.7 ¹·⁰	**64** ⁹⁷	6,681 ⁹⁶	1172.0 ³	905.9 ⁷	78.94 ⁷	826.9 ⁶	266.2 ¹·⁰	1.0076
0.42899 ¹³²	1.6288 ¹³	1.1977 ²⁶	.43311 ¹³	757.5 ¹·⁰	**296** ¹·⁰	63.29	6,750 ⁹⁵	1171.8 ³	906.4 ⁷	78.89 ⁸	827.5 ⁷	265.4 ¹·¹	1.0076
0.42860 ¹³²	1.6292 ¹²	1.1985 ²⁵	0.4307 ¹³	756.8 ¹·⁰	295.7 ¹·⁰	**63** ⁹⁶	6,780 ⁹⁹	1171.5 ³	906.6 ⁷	78.87 ⁷	827.7 ⁸	265.1 ¹·⁰	1.0076
0.42767 ¹³²	1.6299 ¹¹	1.2002 ²⁶	0.4298 ¹³	756.5 ¹·⁰	**295** ¹·¹	62.33	6,847 ¹⁰²	1171.4 ³	907.1 ⁷	78.82 ⁸	828.3 ⁶	264.4 ¹·⁰	1.0076
0.42714 ¹⁴⁶	1.6304 ¹²	1.2011 ²⁵	0.4293 ¹⁴	755.8 ¹·⁰	294.6 ¹·¹	**62** ⁹⁵	6,882 ¹⁰²	1171.2 ³	907.3 ⁷	78.79 ⁷	828.6 ⁷	264.1 ¹·¹	1.0076
0.42635 ¹³²	1.6311 ¹²	1.2027 ²⁶	0.4284 ¹⁴	755.4 ¹·¹	**294** ¹·⁰	61.38	6,946 ⁹⁹	1171.1 ³	907.8 ⁸	78.74 ⁸	829.1 ⁸	263.4 ¹·¹	1.0076
0.42582 ¹³²	1.6316 ¹²	1.2037 ²⁵	0.4279 ¹⁴	754.8 ¹·⁰	293.6 ¹·⁰	**61** ⁹⁴	6,986 ¹⁰⁴	1171.1 ³	908.1 ⁷	78.71 ⁷	829.4 ⁷	263.0 ¹·⁰	1.0076
0.42502 ¹³³	1.6323 ¹³	1.2052 ²⁷	0.4271 ¹⁴	754.4 ¹·⁰	**293** ¹·¹	60.44	7,046 ¹⁰⁰	1170.9 ³	908.5 ⁷	78.67 ⁸	829.8 ⁸	262.4 ¹·¹	1.0076
0.42436 ¹⁴⁶	1.6329	1.2064	0.4265	753.8 ¹·¹	292.5	**60**	7,094 ¹⁰⁸	1170.8	908.8	78.63	830.2	261.9	1.0076

C1	C2	C3	C4	C5	C6	C7	C8	C9	C10	C11	C12	C13	C14
0.42370	1.6335	1.2077	0.4258	752.8	292	59.51	7.148	1170.6	909.2	78.59	830.6	261.4	1.0076
0.42290	1.6342	1.2092	0.4250	752.2	291.4	59	7.206	1170.4	909.6	78.54	831.0	260.8	1.0076
0.42237	1.6347	1.2103	0.4244	751.8	291	58.60	7.252	1170.3	909.9	78.51	831.4	260.4	1.0076
0.42143	1.6355	1.2120	0.4235	751.1	290.3	58	7.323	1170.1	910.4	78.45	831.9	259.7	1.0076
0.42103	1.6359	1.2128	0.4231	750.8	290	57.70	7.358	1170.0	910.6	78.43	832.2	259.4	1.0076
0.41997	1.6369	1.2149	0.4220	750.0	289.2	57	7.443	1169.8	911.2	78.37	832.7	258.6	1.0076
0.41970	1.6371	1.2154	0.4217	749.8	289	56.81	7.465	1169.7	911.3	78.36	832.9	258.4	1.0076
0.41837	1.6383	1.2178	0.4205	748.8	288.0	56	7.566	1169.4	912.0	78.29	833.6	257.4	1.0076
0.41837	1.6384	1.2180	0.4204	748.8	288	55.94	7.574	1169.4	912.0	78.28	833.7	257.4	1.0076
0.41703	1.6396	1.2205	0.4190	747.8	286.9	55.07	7.685	1169.1	912.7	78.21	834.5	256.4	1.0076
0.41690	1.6397	1.2208	0.4189	747.7	287	55	7.695	1169.0	912.8	78.20	834.6	256.3	1.0076
0.41569	1.6408	1.2231	0.4177	746.8	285.7	54.22	7.798	1168.7	913.4	78.13	835.3	255.3	1.0076
0.41529	1.6411	1.2238	0.4173	746.5	286	54	7.828	1168.6	913.6	78.11	835.5	255.1	1.0076
0.41435	1.6420	1.2257	0.4163	745.8	285	53.38	7.913	1168.4	914.1	78.06	836.0	254.3	1.0076
0.41368	1.6426	1.2269	0.4157	745.3	284.5	53	7.967	1168.3	914.4	78.02	836.4	253.9	1.0076
0.41301	1.6433	1.2283	0.4150	744.8	284	52.55	8.030	1168.1	914.8	77.98	836.8	253.3	1.0076
0.41207	1.6441	1.2300	0.4141	744.1	283.3	52	8.110	1167.9	915.3	77.93	837.3	252.7	1.0076
0.41167	1.6445	1.2309	0.4136	743.8	283	51.73	8.149	1167.8	915.5	77.91	837.6	252.3	1.0076
0.41046	1.6457	1.2332	0.4124	742.9	282.1	51	8.259	1167.5	916.1	77.84	838.3	251.4	1.0076

¹³⁵ 0.41032	¹³ 1.6458	²⁶ 1.2335	¹³ 0.4123	¹·⁰ 742.8	**282**	⁸¹ 50.92	¹²² 8.271	³ 1167.5	⁷ 916.2	⁸ 77.83	⁸ 838.4	¹·⁰ 251.3	1.0076
¹³⁵ 0.40897	¹² 1.6470	²⁶ 1.2361	¹⁴ 0.4109	¹·⁰ 741.8	**281** ¹·³	⁸⁰ 50.12	¹²⁴ 8.395	³ 1167.2	⁷ 916.9	⁷ 77.76	⁷ 839.1	¹·⁰ 250.3	1.0076
¹⁷⁵ 0.40871	¹⁵ 1.6472	³³ 1.2365	¹⁷ 0.4107	¹·³ 741.6	280.8	**50** ⁷⁹ 49.33	¹⁵⁵ 8.414	³ 1167.2	⁹ 917.0	⁹ 77.75	⁹ 839.2	¹·⁰ 250.2	1.0076
¹³⁵ 0.40702	¹⁶ 1.6483	²⁶ 1.2387	¹³ 0.4096	¹·⁰ 740.8	**280** ¹·²	⁴⁹ ⁷⁸ 48.55	¹²⁶ 8.521	³ 1166.9	⁷ 917.6	⁷ 77.68	⁸ 839.9	¹·³ 249.3	1.0076
¹⁶² 0.40709	¹⁶ 1.6488	³³ 1.2398	¹⁷ 0.4090	¹·² 740.4	279.6	⁴⁸ ⁷⁷ 47.78	¹⁶² 8.576	⁸ 1166.8	⁹ 917.9	¹⁰ 77.65	¹·⁰ 840.2	¹·³ 248.9	1.0076
¹³⁵ 0.40627	¹³ 1.6496	²⁶ 1.2413	¹⁴ 0.4082	¹·⁰ 739.8	**279** ¹·³	⁷⁶ 47.02	¹²⁸ 8.649	³ 1166.6	⁹ 918.3	⁷ 77.61	⁸ 840.7	¹·⁰ 248.3	1.0076
¹⁷⁶ 0.40533	¹³ 1.6505	³⁴ 1.2432	¹⁶ 0.4074	¹·³ 739.1	278.3	**47** ⁷⁵ 46.27	¹⁶⁶ 8.742	⁴ 1166.4	⁸ 918.8	¹⁰ 77.55	⁸ 841.2	¹·³ 247.6	1.0076
¹³⁵ 0.40492	¹³ 1.6509	²⁷ 1.2440	¹³ 0.4069	¹·⁰ 738.8	**278**	⁴⁶ ⁷⁴ 45.53	¹³¹ 8.780	³ 1166.3	⁷ 919.0	⁸ 77.53	⁷ 841.5	¹·⁰ 247.3	1.0076
¹³⁵ 0.40357	¹³ 1.6522	²⁶ 1.2466	¹³ 0.4056	¹·⁰ 737.8	**277** ¹·³	⁴⁵ ⁷³ 44.80	¹³³ 8.913	³ 1166.0	⁷ 919.7	⁷ 77.46	⁷ 842.2	¹·⁰ 246.3	1.0076
¹⁷⁶ 0.40357	¹³ 1.6522	³⁵ 1.2467	¹⁸ 0.4056	¹·⁰ 737.8	277.0	⁷² 44.08	¹⁷⁵ 8.917	⁴ 1166.0	⁹ 919.7	⁹ 77.46	⁸ 842.2	¹·³ 246.2	1.0076
¹³⁶ 0.40221	¹³ 1.6535	²⁶ 1.2492	¹⁴ 0.4042	¹·⁰ 736.8	**276** ¹·⁴	**44** ⁷¹ 43.37	¹³⁶ 9.049	³ 1165.7	⁷ 920.4	⁸ 77.38	⁸ 843.0	¹·⁰ 245.3	1.0076
¹⁹⁰ 0.40167	¹⁷ 1.6539	³⁵ 1.2502	¹⁹ 0.4037	¹·⁴ 736.4	275.6	**43**	¹⁸³ 9.100	⁶ 1165.6	¹·⁰ 920.7	¹⁰ 77.36	¹·¹ 843.3	¹·³ 244.9	1.0076
¹³⁶ 0.40085	¹² 1.6547	²⁷ 1.2519	¹⁴ 0.4028	¹·⁰ 735.8	**275** ¹·³		¹³⁸ 9.187	⁴ 1165.4	⁷ 921.1	⁷ 77.31	⁸ 843.8	¹·⁰ 244.3	1.0076
¹⁷⁷ 0.39990	¹⁷ 1.6556	³⁶ 1.2538	¹⁹ 0.4018	¹·³ 735.1	274.3		¹⁹⁰ 9.290	⁴ 1165.2	⁹ 921.6	¹¹ 77.25	¹¹ 844.4	¹·⁴ 243.5	1.0076
¹³⁶ 0.39949	¹³ 1.6560	²⁶ 1.2545	¹³ 0.4015	¹·⁰ 734.8	**274**		¹⁴¹ 9.328	³ 1165.1	⁷ 921.8	⁸ 77.23	⁸ 844.6	¹·⁰ 243.3	1.0076
¹³⁶ 0.39813	¹³ 1.6573	²⁷ 1.2572	¹⁴ 0.4001	¹·⁰ 733.8	**273** ¹·⁴		¹⁴³ 9.471	⁸ 1164.8	¹·⁰ 922.5	⁷ 77.16	⁷ 845.3	¹·⁰ 242.3	1.0076
¹⁹⁰ 0.39800	¹⁸ 1.6574	³⁷ 1.2575	¹⁹ 0.3999	¹·⁴ 733.7	272.9		¹⁹⁷ 9.487	⁵ 1164.7	⁷ 922.6	¹⁰ 77.15	¹·⁰ 845.4	¹·⁴ 242.1	1.0076
¹³⁶ 0.39677	¹³ 1.6586	²⁷ 1.2599	¹⁴ 0.3987	¹·⁰ 732.8	**272** ¹·⁴		¹⁴⁶ 9.617	⁴ 1164.4	¹·⁰ 923.2	⁸ 77.08	⁸ 846.1	¹·¹ 241.2	1.0076
¹⁹¹ 0.39609	¹⁹ 1.6593	³⁸ 1.2613	¹⁹ 0.3980	¹·⁴ 732.3	271.5		²⁰⁹ 9.696	³ 1164.3	¹·⁰ 923.6	¹¹ 77.04	¹¹ 846.5	¹·⁴ 240.7	1.0076

0.39540	1.6599	1.2625	0.3973	731.8	**271**	42.67	9.766	1164.1	923.9	77.01	846.9	240.2	1.0076
0.39403	1.6612	1.2652	0.3960	730.8	270.0	41.98	9.913	1163.8	924.6	76.93	847.6	239.3	1.0076
0.39403	1.6612	1.2652	0.3960	730.8	**270**	41.30	9.918	1163.8	924.6	76.93	847.7	239.2	1.0076
0.39266	1.6625	1.2679	0.3946	729.8	**269**	40.63	10.07	1163.5	925.3	76.86	848.4	238.2	1.0076
0.39198	1.6631	1.2691	0.3940	729.3	268.5	40.63	10.14	1163.4	925.6	76.82	848.7	237.8	1.0076
0.39129	1.6638	1.2706	0.3932	728.8	**268**	39.96	10.23	1163.2	926.0	76.78	849.2	237.2	1.0076
0.38992	1.6651	1.2732	0.3919	727.8	267.0	39.30	10.38	1162.9	926.7	76.71	849.9	236.3	1.0076
0.38992	1.6651	1.2733	0.3918	727.8	**267**	39.96	10.39	1162.9	926.7	76.71	850.0	236.2	1.0076
0.38855	1.6665	1.2760	0.3904	726.8	**266**	39.30	10.55	1162.6	927.4	76.63	850.8	235.2	1.0076
0.38786	1.6671	1.2773	0.3898	726.3	265.5	38.65	10.63	1162.5	927.7	76.60	851.1	234.7	1.0076
0.38717	1.6678	1.2787	0.3891	725.8	**265**	38.01	10.72	1162.3	928.1	76.56	851.5	234.2	1.0076
0.38579	1.6692	1.2815	0.3877	724.8	**264**	38.65	10.89	1162.0	928.8	76.48	852.3	233.2	1.0076
0.38579	1.6692	1.2815	0.3877	724.8	264.0	38.01	10.89	1162.0	928.8	76.48	852.3	233.2	1.0076
0.38441	1.6705	1.2842	0.3863	723.8	**263**	37.38	11.06	1161.7	929.5	76.41	853.1	232.2	1.0076
0.38358	1.6713	1.2859	0.3854	723.2	262.4	36.76	11.17	1161.5	929.9	76.36	853.6	231.6	1.0076
0.38303	1.6719	1.2870	0.3849	722.8	**262**	36.15	11.24	1161.4	930.2	76.33	853.9	231.2	1.0076
0.38164	1.6732	1.2897	0.3835	721.8	**261**	36.76	11.42	1161.1	930.9	76.26	854.6	230.2	1.0076
0.38123	1.6736	1.2904	0.3831	721.5	260.7	36.15	11.47	1161.0	931.1	76.24	854.8	229.9	1.0076
0.38026	1.6746	1.2925	0.3821	720.8	**260**	35.55	11.60	1160.8	931.6	76.18	855.4	229.2	1.0076

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)
0.37901 ²²²	1.6759 ²³	1.2951 ⁴⁷	0.3808 ²³	719.9 ¹·⁶	259.1 ¹·⁶	**35**	11.78 ³¹	1160.4 ⁶	932.3 ¹·²	76.11 ¹³	856.1 ¹·³	228.2 ¹·⁷	1.0076
0.37887 ¹³⁹	1.6760 ¹⁴	1.2953 ²⁸	0.3807 ¹⁴	719.8 ¹·⁰	**259**	34.96 ⁵⁹	11.78 ¹⁸	1160.4 ⁴	932.3 ⁷	76.11 ⁷	856.2 ⁸	228.1 ¹·¹	1.0076
0.37748 ¹³⁹	1.6774 ¹⁴	1.2981 ²⁸	0.3793 ¹⁴	718.8 ¹·⁰	**258**	34.38 ⁵⁸	11.98 ¹⁹	1160.1 ³	933.0 ⁷	76.03 ⁸	857.0 ⁸	227.1 ¹·⁷	1.0076
0.37664 ²³⁷	1.6783 ²¹	1.2999 ²⁸	0.3784 ²⁴	718.2 ¹·⁷	257.4 ¹·⁷	**34**	12.11 ³³	1159.9 ⁵	933.5 ⁸	75.98 ¹³	857.5 ¹·⁴	226.5 ¹·⁵	1.0076
0.37609 ¹³⁹	1.6788 ¹⁴	1.3009 ²⁸	0.3779 ¹⁴	717.8 ¹·⁰	**257**	33.80 ⁵⁸	12.17 ¹⁹	1159.8 ⁸	933.7 ⁷	75.95 ⁸	857.8 ⁸	226.1 ¹·⁰	1.0076
0.37469 ¹⁴⁰	1.6802 ¹⁴	1.3037 ²⁸	0.3765 ¹⁴	716.8 ¹·⁰	**256**	33.23 ⁵⁷	12.37 ²⁰	1159.6 ⁶	934.5 ⁸	75.88 ⁸	858.6 ⁸	225.1 ¹·⁰	1.0076
0.37413 ²⁵¹	1.6807 ²⁴	1.3048 ⁴⁹	0.3759 ²⁵	716.4 ¹·⁸	255.6 ¹·⁸	**33**	12.46 ³⁵	1159.4 ⁵	934.8 ⁷	75.85 ¹³	858.8 ¹·⁴	224.7 ¹·⁰	1.0076
0.37330 ¹³⁹	1.6816 ¹⁴	1.3065 ²⁸	0.3751 ¹⁴	715.8 ¹·⁰	**255**	32.66 ⁵⁷	12.57 ²⁰	1159.3 ³	935.2 ⁷	75.80 ⁸	859.4 ⁸	224.1 ¹·⁰	1.0076
0.37190 ¹⁴⁰	1.6830 ¹⁴	1.3093 ⁵⁰	0.3737 ²⁵	714.8 ¹·⁰	**254**	32.10 ⁵⁶	12.78 ²¹	1159.0 ³	935.9 ⁷	75.72 ⁸	860.2 ¹·⁴	223.1 ¹·⁸	1.0076
0.37162 ²⁵¹	1.6832 ²⁵	1.3098 ²⁸	0.3734 ¹⁴	714.6 ¹·⁰	253.8	**32**	12.82 ³⁶	1158.9 ³	936.0 ⁶	75.71 ⁸	860.3 ¹·⁴	222.9 ¹·⁴	1.0076
0.37050 ¹⁴⁰	1.6844 ¹⁴	1.3121 ²⁸	0.3723 ²⁵	713.8 ¹·⁰	**253**	31.54 ⁵⁶	12.99 ²¹	1158.7 ⁵	936.6 ¹·³	75.64 ⁸	861.0 ⁸	222.1 ¹·⁸	1.0076
0.36910 ²⁵²	1.6858 ²⁶	1.3149 ⁵¹	0.3709 ²⁵	712.8 ¹·⁸	252.0 ¹·⁸	**31**	13.20 ³⁸	1158.4 ⁴	937.3 ¹·³	75.56 ¹⁵	861.7 ¹·⁴	221.1 ¹·¹	1.0076
0.36910 ¹⁴⁰	1.6858 ¹⁴	1.3150 ²⁹	0.3708 ¹⁵	712.8 ¹·⁸	**252**	30.98 ⁵⁶	13.21 ²²	1158.4 ³	937.3 ⁷	75.56 ⁸	861.7 ⁸	221.1 ¹·⁰	1.0076
0.36769 ¹⁴¹	1.6872 ¹⁴	1.3178 ²⁸	0.3694 ¹⁴	711.8 ¹·⁰	**251**	30.43 ⁵⁵	13.43 ²²	1158.1 ⁵	938.0 ⁷	75.48 ⁸	862.5 ⁸	220.1 ¹·⁰	1.0076
0.36657 ²⁵³	1.6884 ²⁶	1.3201 ⁵²	0.3683 ²⁶	711.0 ¹·⁰	250.2 ¹·⁸	**30**	13.61 ⁴¹	1157.9 ⁵	938.6 ¹·³	75.42 ¹⁴	863.1 ¹·⁴	219.3 ¹·⁸	1.0076
0.36629 ¹⁴⁰	1.6886 ¹⁴	1.3206 ²⁸	0.3680 ¹⁴	710.8 ¹·⁰	**250**	29.89 ⁵⁴	13.65 ²²	1157.8 ³	938.7 ⁸	75.40 ⁸	863.3 ⁸	219.1 ¹·⁰	1.0076
0.36488 ¹⁴¹	1.6901 ¹⁵	1.3235 ²⁹	0.3666 ¹⁴	709.8 ¹·⁰	**249**	29.36 ⁵³	13.88 ²³	1157.5 ³	939.4 ⁷	75.32 ⁸	864.1 ¹·⁴	218.1 ¹·⁰	1.0076
0.36389 ²⁶⁸	1.6911 ²⁷	1.3254 ⁵³	0.3656 ²⁷	709.1 ¹·⁹	248.3 ¹·⁹	**29**	14.04 ⁴³	1157.3 ⁶	939.9 ¹·³	75.27 ¹⁵	864.6 ¹·⁵	217.4 ¹·⁹	1.0076
0.36347 ¹⁴¹	1.6915 ¹⁴	1.3264 ²⁹	0.3652 ¹⁴	708.8 ¹·⁰	**248**	28.84 ⁵²	14.11 ²³	1157.2 ³	940.1 ⁷	75.24 ⁸	864.9 ⁸	217.1 ¹·⁰	1.0076

0.36206	1.6930	1.3292	0.3638	707.8	**247**	28.32	14.34	1156.9	940.8	75.16	865.6	216.1
0.36107	1.6939	1.3310	0.3629	707.1	246.3	**28**	14.49	1156.6	941.3	75.11	866.1	215.4
0.36064	1.6944	1.3321	0.3623	706.8	**246**	27.82	14.58	1156.5	941.5	75.08	866.4	215.0
0.35923	1.6959	1.3350	0.3609	705.8	**245**	27.33	14.83	1156.2	942.2	75.00	867.2	214.0
0.35823	1.6969	1.3369	0.3600	705.1	244.3	**27**	15.00	1156.0	942.7	74.95	867.7	213.3
0.35781	1.6974	1.3379	0.3595	704.8	**244**	26.84	15.08	1155.9	942.9	74.92	868.0	213.0
0.35639	1.6988	1.3408	0.3580	703.8	**243**	26.36	15.34	1155.6	943.6	74.84	868.8	212.0
0.35525	1.7000	1.3431	0.3569	703.0	242.2	**26**	15.54	1155.4	944.2	74.78	869.3	211.2
0.35497	1.7003	1.3437	0.3566	702.8	**242**	25.90	15.60	1155.3	944.3	74.76	869.5	211.0
0.35354	1.7018	1.3466	0.3552	701.8	**241**	25.44	15.87	1155.0	945.0	74.68	870.3	210.0
0.35212	1.7033	1.3495	0.3537	700.8	240.0	**25**	16.14	1154.7	945.7	74.60	871.0	209.0
0.35212	1.7033	1.3495	0.3537	700.8	**240**	24.99	16.14	1154.7	945.7	74.60	871.1	209.0
0.35069	1.7048	1.3525	0.3523	699.8	**239**	24.56	16.42	1154.4	946.4	74.52	871.9	208.0
0.34926	1.7063	1.3554	0.3509	698.8	**238**	24.12	16.70	1154.1	947.1	74.44	872.7	207.0
0.34883	1.7067	1.3562	0.3505	698.5	237.7	**24**	16.78	1154.0	947.3	74.42	872.8	206.7
0.34783	1.7078	1.3583	0.3494	697.8	**237**	23.69	16.99	1153.8	947.8	74.36	873.4	206.0
0.34639	1.7093	1.3613	0.3480	696.8	**236**	23.26	17.29	1153.5	948.5	74.28	874.2	205.0
0.34553	1.7102	1.3630	0.3471	696.2	235.4	**23**	17.46	1153.3	948.9	74.23	874.7	204.4
0.34496	1.7108	1.3643	0.3465	695.8	**235**	22.83	17.59	1153.2	949.2	74.20	875.0	204.0

(final column, all rows: 1.0076)

¹⁴⁴ 0.34352	¹⁵ 1.7123	³⁰ 1.3673	¹⁴ 0.3451	1.0 694.8	**234**	⁴² 22.41	³¹ 17.90	³ 1152.9	⁷ 949.9	⁸ 74.12	⁸ 875.8	1.0 203.0	1.0076
¹⁴⁴ 0.34208	¹⁶ 1.7139	²⁹ 1.3702	¹⁵ 0.3436	1.0 693.8	**233**	⁴¹ 22.00	³² 18.22	⁴ 1152.5	⁷ 950.6	⁸ 74.04	⁸ 876.6	1.1 201.9	1.0076
³⁴⁵ 0.34208	³⁷ 1.7139	⁷² 1.3702	³⁵ 0.3436	2 693.8	2.4 233.0	**22** ⁴¹ 21.59	⁷⁶ 18.22	⁸ 1152.5	1.7 950.6	¹⁹ 74.04	1.9 876.6	2.5 201.9	1.0076
¹⁴⁴ 0.34464	¹⁵ 1.7154	³⁰ 1.3732	¹⁴ 0.3422	1.0 692.8	**232**	⁴⁰ 21.19	³² 18.54	³ 1152.2	⁷ 951.3	⁸ 73.96	⁸ 877.3	1.0 200.9	1.0076
¹⁴⁵ 0.33919	¹⁵ 1.7169	³⁰ 1.3762	¹⁵ 0.3407	1.0 691.8	**231**	**21** ⁴⁰ 20.79	³³ 18.87	³ 1151.9	⁷ 952.0	⁸ 73.88	⁸ 878.1	1.0 199.9	1.0076
³⁶¹ 0.33847	³⁸ 1.7177	⁷⁵ 1.3777	³⁶ 0.3400	2.5 691.3	2.5 230.5	³⁹ 20.40	⁸¹ 19.03	⁸ 1151.8	1.7 952.3	²⁰ 73.84	1.9 878.5	2.5 199.4	1.0076
¹⁴⁴ 0.33775	¹⁶ 1.7185	³⁰ 1.3792	¹⁵ 0.3392	1.0 690.8	**230**	³⁸ 20.02	³³ 19.20	³ 1151.6	⁷ 952.7	⁸ 73.80	⁸ 878.9	1.0 198.9	1.0076
¹⁴⁵ 0.33630	¹⁶ 1.7200	³⁰ 1.3822	¹⁴ 0.3378	1.0 689.8	**229**	**20** ³⁷ 19.65	³⁴ 19.54	³ 1151.3	⁷ 953.4	⁸ 73.72	⁸ 879.7	1.0 197.9	1.0076
¹⁴⁵ 0.33485	³⁹ 1.7216	³⁰ 1.3852	¹⁵ 0.3363	2.6 688.8	**228**	³⁷ 19.28	³⁵ 19.89	³ 1151.0	⁷ 954.1	⁹ 73.63	⁸ 880.5	1.0 196.9	1.0076
³⁷⁷ 0.33470	³⁹ 1.7216	⁷⁷ 1.3854	³⁸ 0.3362	2.6 688.7	2.6 227.9	¹⁹ 18.92	⁸⁸ 19.91	⁸ 1151.0	1.8 954.1	²¹ 73.63	2.0 880.5	2.6 196.8	1.0076
¹⁴⁶ 0.33339	¹⁶ 1.7231	³⁰ 1.3883	¹⁴ 0.3348	1.0 687.8	**227**	³⁶ 18.56	³⁶ 20.25	³ 1150.7	⁷ 954.8	⁹ 73.55	⁸ 881.3	1.0 195.9	1.0076
¹⁴⁵ 0.33194	¹⁶ 1.7247	³⁰ 1.3913	¹⁵ 0.3334	1.0 686.8	**226**	³⁵ 18.21	³⁷ 20.62	⁴ 1150.4	⁷ 955.5	⁹ 73.47	⁷ 882.0	1.0 194.9	1.0076
³⁹³ 0.33077	⁴³ 1.7259	⁸² 1.3936	³⁹ 0.3323	2.7 686.0	2.7 225.2	**18** ³⁴ 17.87	⁹⁹ 20.90	⁸ 1150.2	1.9 956.0	²² 73.41	2.1 882.6	2.7 194.1	1.0076
¹⁴⁶ 0.33048	¹⁶ 1.7263	³¹ 1.3944	¹⁴ 0.3319	1.0 685.8	**225**	³⁶ 18.56	³⁷ 20.99	³ 1150.1	⁷ 956.2	⁹ 73.39	⁸ 882.8	1.0 193.9	1.0076
¹⁴⁶ 0.32902	¹⁶ 1.7279	³⁰ 1.3974	¹⁵ 0.3305	1.0 684.8	**224**	³⁵ 18.21	³⁸ 21.37	³ 1149.8	⁷ 956.9	⁹ 73.30	⁸ 883.6	1.0 192.9	1.0076
¹⁴⁶ 0.32756	¹⁶ 1.7295	³¹ 1.4005	¹⁵ 0.3290	1.0 683.8	**223**	³⁴ 17.87	³⁹ 21.76	³ 1149.5	⁷ 957.6	⁸ 73.22	⁸ 884.4	1.0 191.9	1.0076
⁴⁰⁹ 0.32668	⁴⁶ 1.7305	⁸⁸ 1.4024	⁴² 0.3281	2.8 683.2	2.8 222.4	³⁴ 17.53	1.11 22.01	³ 1149.3	2.0 958.0	²⁴ 73.17	2.3 884.9	2.9 191.2	1.0076
¹⁴⁶ 0.32610	¹⁶ 1.7311	³¹ 1.4036	¹⁵ 0.3275	1.0 682.8	**222**	22.01	⁴⁰ 22.16	³ 1149.2	⁷ 958.3	⁹ 73.14	⁸ 885.2	1.0 190.9	1.0076
¹⁴⁷ 0.32463	¹⁶ 1.7327	³¹ 1.4067	¹⁵ 0.3260	1.0 681.8	**221**	17.53	⁴¹ 22.57	³ 1148.9	⁷ 959.0	⁹ 73.05	⁸ 886.0	1.0 189.9	1.0076

Top section (indices 220–213):

					Index	Inches of Mercury							
0.32316 (147)	1.7343 (16)	1.4098 (31)	0.3245 (15)	680.8 (1.0)	**220**	17.20 (33)	22.99 (42)	1148.6 (3.6)	959.7 (7)	72.97 (8)	886.8 (8)	188.8 (1.1)	1.0076
0.32228 (440)	1.7353 (48)	1.4117 (93)	0.3236 (45)	680.2 (3.0)	219.4	**17**	23.25 (1.24)	1148.4 (9)	960.1 (2.1)	72.91 (26)	887.3 (2.4)	188.2 (3.0)	1.0076
0.32169 (147)	1.7359 (16)	1.4129 (31)	0.3231 (14)	679.8 (1.0)	**219**	16.88 (32)	23.41 (42)	1148.3 (3)	960.5 (8)	72.88 (9)	887.6 (8)	187.8 (1.0)	1.0076
0.32022 (147)	1.7376 (17)	1.4160 (31)	0.3216 (15)	678.8 (1.0)	**218**	16.56 (32)	23.84 (43)	1148.0 (8)	961.2 (7)	72.80 (8)	888.4 (8)	186.8 (1.0)	1.0076
0.31875 (147)	1.7392 (16)	1.4191 (31)	0.3201 (15)	677.8 (1.0)	**217**	16.24 (32)	24.28 (44)	1147.7 (3)	961.9 (7)	72.71 (9)	889.2 (8)	185.8 (1.0)	1.0076
0.31771 (457)	1.7405 (52)	1.4215 (98)	0.3190 (46)	677.1 (3.1)	216.3	**16**	24.62 (1.37)	1147.5 (9)	962.4 (2.3)	72.64 (27)	889.8 (2.5)	185.1 (3.1)	1.0076
0.31727 (148)	1.7409 (16)	1.4222 (32)	0.3186 (15)	676.8 (1.0)	**216**	15.92 (32)	24.73 (45)	1147.4 (3)	962.6 (7)	72.62 (8)	890.0 (8)	184.8 (1.0)	1.0076
0.31579 (148)	1.7425 (17)	1.4254 (32)	0.3171 (15)	675.8 (1.0)	**215**	15.61 (31)	25.19 (46)	1147.1 (3)	963.3 (7)	72.54 (9)	890.8 (8)	183.8 (1.0)	1.0076
0.31431 (148)	1.7442 (17)	1.4286 (32)	0.3156 (15)	674.8 (1.0)	**214**	15.30 (31)	25.66 (47)	1146.8 (3)	964.0 (7)	72.45 (9)	891.6 (8)	182.8 (1.0)	1.0076
0.31283 (488)	1.7459 (54)	1.4317 (102)	0.3141 (49)	673.8 (3.3)	213.0	**15**	26.13 (1.51)	1146.5 (1.0)	964.7 (2.3)	72.36 (54)	892.3 (2.5)	181.8 (3.3)	1.0076

Bottom section (indices 213–209):

					Index	Inches of Mercury.							
0.31283 (148)	1.7459 (17)	1.4317 (31)	0.3141 (15)	673.8 (1.0)	**213**	30.53 (62)	26.14 (48)	1146.5 (3)	964.7 (7)	72.36 (9)	892.3 (7)	181.8 (1.0)	1.0076
0.31149 (238)	1.7473 (28)	1.4345 (51)	0.3128 (24)	672.9 (1.6)	212.1	**30**	26.56 (77)	1146.2 (5)	965.3 (1.2)	72.29 (15)	893.0 (1.4)	180.9 (1.6)	1.0076
0.31134 (149)	1.7475 (16)	1.4349 (32)	0.3126 (15)	672.0 (1.0)	**212**	29.92 (61)	26.62 (48)	1146.2 (3)	965.4 (7)	72.27 (9)	893.1 (8)	180.8 (1.0)	1.0076
0.30986 (148)	1.7492 (17)	1.4381 (32)	0.3111 (15)	671.8 (1.0)	**211**	29.32 (60)	27.11 (49)	1145.9 (3)	966.1 (7)	72.19 (9)	893.9 (8)	179.8 (1.0)	1.0076
0.30896 (253)	1.7502 (29)	1.4399 (54)	0.3103 (25)	671.2 (1.7)	210.4	**29**	27.40 (84)	1145.7 (5)	966.5 (1.2)	72.14 (15)	894.4 (1.4)	179.2 (1.7)	1.0076
0.30837 (149)	1.7509 (17)	1.4413 (32)	0.3096 (15)	670.8 (1.0)	**210**	28.74 (58)	27.61 (50)	1145.6 (3)	966.8 (7)	72.10 (9)	894.7 (8)	178.8 (1.0)	1.0076
0.30687 (150)	1.7526 (54)	1.4445 (32)	0.3081 (15)	669.8 (1.0)	**209**	28.18 (56)	28.13 (52)	1145.3 (3)	967.5 (7)	72.01 (9)	895.5 (7)	177.8 (1.0)	1.0076

(253) 0.30643	(30) 1.7532	(56) 1.4455	(27) 0.3076	(1.7) 669.5	(1.7) 208.7	(56) 28 27.62	(90) 28.30	(5) 1145.2	(1.2) 967.7	(16) 71.98	(1.4) 895.8	(1.8) 177.4	1.0076
(149) 0.30538	(17) 1.7543	(32) 1.4477	(15) 0.3066	(1.0) 668.8	208	27.06	(53) 28.66	(5) 1145.0	(1.3) 968.2	(9) 71.92	(1.4) 896.3	(1.0) 176.8	1.0076
0.30388	(31) 1.7561	(56) 1.4509	(15) 0.3051	(1.8) 667.8	207	(56) 27 26.52	(54) 29.20	(5) 1144.7	(1.4) 968.9	(9) 71.83	(1.5) 897.1	(1.1) 175.7	1.0076
(270) 0.30373	(17) 1.7563	(58) 1.4513	(27) 0.3049	(1.8) 667.7	(1.8) 206.9	(54) 26.52	(97) 29.27	(6) 1144.7	(1.3) 969.0	(16) 71.82	(1.6) 897.2	(1.8) 175.6	1.0076
(150) 0.30238	(32) 1.7578	(33) 1.4542	(15) 0.3036	(1.9) 666.8	206	(53) 26 25.99	(56) 29.76	(3) 1144.4	(1.5) 969.6	(9) 71.74	(1.6) 897.9	(1.0) 174.7	1.0076
(285) 0.30088	(17) 1.7595	(61) 1.4574	(28) 0.3021	(1.0) 665.8	(1.9) 205.0	(53) 25.46	(1.05) 30.32	(3) 1144.1	(1.4) 970.4	(9) 71.65	(1.6) 898.7	(1.0) 173.7	1.0076
(150) 0.30088	(17) 1.7595	(32) 1.4574	(15) 0.3021	(1.0) 665.8	205	(51) 25 24.95	(57) 30.33	(6) 1144.1	(1.7) 970.4	(9) 71.65	(1.6) 898.7	(1.0) 173.7	1.0076
(150) 0.29938	(34) 1.7613	(33) 1.4607	(15) 0.3006	(1.0) 664.8	204	(51) 24.95	(59) 30.92	(3) 1143.8	(1.8) 971.1	(9) 71.56	(1.7) 899.5	(1.0) 172.7	1.0076
(285) 0.29803	(17) 1.7629	(63) 1.4637	(29) 0.2992	(1.0) 663.9	(1.9) 203.1	(50) 25 24.44	(1.14) 31.46	(3) 1143.5	(7) 971.7	(8) 71.48	900.2	(1.0) 171.8	1.0076
(150) 0.29788	(17) 1.7630	(33) 1.4640	(15) 0.2991	(1.0) 663.8	203	(51) 24 23.94	(60) 31.52	(3) 1143.5	(1.4) 971.7	(9) 71.47	(1.7) 900.3	(1.0) 171.7	1.0076
(151) 0.29637	(18) 1.7648	(32) 1.4672	(15) 0.2976	(2.0) 662.8	202	(50) 23.94	(61) 32.13	(3) 1143.2	(7) 972.5	(9) 71.38	(1.6) 901.1	(2.0) 170.7	1.0076
(302) 0.29501	(34) 1.7663	(65) 1.4702	(30) 0.2962	(1.8) 661.9	(2.0) 201.1	(49) 23 23.45	(1.23) 32.69	(6) 1142.9	(1.4) 973.1	(9) 71.30	(1.8) 901.8	(2.0) 169.8	1.0076
(151) 0.29486	(17) 1.7665	(33) 1.4705	(16) 0.2960	(1.0) 661.8	201	(49) 23.45	(63) 32.76	(3) 1142.6	(7) 973.2	(9) 71.29	(1.9) 901.9	(1.0) 169.7	1.0076
(151) 0.29335	(18) 1.7683	(33) 1.4738	(15) 0.2945	(1.0) 660.8	200	(48) 23 22.97	(64) 33.40	(6) 1142.6	(7) 973.9	(8) 71.20	(1.7) 902.7	(2.0) 168.7	1.0076
(303) 0.29198	(37) 1.7700	(67) 1.4769	(31) 0.2931	(2.0) 659.9	(2.1) 199.1	(47) 22.50	(1.33) 34.02	(3) 1142.3	(1.5) 974.5	(9) 71.12	(1.6) 903.4	(1.7) 167.8	1.0076
(152) 0.29183	(18) 1.7701	(33) 1.4771	(15) 0.2930	(1.0) 659.8	199	(48) 22.97	(66) 34.06	(3) 1142.3	(7) 974.6	(9) 71.11	(1.6) 903.5	(1.0) 167.7	1.0076
(151) 0.29032	(18) 1.7719	(33) 1.4804	(15) 0.2915	(1.0) 658.8	198	(47) 22.50	(67) 34.73	(3) 1142.0	(7) 975.3	(9) 71.02	(1.7) 904.3	(1.0) 166.7	1.0076
(152) 0.28880	(17) 1.7736	(33) 1.4837	(16) 0.2899	(2.2) 657.8	(2.2) 196.9	(47) 22.03	(69) 35.42	(3) 1141.7	(1.5) 976.0	(9) 70.93	(1.7) 905.1	(2.2) 165.7	1.0076
(334) 0.28864	(37) 1.7737	(70) 1.4839	(33) 0.2898	657.7	22	22.03	(1.45) 35.47	1141.7	(1.5) 976.0	(19) 70.93	905.1	165.6	1.0076

153 0.28727	18 1.7754	33 1.4870	15 0.2884	1.0 656.8	**196**	46 21.57	71 36.13	3 1141.4	7 976.7	9 70.84	8 905.9	1.0 164.7	1.0076
152 0.2875	18 1.7772	34 1.4904	15 0.2869	1.0 655.8	**195**	44 21.13	72 36.85	3 1141.1	7 977.4	9 70.75	8 906.7	1.0 163.7	1.0076
335 0.28529	40 1.7777	74 1.4913	34 0.2884	2.2 655.5	2.2 **1947**	**21**	1.57 37.04	3 1141.0	1.6 977.6	20 70.73	1.6 906.9	2.2 163.4	1.0076
153 0.28422	18 1.7790	33 1.4937	16 0.2853	1.0 654.8	**194**	44 20.69	74 37.59	8 1140.8	7 978.1	9 70.66	7 907.4	1.0 162.7	1.0076
152 0.28270	19 1.7809	34 1.4971	15 0.2838	1.0 653.8	**193**	44 20.25	76 38.35	4 1140.4	8 978.8	9 70.57	8 908.2	1.1 161.6	1.0076
351 0.28178	43 1.7820	78 1.4991	36 0.2828	2.3 653.2	2.3 **1924**	**20**	1.78 38.82	8 1140.2	1.6 979.2	21 70.52	1.8 908.7	2.4 161.0	1.0076
153 0.28117	18 1.7827	34 1.5005	16 0.2822	1.0 652.8	**192**	42 19.83	78 39.13	3 1140.1	7 979.5	9 70.48	8 909.0	1.0 160.6	1.0076
154 0.27963	18 1.7845	33 1.5038	15 0.2807	1.0 651.8	**191**	42 19.41	79 39.92	3 1139.8	7 980.2	9 70.39	8 909.8	1.0 159.6	1.0076
368 0.27810	44 1.7864	81 1.5072	37 0.2791	2.4 650.8	2.4 **190.0**	**19**	1.90 40.72	8 1139.5	1.7 980.9	22 70.30	1.9 910.6	2.4 158.6	1.0076
153 0.27810	19 1.7864	34 1.5072	16 0.2791	1.0 650.8	**190**	41 19.00	81 40.73	3 1139.5	7 980.9	9 70.30	8 910.6	1.0 158.6	1.0076
154 0.27656	18 1.7882	34 1.5106	15 0.2776	1.0 649.8	**189**	41 18.59	83 41.56	3 1139.2	7 981.6	9 70.21	8 911.4	1.0 157.6	1.0076
154 0.27502	19 1.7901	34 1.5140	16 0.2760	1.0 648.8	**188**	40 18.19	85 42.41	9 1138.9	7 982.3	9 70.12	8 912.2	1.0 156.6	1.0076
385 0.27425	46 1.7910	85 1.5157	38 0.2753	2.5 648.3	2.5 **187.5**	**18**	2.12 42.84	8 1138.7	1.8 982.7	22 70.08	2.0 912.6	2.5 156.1	1.0076
154 0.27348	19 1.7920	35 1.5175	15 0.2745	1.0 647.8	**187**	39 17.80	87 43.28	3 1138.6	7 983.0	9 70.03	8 913.0	1.0 155.6	1.0076
155 0.27193	19 1.7938	34 1.5209	16 0.2729	1.0 646.8	**186**	38 17.42	89 44.17	3 1138.3	7 983.7	9 69.94	8 913.8	1.0 154.6	1.0076
155 0.27038	19 1.7957	34 1.5243	15 0.2714	1.0 645.8	**185**	37 17.05	92 45.09	8 1138.0	8 984.4	9 69.85	8 914.6	1.0 153.6	1.0076
402 0.26023	50 1.7960	91 1.5248	41 0.2712	2.6 645.7	2.6 **184.9**	**17**	2.38 45.22	8 1137.9	1.8 984.5	24 69.84	2.1 914.7	2.7 153.4	1.0076
155 0.26883	19 1.7976	35 1.5278	16 0.2698	1.0 644.8	**184**	36 16.69	94 46.03	3 1137.7	7 985.1	9 69.76	7 915.3	1.0 152.6	1.0076
155 0.26728	19 1.7995	34 1.5312	16 0.2682	1.0 643.8	**183**	36 16.33	96 46.99	3 1137.4	7 985.8	9 69.67	8 916.1	1.0 151.6	1.0076

1.0076	150.6	916.9	69.59	986.4	1137.1	47.91	**16**	**182.1**	642.9	0.2668	1.5345	1.8013	0.26588
1.0076	150.6	916.6	69.58	986.5	1137.1	47.97	15.98	**182**	642.8	0.2667	1.5347	1.8014	0.26573
1.0076	149.6	917.7	69.49	987.2	1136.8	48.98	15.63	**181**	641.8	0.2651	1.5382	1.8033	0.26417
1.0076	148.5	918.5	69.40	987.9	1136.4	50.01	15.29	**180**	640.8	0.2635	1.5417	1.8052	0.26261
1.0076	147.6	919.2	69.32	988.5	1136.2	50.93	**15**	179.1	639.9	0.2622	1.5448	1.8069	0.26121
1.0076	147.5	919.3	69.31	988.6	1136.1	51.06	14.96	**179**	639.8	0.2620	1.5452	1.8072	0.26105
1.0076	146.5	920.1	69.22	989.3	1135.8	52.14	14.63	**178**	638.8	0.2604	1.5487	1.8091	0.25949
1.0076	145.5	920.9	69.13	990.0	1135.5	53.26	14.31	**177**	637.8	0.2588	1.5522	1.8110	0.25792
1.0076	144.5	921.6	69.04	990.7	1135.2	54.37	**14**	176.0	636.8	0.2573	1.5557	1.8129	0.25635
1.0076	144.5	921.7	69.04	990.7	1135.2	54.40	13.99	**176**	636.8	0.2572	1.5558	1.8130	0.25635
1.0076	143.5	922.5	68.95	991.4	1134.9	55.56	13.67	**175**	635.8	0.2556	1.5593	1.8150	0.25478
1.0076	142.5	923.3	68.85	992.1	1134.6	56.76	13.35	**174**	634.8	0.2541	1.5629	1.8169	0.25321
1.0076	141.5	924.0	68.76	992.8	1134.3	57.99	13.04	**173**	633.8	0.2525	1.5664	1.8189	0.25163
1.0076	141.3	924.1	68.75	992.9	1134.2	58.17	**13**	172.8	633.6	0.2521	1.5670	1.8192	0.25131
1.0076	140.5	924.8	68.67	993.5	1134.0	59.25	12.75	**172**	632.8	0.2509	1.5700	1.8209	0.25005
1.0076	139.5	925.6	68.58	994.2	1133.7	60.53	12.46	**171**	631.8	0.2493	1.5736	1.8229	0.24847
1.0076	138.5	926.4	68.49	994.9	1133.4	61.85	12.18	**170**	630.8	0.2477	1.5772	1.8249	0.24688
1.0076	137.9	926.9	68.43	995.3	1133.2	62.73	**12**	169.4	630.2	0.2462	1.5796	1.8262	0.24593
1.0076	137.5	927.2	68.40	995.6	1133.1	63.22	11.90	**169**	629.8	0.2461	1.5809	1.8270	0.24530

1.0076	136.5 (1.0)	928.0 (8)	68.30 (10)	996.3 (7)	1132.8 (3)	64.62 (1.40)	11.62 (28)	**168**	628.8 (1.0)	0.2445 (16)	1.5845 (36)	1.8290 (20)	0.24371 (159)
1.0076	135.4 (1.1)	928.8 (8)	68.21 (9)	997.0 (7)	1132.5 (3)	66.05 (1.43)	11.36 (26)	**167**	627.8 (1.0)	0.2429 (16)	1.5882 (37)	1.8310 (20)	0.24212 (159)
1.0076	134.4 (1.0)	929.7 (9)	68.12 (9)	997.8 (8)	1132.2 (3)	67.51 (1.46)	11.10 (26)	**166**	626.8 (1.0)	0.2413 (16)	1.5918 (36)	1.8331 (21)	0.24052 (160)
1.0076	134.0 (3.9)	930.0 (3.1)	68.08 (35)	998.1 (2.8)	1132.1 (1.1)	68.12 (5.39)	**11**	165.6 (3.8)	626.4 (3.8)	0.2406 (60)	1.5933 (137)	1.8340 (78)	0.23988 (605)
1.0076	133.4 (1.0)	930.5 (8)	68.03 (9)	998.5 (7)	1131.9 (3)	69.01 (1.50)	10.85 (25)	**165**	625.8 (1.0)	0.2397 (16)	1.5955 (37)	1.8352 (21)	0.23893 (159)
1.0076	132.4 (1.0)	931.3 (8)	67.93 (10)	999.2 (7)	1131.6 (3)	70.56 (1.55)	10.60 (25)	**164**	624.8 (1.0)	0.2381 (16)	1.5992 (37)	1.8373 (21)	0.23733 (160)
1.0076	131.4 (1.0)	932.1 (8)	67.84 (9)	999.9 (7)	1131.3 (3)	72.14 (1.58)	10.35 (25)	**163**	623.8 (1.0)	0.2364 (17)	1.6029 (37)	1.8393 (20)	0.23572 (161)
1.0076	130.4 (1.0)	932.9 (8)	67.75 (10)	1000.6 (6)	1131.0 (3)	73.77 (1.63)	10.11 (24)	**162**	622.8 (1.0)	0.2348 (16)	1.6066 (37)	1.8414 (21)	0.23412 (160)
1.0076	129.9 (4.1)	933.3 (3.3)	67.70 (38)	1000.9 (2.8)	1130.9 (1.2)	74.53 (6.41)	**10**	161.5 (4.1)	622.3 (4.1)	0.2341 (65)	1.6083 (150)	1.8424 (84)	0.23332 (656)
1.0076	129.4 (1.0)	933.7 (7)	67.65 (10)	1001.3 (7)	1130.7 (3)	75.43 (1.66)	9.87 (24)	**161**	621.8 (1.0)	0.2332 (16)	1.6103 (37)	1.8435 (21)	0.23251 (161)
1.0076	128.4 (1.0)	934.4 (8)	67.56 (9)	1002.0 (7)	1130.4 (3)	77.14 (1.71)	9.63 (24)	**160**	620.8 (1.0)	0.2316 (16)	1.6140 (37)	1.8456 (21)	0.23090 (161)
1.0076	127.4 (1.0)	935.2 (8)	67.46 (10)	1002.7 (7)	1130.1 (3)	78.90 (1.76)	9.40 (23)	**159**	619.8 (1.0)	0.2300 (16)	1.6177 (37)	1.8477 (21)	0.22929 (161)
1.0076	126.4 (1.0)	936.0 (8)	67.37 (9)	1003.4 (7)	1129.8 (3)	80.70 (1.80)	9.17 (23)	**158**	618.8 (1.0)	0.2283 (17)	1.6215 (38)	1.8498 (21)	0.22768 (161)
1.0076	125.0 (4.3)	936.6 (3.3)	67.29 (41)	1003.9 (3.0)	1129.6 (1.3)	82.13 (7.60)	**9**	157.2 (4.3)	618.0 (4.3)	0.2271 (70)	1.6244 (161)	1.8514 (90)	0.22638 (604)
1.0076	125.4 (1.0)	936.8 (8)	67.27 (10)	1004.1 (7)	1129.5 (3)	82.56 (1.86)	8.95 (22)	**157**	617.8 (1.0)	0.2267 (16)	1.6252 (37)	1.8519 (21)	0.22606 (162)
1.0076	124.4 (1.0)	937.6 (8)	67.17 (10)	1004.8 (8)	1129.2 (3)	84.47 (1.91)	8.74 (21)	**156**	616.8 (1.0)	0.2251 (16)	1.6290 (38)	1.8541 (21)	0.22444 (162)
1.0076	123.4 (1.0)	938.4 (8)	67.08 (10)	1005.5 (7)	1128.9 (3)	86.42 (1.95)	8.53 (21)	**155**	615.8 (1.0)	0.2234 (17)	1.6328 (38)	1.8562 (21)	0.22282 (162)
1.0076	122.3 (1.1)	939.2 (8)	66.98 (10)	1006.2 (7)	1128.5 (4)	88.43 (2.01)	8.33 (20)	**154**	614.8 (1.0)	0.2218 (16)	1.6366 (38)	1.8584 (22)	0.22119 (163)
1.0076	121.3 (1.0)	940.0	66.88	1006.9	1128.2	90.49 (2.06)	8.13	**153**	613.8	0.2202	1.6404	1.8606 (22)	0.21956 (163)

^{796}0.21842	^{107}1.8621	^{186}1.6430	^{81}0.2190	$^{4.9}$613.1	$^{4.9}$152.3	8	$^{9.82}$91.95	$^{1.6}$1128.0	$^{3.5}$1007.4	^{48}66.81	$^{3.9}$940.5	$^{5.0}$120.6	1.0076
^{163}0.21793	^{22}1.8628	^{38}1.6442	^{17}0.2185	$^{1.0}$612.8	152	^{19}7.94	$^{2.12}$92.61	^{3}1127.9	^{7}1007.6	^{10}66.78	^{8}940.8	$^{1.0}$120.3	1.0076
^{163}0.21630	^{22}1.8650	^{39}1.6481	^{16}0.2169	$^{1.0}$611.8	151	^{19}7.75	$^{2.18}$94.79	^{3}1127.6	^{7}1008.3	^{9}66.69	^{8}941.6	$^{1.0}$119.3	1.0076
^{164}0.21466	^{22}1.8672	^{38}1.6519	^{17}0.2152	$^{1.0}$610.8	150	^{19}7.56	$^{2.24}$97.03	^{3}1127.3	^{7}1009.0	^{10}66.59	^{8}942.4	$^{1.0}$118.3	1.0076
^{163}0.21303	^{22}1.8694	^{39}1.6558	^{16}0.2136	$^{1.0}$609.8	149	^{19}7.37	$^{2.30}$99.33	^{3}1127.0	^{7}1009.7	^{10}66.49	^{8}943.2	$^{1.0}$117.3	1.0076
^{165}0.21138	^{22}1.8716	^{39}1.6597	^{17}0.2119	$^{5.3}$608.8	148	^{19}7.18	$^{2.4}$101.7	^{3}1126.7	^{7}1010.4	^{10}66.39	^{8}944.0	$^{1.0}$116.3	1.0076
^{868}0.20974	^{116}1.8737	^{204}1.6634	^{87}0.2103	$^{1.0}$607.8	$^{5.3}$147.0	7	$^{12.1}$104.0	$^{1.6}$1126.4	$^{3.7}$1011.1	^{51}66.30	$^{4.3}$944.8	$^{5.3}$115.3	1.0076
^{164}0.20974	^{22}1.8738	^{39}1.6636	^{16}0.2103	$^{1.0}$607.8	147	^{18}7.00	$^{2.4}$104.1	^{3}1126.4	^{7}1011.1	^{10}66.29	^{8}944.8	$^{1.0}$115.3	1.0076
^{165}0.20809	^{23}1.8761	^{39}1.6675	^{17}0.2086	$^{1.0}$606.8	146	^{18}6.82	$^{2.5}$106.6	^{3}1126.1	^{7}1011.8	^{9}66.20	^{8}945.6	$^{1.0}$114.3	1.0076
^{165}0.20644	^{22}1.8783	^{39}1.6714	^{17}0.2069	$^{1.0}$605.8	145	^{17}6.65	$^{2.6}$109.2	^{3}1125.8	^{7}1012.5	^{10}66.10	^{8}946.4	$^{1.0}$113.3	1.0076
^{165}0.20479	^{23}1.8806	^{39}1.6753	^{16}0.2053	$^{1.0}$604.8	144	^{16}6.49	$^{2.6}$111.8	^{3}1125.5	^{7}1013.2	^{10}66.00	^{8}947.2	$^{1.0}$112.3	1.0076
^{165}0.20314	^{23}1.8829	^{40}1.6793	^{17}0.2036	$^{1.0}$603.8	143	^{16}6.33	$^{2.7}$114.5	^{3}1125.2	^{8}1014.0	^{10}65.90	^{9}948.1	$^{1.0}$111.3	1.0076
^{166}0.20148	^{23}1.8852	^{39}1.6832	^{17}0.2019	$^{1.0}$602.8	142	^{15}6.18	$^{2.8}$117.3	^{3}1124.9	^{7}1014.7	^{10}65.80	^{8}948.9	$^{1.0}$110.3	1.0076
^{166}0.19982	^{23}1.8875	^{40}1.6872	^{16}0.2003	$^{1.0}$601.8	141	^{16}6.02	$^{2.9}$120.2	^{3}1124.6	^{7}1015.4	^{10}65.70	^{8}949.7	$^{1.1}$109.2	1.0076
^{1009}0.19965	^{141}1.8878	^{243}1.6877	^{102}0.2001	$^{6.1}$601.7	$^{6.1}$140.9	6	$^{16.6}$120.6	$^{1.8}$1124.6	$^{4.4}$1015.5	^{61}65.69	$^{5.0}$949.8	$^{6.2}$109.1	1.0076
^{166}0.19816	^{23}1.8898	^{40}1.6912	^{17}0.1986	$^{1.0}$600.8	140	^{16}5.86	$^{3.0}$123.2	^{3}1124.3	^{7}1016.1	^{10}65.60	^{8}950.5	$^{1.0}$108.2	1.0076
^{167}0.19649	^{23}1.8921	^{40}1.6952	^{17}0.1969	$^{1.0}$599.8	139	^{15}5.71	$^{3.0}$126.2	^{3}1124.0	^{7}1016.8	^{10}65.50	^{8}951.3	$^{1.0}$107.2	1.0076
^{167}0.19482	^{23}1.8944	^{40}1.6992	^{16}0.1952	$^{1.0}$598.8	138	^{15}5.56	$^{3.1}$129.3	^{3}1123.7	^{7}1017.5	^{10}65.40	^{8}952.1	$^{1.0}$106.2	1.0076
^{167}0.19315	^{23}1.8967	^{40}1.7032	^{16}0.1936	$^{1.0}$597.8	137	^{14}5.42	$^{3.2}$132.5	^{3}1123.4	^{7}1018.2	^{10}65.30	^{8}952.9	$^{1.0}$105.2	1.0076

0.19148	1.8991	1.7072	0.1919	596.8	**136**	5.28	135.8	1123.1	1018.9	65.20	953.7	104.2	1.0076
0.18980	1.9014	1.7112	0.1902	595.8	**135**	5.14	139.2	1122.8	1019.6	65.10	954.5	103.2	1.0076
0.18812	1.9037	1.7153	0.1885	594.8	**134**	5.01	142.8	1122.4	1020.2	65.00	955.2	102.2	1.0076
0.18795	1.9039	1.7156	0.1883	594.7	133.9	**5**	143.1	1122.4	1020.3	64.99	955.3	102.1	1.0076
0.18644	1.9061	1.7193	0.1868	593.8	**133**	4.88	146.4	1122.1	1020.9	64.90	956.0	101.2	1.0076
0.18475	1.9085	1.7234	0.1851	592.8	**132**	4.75	150.1	1121.8	1021.6	64.80	956.8	100.2	1.0076
0.18306	1.9109	1.7275	0.1834	591.8	**131**	4.63	153.9	1121.5	1022.3	64.70	957.6	99.17	1.0076
0.18137	1.9133	1.7316	0.1817	590.8	**130**	4.50	157.8	1121.2	1023.0	64.60	958.4	98.16	1.0075
0.17968	1.9157	1.7357	0.1800	589.8	**129**	4.38	161.9	1120.9	1023.7	64.50	959.2	97.16	1.0074
0.17798	1.9181	1.7398	0.1783	588.8	**128**	4.27	166.1	1120.6	1024.4	64.40	960.0	96.15	1.0073
0.17628	1.9205	1.7440	0.1765	587.8	**127**	4.15	170.4	1120.2	1025.1	64.30	960.8	95.14	1.0072
0.17458	1.9230	1.7481	0.1748	586.8	**126**	4.04	174.8	1119.9	1025.8	64.20	961.6	94.14	1.0070
0.17390	1.9240	1.7498	0.1741	586.4	125.6	**4**	176.7	1119.8	1026.1	64.16	961.9	93.73	1.0069
0.17287	1.9254	1.7553	0.1731	585.8	**125**	3.93	179.4	1119.6	1026.5	64.10	962.4	93.13	1.0068
0.17117	1.9279	1.7565	0.1714	584.8	**124**	3.83	184.1	1119.3	1027.2	64.00	963.2	92.12	1.0066
0.16945	1.9304	1.7607	0.1697	583.8	**123**	3.72	188.9	1119.0	1027.9	63.90	964.0	91.12	1.0063
0.16774	1.9329	1.7650	0.1680	582.8	**122**	3.62	193.9	1118.7	1028.6	63.80	964.8	90.11	1.0061
0.16602	1.9354	1.7692	0.1662	581.8	**121**	3.52	199.1	1118.4	1029.3	63.70	965.6	89.10	1.0058
0.16430	1.9380	1.7735	0.1645	580.8	**120**	3.42	204.4	1118.1	1030.0	63.60	966.4	88.10	1.0055

0.16258 ¹⁷²	1.9405 ²⁵	1.7777 ⁴²	0.1628 ¹⁷	579.8 ¹·⁰	**119**	3.33 ⁹	209.9 ⁵·⁵	1117.8 ³	1030.7 ⁷	63.50 ¹⁰	967.2 ⁸	87.09 ¹·⁰¹	1.0052 ³
0.16085 ¹⁷³	1.9430 ²⁵	1.7820 ⁴³	0.1610 ¹⁸	578.8 ¹·⁰	**118**	3.24 ⁹	215.5 ⁵·⁶	1117.5 ³	1031.4 ⁸	63.40 ¹⁰	968.0 ⁸	86.09 ¹·⁰⁰	1.0049 ²
0.15912 ¹⁷³	1.9456 ²⁶	1.7863 ⁴³	0.1593 ¹⁷	577.8 ¹·⁰	**117**	3.15 ⁹	221.3 ⁵·⁸	1117.3 ²	1032.2 ⁷	63.30 ¹⁰	968.9 ⁹	85.08 ¹·⁰¹	1.0047 ³
0.15739 ¹⁷³	1.9482 ²⁶	1.7906 ⁴³	0.1575 ¹⁸	576.8 ¹·⁰	**116** 10.3 / 115.3	3.06 ³	227.3 ⁶·⁰	1117.0 ³	1032.9 ⁷	63.20 ¹·⁰³	969.7 ⁸·⁴	84.08 ¹·⁰⁰	1.0044 ²⁷
0.15618 ⁷⁷²	1.9500 ²⁶⁰	1.7937 ⁴³⁹	0.1563 ¹⁷⁸	576.1 ¹⁰·³	**3**		231.7 ⁵⁵·⁰	1116.8 ³·⁰	1033.4 ⁷·³	63.13 ¹·⁰³	970.3 ⁸	83.36 ¹⁰·³⁷	1.0042 ³
0.15566 ¹⁷³	1.9508 ²⁶	1.7950 ⁴³	0.1558 ¹⁷	575.8 ¹·⁰	**115**	2.97 ⁹	233.5 ⁶·²	1116.7 ²	1033.6 ⁷	63.10 ¹⁰	970.5 ⁸	83.07 ¹·⁰⁰	1.0041 ²
0.15392 ¹⁷⁴	1.9534 ²⁶	1.7993 ⁴⁴	0.1541 ¹⁸	574.8 ¹·⁰	**114**	2.89 ⁸	239.9 ⁶·⁴	1116.4 ³	1034.3 ⁷	63.00 ¹⁰	971.3 ⁸	82.07 ¹·⁰⁰	1.0039 ³
0.15218 ¹⁷⁴	1.9560 ²⁶	1.8037 ⁴⁴	0.1523 ¹⁷	573.8 ¹·⁰	**113**	2.81 ⁸	246.5 ⁶·⁶	1116.1 ³	1035.0 ⁷	62.90 ¹⁰	972.1 ⁸	81.07 ¹·⁰⁰	1.0036 ³
0.15043 ¹⁷⁵	1.9586 ²⁷	1.8081 ⁴⁴	0.1506 ¹⁸	572.8 ¹·⁰	**112**	2.73 ⁸	253.3 ⁶·⁸	1115.8 ³	1035.7 ⁷	62.80 ¹⁰	972.9 ⁸	80.06 ¹·⁰⁰	1.0033 ³
0.14868 ¹⁷⁵	1.9613 ²⁶	1.8125 ⁴⁴	0.1488 ¹⁸	571.8 ¹·⁰	**111**	2.65 ⁸	260.3 ⁷·⁰	1115.5 ³	1036.4 ⁷	62.70 ¹⁰	973.7 ⁸	79.06 ¹·⁰⁰	1.0031 ³
0.14693 ¹⁷⁵	1.9639 ²⁷	1.8169 ⁴⁴	0.1470 ¹⁷	570.8 ¹·⁰	**110**	2.58 ⁸	267.5 ⁷·²	1115.2 ³	1037.1 ⁷	62.60 ¹⁰	974.5 ⁸	78.06 ¹·⁰⁰	1.0028 ³
0.14518 ¹⁷⁶	1.9666 ²⁷	1.8213 ⁴⁴	0.1453 ¹⁸	569.8 ¹·⁰	**109**	2.50 ⁷	275.0 ⁷·⁵	1114.9 ³	1037.8 ⁷	62.50 ¹⁰	975.3 ⁸	77.06 ¹·⁰⁰	1.0026 ²
0.14342 ¹⁷⁶	1.9693 ²⁷	1.8257 ⁴⁵	0.1435 ¹⁷	568.8 ¹·⁰	**108**	2.43 ⁷	282.7 ⁷·⁷	1114.6 ⁴	1038.5 ⁷	62.40 ¹⁰	976.1 ⁸	76.05 ¹·⁰¹	1.0024 ²
0.14166 ¹⁷⁶	1.9720 ²⁷	1.8302 ⁴⁴	0.1418 ¹⁸	567.8 ¹·⁰	**107**	2.36 ⁷	290.6 ⁷·⁹	1114.2 ⁴	1039.2 ⁷	62.30 ¹⁰	976.9 ⁸	75.05 ¹·⁰⁰	1.0021 ²
0.13990 ¹⁷⁶	1.9746 ²⁷	1.8346 ⁴⁵	0.1400 ¹⁸	566.8 ¹·⁰	**106**	2.29 ⁶	298.8 ⁸·²	1113.9 ³	1039.9 ⁷	62.20 ¹⁰	977.7 ⁸	74.05 ¹·⁰⁰	1.0019 ²
0.13814 ¹⁷⁶	1.9773 ²⁸	1.8391 ⁴⁵	0.1382 ¹⁷	565.8 ¹·⁰	**105**	2.23 ⁷	307.3 ⁸·⁵	1113.6 ³	1040.6 ⁷	62.10 ¹⁰	978.5 ⁸	73.05 ¹·⁰⁰	1.0016 ²
0.13637 ¹⁷⁷	1.9801 ²⁷	1.8436 ⁴⁵	0.1365 ¹⁸	564.8 ¹·⁰	**104**	2.16 ⁶	316.1 ⁸·⁸	1113.3 ³	1041.3 ⁷	62.00 ¹⁰	979.3 ⁸	72.05 ¹·⁰¹	1.0014 ⁴
0.13459 ¹⁷⁸	1.9828 ²⁷	1.8481 ⁴⁵	0.1347 ¹⁸	563.8 ¹·⁰	**103**	2.10 ⁷	325.2 ⁹·¹	1113.0 ³	1042.0 ⁷	61.90 ¹⁰	980.1 ⁸	71.04 ¹·⁰¹	1.0012 ²
0.13282 ¹⁷⁷	1.9855	1.8526	0.1329	562.8	**102**	2.03	334.5 ⁹·³	1112.7	1042.7	61.80	980.9	70.04 ¹·⁰⁰	1.0010

2443	371	615	244	13.9	13.9	2	108.3	4.3	9.7	1.38	11.1	13.89	33
0.13175 178	1.9871 28	1.8552 46	0.1319 18	562.2 1.0	101.4		340.0 9.6	1112.5 3	1043.1 7	61.75 9	981.4 8	69.47 1.00	1.0009 9
0.13104 178	1.9883 27	1.8572 45	0.1311 18	561.8 1.0	**101**	1.97 6	344.1 9.9	1112.4 3	1043.4 7	61.71 10	981.7 8	69.04 1.00	1.0008 8
0.12926 179	1.9910 28	1.8617 46	0.1293 17	560.8 1.0	**100**	1.91 5	354.0 10.4	1112.1 4	1044.1 6	61.61 10	982.5 8	68.04 1.00	1.0007 1
0.12747 178	1.9938 28	1.8663 45	0.1276 18	559.8 1.0	**99**	1.86 6	364.4 10.7	1111.7 3	1044.7 7	61.51 10	983.2 8	67.04 1.00	1.0007 1
0.12569 179	1.9966 28	1.8708 46	0.1258 18	558.8 1.0	**98**	1.80 5	375.1 11.0	1111.4 3	1045.4 7	61.41 9	984.0 8	66.04 1.00	1.0005 2
0.12390 179	1.9994 28	1.8754 46	0.1240 18	557.8 1.0	**97**	1.75 6	386.1 11.4	1111.1 3	1046.1 7	61.32 10	984.8 8	65.04 1.00	1.0003 2
0.12210 180	2.0022 28	1.8800 46	0.1222 18	556.8 1.0	**96**	1.69 5	397.5 11.8	1110.8 3	1046.8 7	61.22 9	985.6 8	64.04 1.00	1.0001 2
0.12030 180	2.0050 28	1.8846 46	0.1204 18	555.8 1.0	**95**	1.64 5	409.3 12.2	1110.5 3	1047.5 7	61.13 10	986.4 8	63.04 1.00	0.9999 1
0.11850 180	2.0078 29	1.8893 47	0.1186 18	554.8 1.0	**94**	1.59 5	421.5 12.5	1110.2 3	1048.2 7	61.03 9	987.2 8	62.04 1.00	0.9997 1
0.11670 181	2.0107 29	1.8939 47	0.1168 18	553.8 1.0	**93**	1.54 4	434.0 13.1	1109.9 3	1048.9 7	60.94 9	988.0 7	61.04 1.00	0.9996 2
0.11489 181	2.0136 29	1.8986 47	0.1150 18	552.8 1.0	**92**	1.50 5	447.1 13.6	1109.6 3	1049.6 6	60.85 9	988.7 8	60.04 1.00	0.9994 2
0.11308 181	2.0165 29	1.9033 47	0.1132 18	551.8 1.0	**91**	1.45 5	460.7 13.9	1109.3 4	1050.3 7	60.76 9	989.5 7	59.04 99	0.9992 2
0.11127 182	2.0194 29	1.9080 47	0.1114 19	550.8 1.0	**90**	1.41 4	474.6 14.3	1108.9 3	1050.9 7	60.67 10	990.2 8	58.05 99	0.9990 2
0.10945 182	2.0223 29	1.9127 47	0.1095 18	549.8 1.0	**89**	1.36 4	488.9 14.8	1108.6 3	1051.6 7	60.57 9	991.0 8	57.05 1.00	0.9988 2
0.10763 182	2.0252 29	1.9174 48	0.1077 18	548.8 1.0	**88**	1.32 4	503.7 15.5	1108.3 3	1052.3 7	60.48 9	991.8 8	56.05 1.00	0.9987 1
0.10581 183	2.0281 29	1.9222 47	0.1059 18	547.8 1.0	**87**	1.28 4	519.2 16.0	1108.0 3	1053.0 7	60.39 9	992.6 8	55.05 1.00	0.9985 2
0.10398 183	2.0310 29	1.9269 48	0.1041 18	546.8 1.0	**86**	1.24 4	535.2 16.5	1107.7 3	1053.7 7	60.30 9	993.4 8	54.05 1.00	0.9983 2
0.10215 184	2.0340 30	1.9317 48	0.1023 19	545.8 1.0	**85**	1.20 4	551.7 17.1	1107.4 3	1054.4 7	60.21 9	994.2 8	53.05 99	0.9981 2
0.10031	2.0370	1.9365	0.1004	544.8	**84**	1.16	568.8	1107.1	1055.1	60.12	995.0	52.06	0.9979 2
													0.9977

0.9975	1.00 / 51.06	7 / 995.7	9 / 60.03	6 / 1055.7	1106.8	17.8 / 586.6	3 / 1.13	**83**	1.0 / 543.8	18 / 0.0986	48 / 1.9413	30 / 2.0400	183 / 0.09848
0.9974	1.00 / 50.06	8 / 996.5	8 / 59.95	7 / 1056.4	3 / 1106.5	18.4 / 605.0	4 / 1.09	**82**	1.0 / 542.8	18 / 0.0968	49 / 1.9462	30 / 2.0430	184 / 0.09664
0.9974	1.00 / 49.06	7 / 997.2	9 / 59.86	7 / 1057.1	3 / 1106.2	19.1 / 624.1	3 / 1.06	**81**	1.0 / 541.8	19 / 0.0949	49 / 1.9511	30 / 2.0460	185 / 0.09479
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